ECI 2015
Buenos Aires

Fundamentos lógicos de bases de datos

(Logical foundations of databases)

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CNRS LaBRI

Trading expressiveness for efficiency

Alternation of quantifiers significantly affects complexity
(recall that evaluation of QBF is PSPACE-complete: $\forall x\ \exists y\ \forall z\ \exists w \ldots \phi$).

What happens if we disallow $\forall$ and $\neg$?

The class NP

$$\text{LOGSPACE} \subseteq \text{PTIME} \subseteq \text{PSPACE} \subseteq \text{EXPTIME}$$

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Examples:

- **3-COLORABILITY**: Given a graph G, can we assign a colour from $\{R,G,B\}$ to each node so that adjacent nodes have always different colours ?
- **SAT**: Given a propositional formula, e.g. $(p \vee \neg q \vee r) \wedge (\neg p \vee s) \wedge (\neg s \vee \neg p)$, can we assign a truth value to each variable so that the formula becomes true ?
- **MONEY-CHANGE**: Given an amount of money A and a set of coins $\{B_1, \ldots, B_n\}$, can we find a subset $S \subseteq \{B_1, \ldots, B_n\}$ such that $\sum S = A$?

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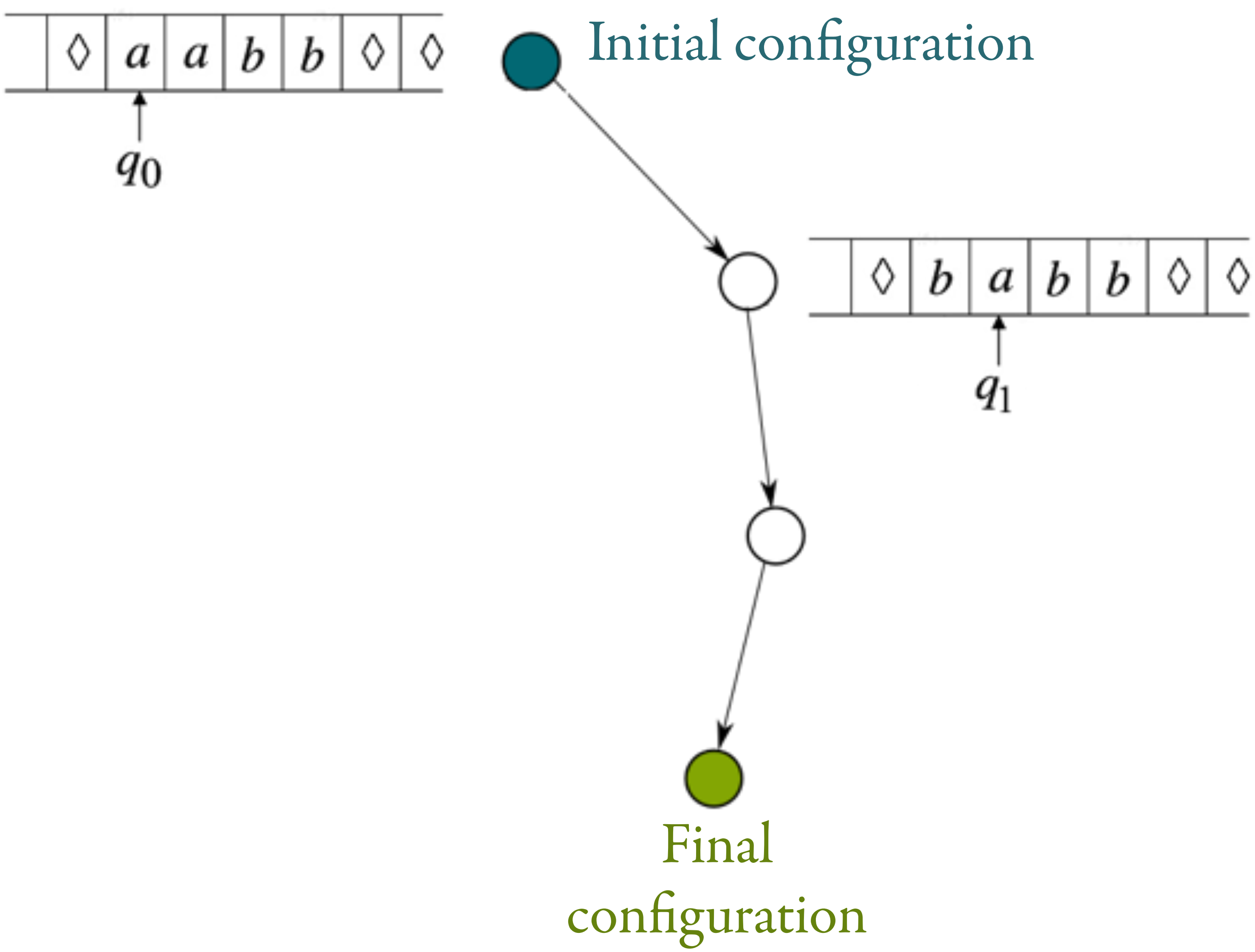

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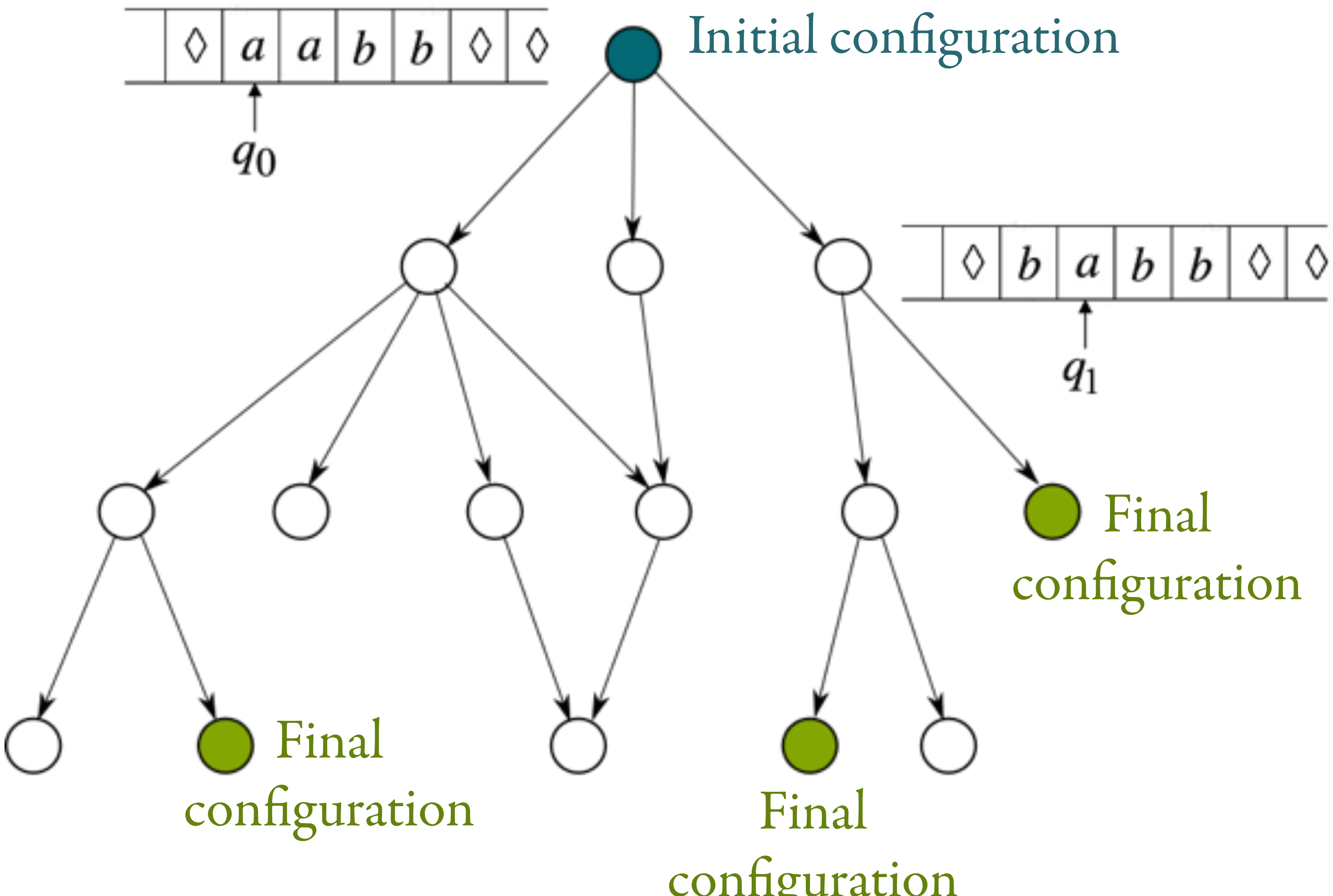

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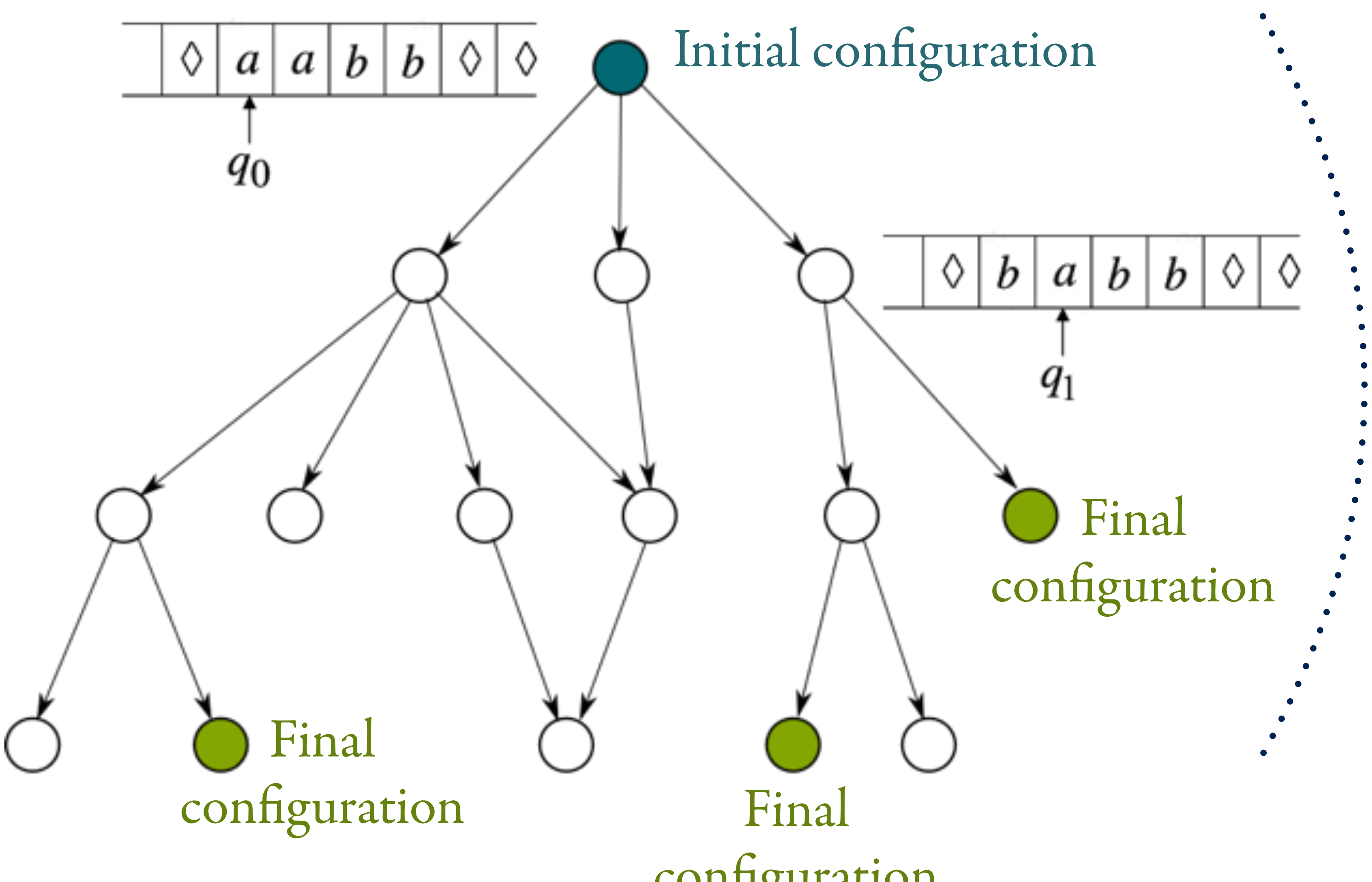

Non-deterministic transitions

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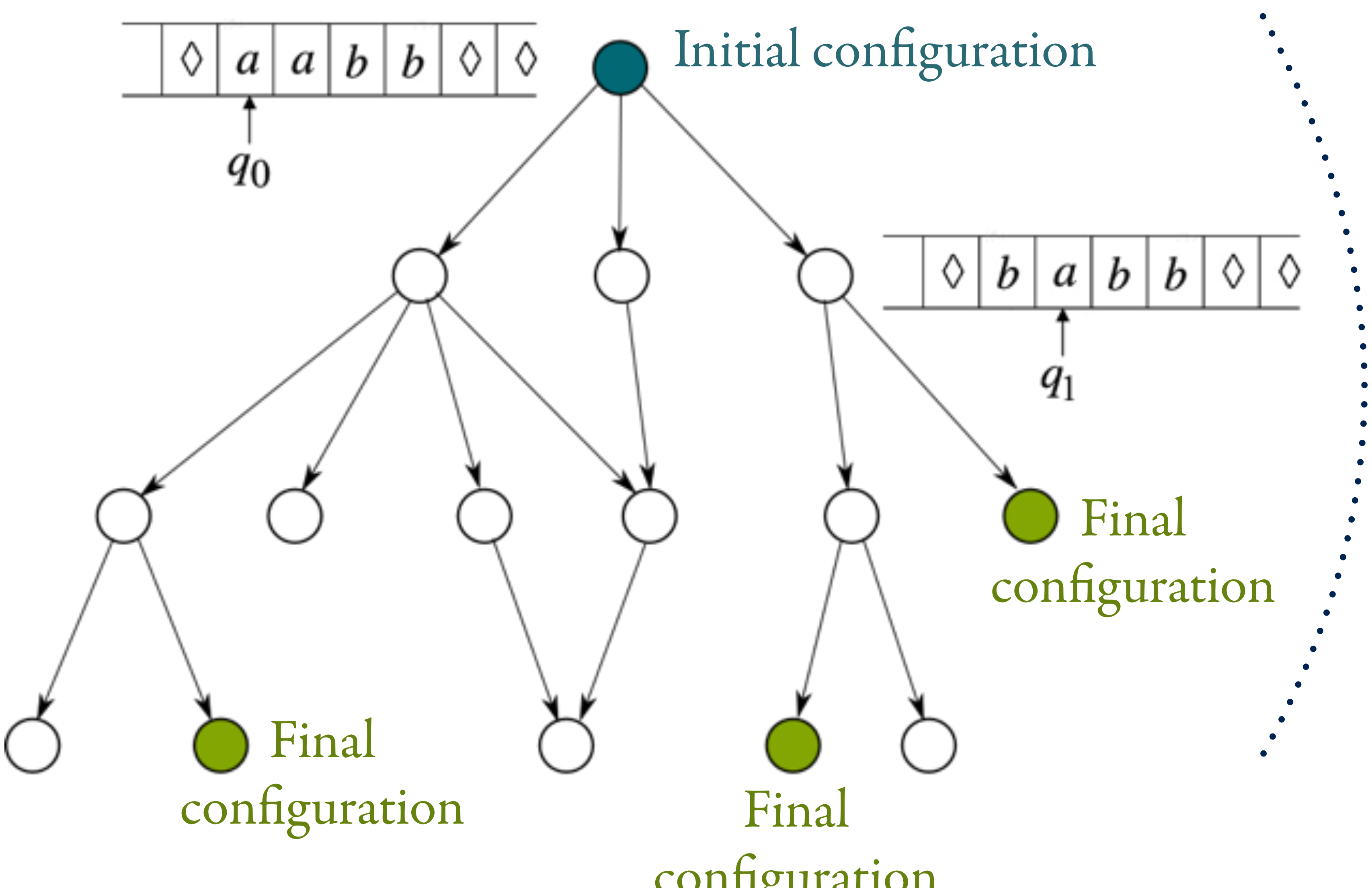

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Many paths, each has length bounded by a **polynomial**

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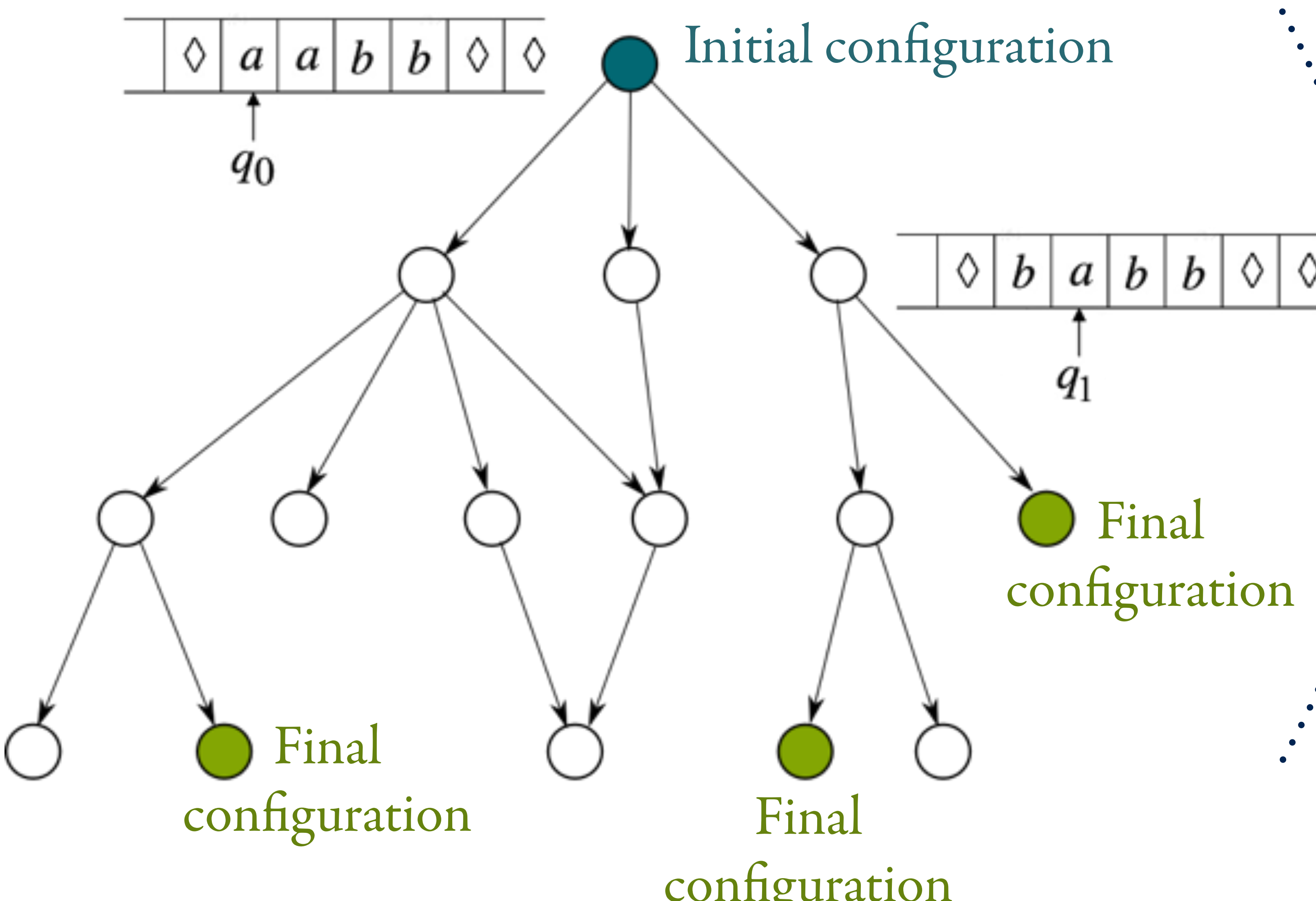

Non-deterministic transitions

Many paths, each has length bounded by a **polynomial**

A solution exists if there is at least a **successful** path.

Question

Consider: **Positive FO = FO without ∀,¬**

E.g. $\phi = \exists x \, \exists y \, \exists z \, . \, \big(E(x,y) \vee E(y,z)\big) \wedge \big(\, y{=}z \vee E(x,z)\big)$

What is the complexity of evaluating Positive FO on graphs ?

Question

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What is the complexity of evaluating Positive FO on graphs ?

Solution

This is in NP: Given ϕ and $G{=}(V,E)$
it suffices to guess a binding $\alpha : \{ x, y, z, \dots \} \to V$
and then verify that the formula holds.

Conjunctive Queries

Def.

CQ = FO without ∀, ¬, ∨

Eg: $\phi(x, y) = \exists z\, .\, (\mathrm{Parent}(x, z) \wedge \mathrm{Parent}(z, y))$

Usual notation: "Grandparent(X,Y) : – Parent(X,Z), Parent(Z,Y)"

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Normal form: " $\exists x_1, \ldots, x_n \, . \, \phi(x_1, \ldots, x_n)$ "

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It corresponds to positive "SELECT-FROM-WHERE" SQL queries

```
Select ...
From ...
Where Z
```

no negation or disjunction

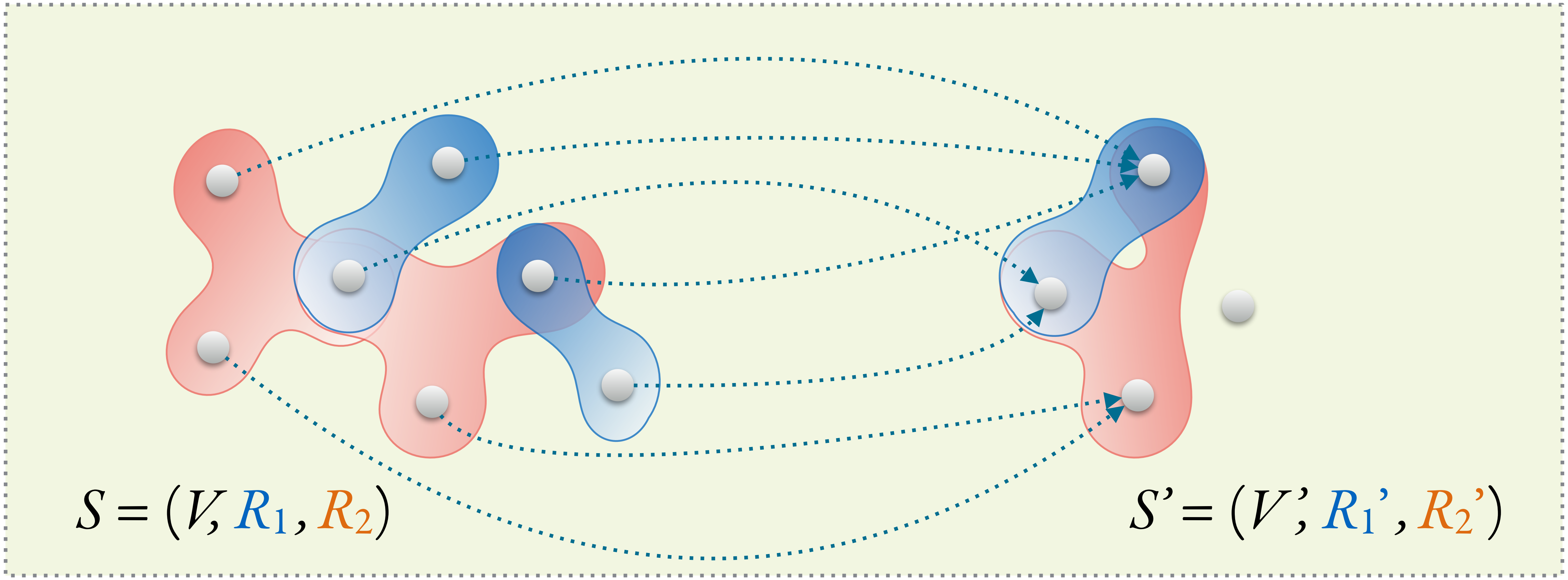
It corresponds to "π-σ-×" RA queries

$\pi_X(\sigma_Z(R_1 \times \cdots \times R_n))$

no negation

Homomorphisms

Homomorphism between structures $S=(V, R_1, \ldots, R_n)$ and $S'=(V', R_1', \ldots, R_n')$
is a function $h : V \longrightarrow V'$ such that
$(x_1, \ldots, x_n) \in R_i$ implies $(h(x_1), \ldots, h(x_n)) \in R_i'$



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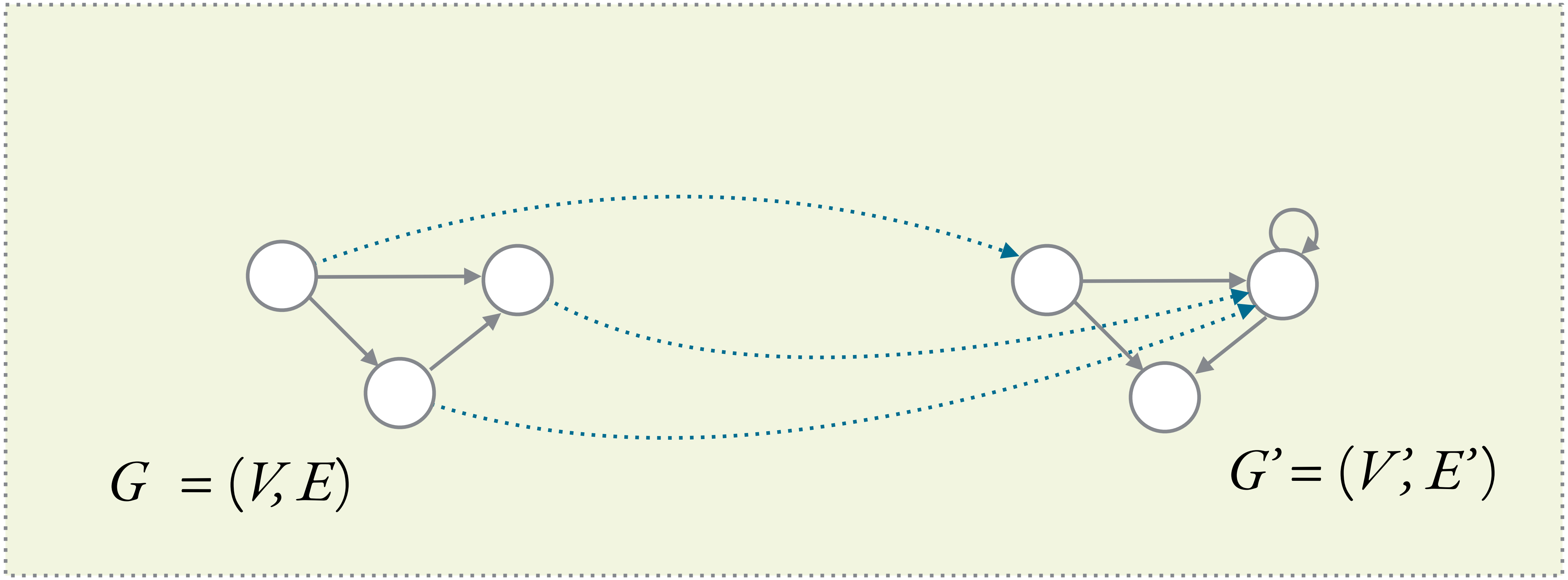
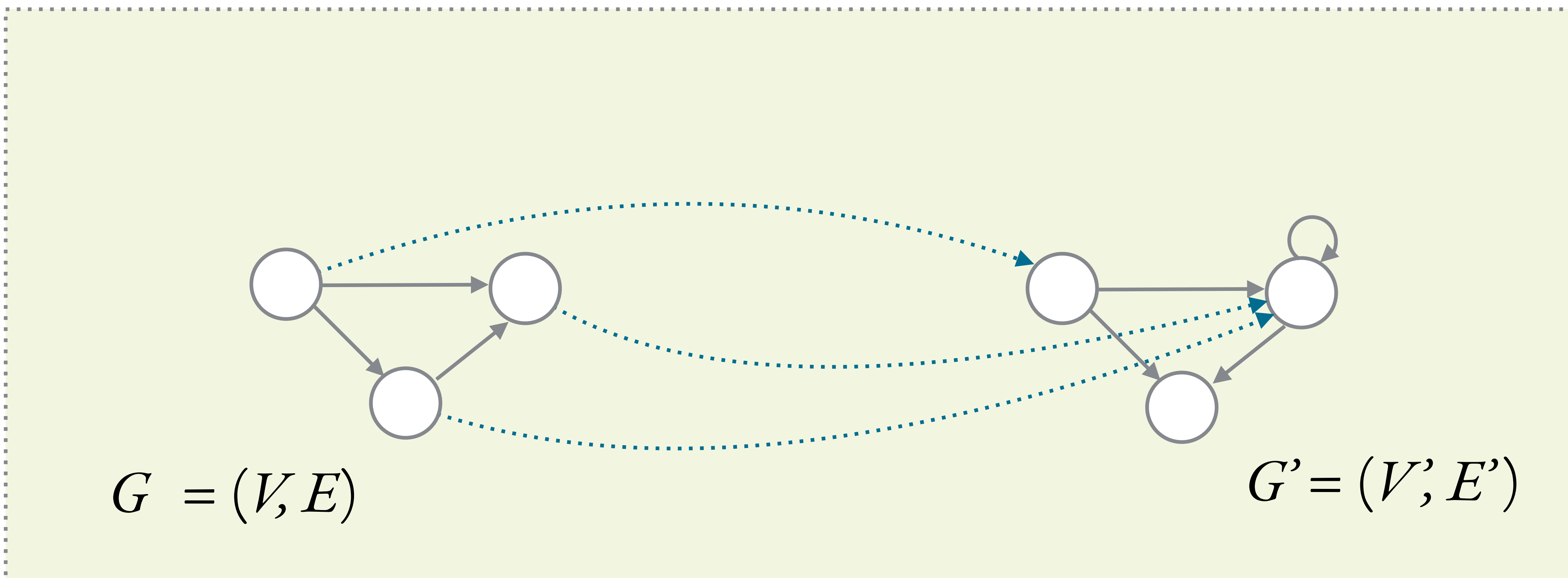


Canonical structures

Canonical structure S_ϕ of a Conjunctive Query ϕ has

- variables as nodes
- tuples $(x_1, \ldots, x_n) \in R_i$
  for all atomic sub-formulas $R_i(x_1, \ldots, x_n)$ of ϕ



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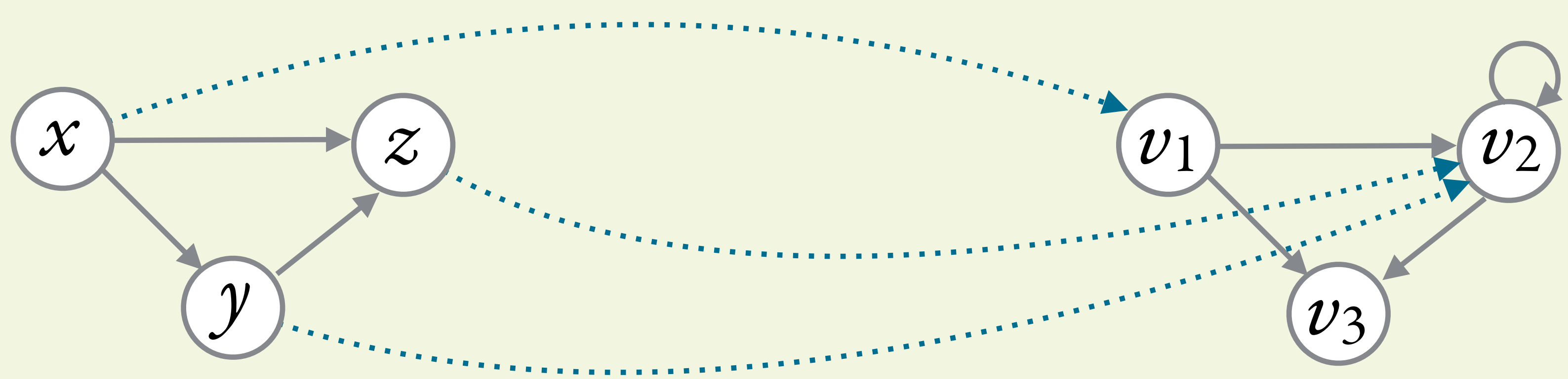

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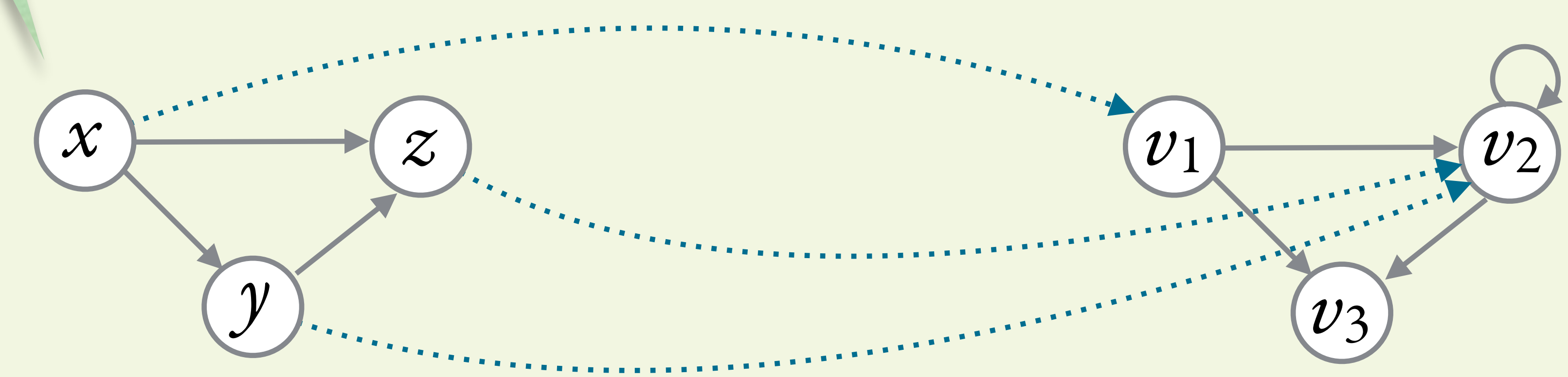

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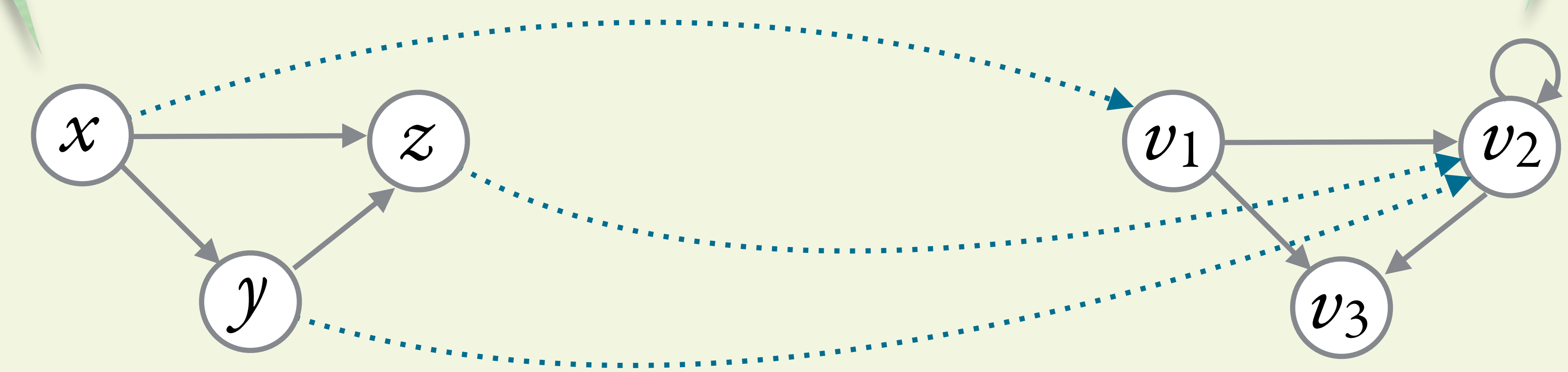

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Fact 3: $G" \vDash \phi$ implies $\exists$ h: $G_\phi \rightarrow G"$

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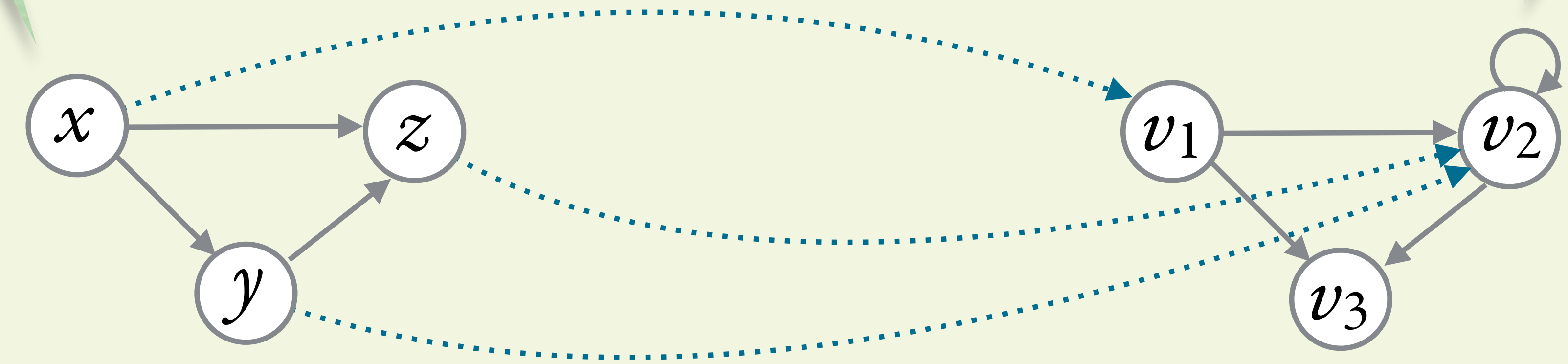

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Evaluation via homomorphisms

Lemma. The evaluation of a CQ $\phi(x_1, \ldots, x_n)$ on S' returns the set

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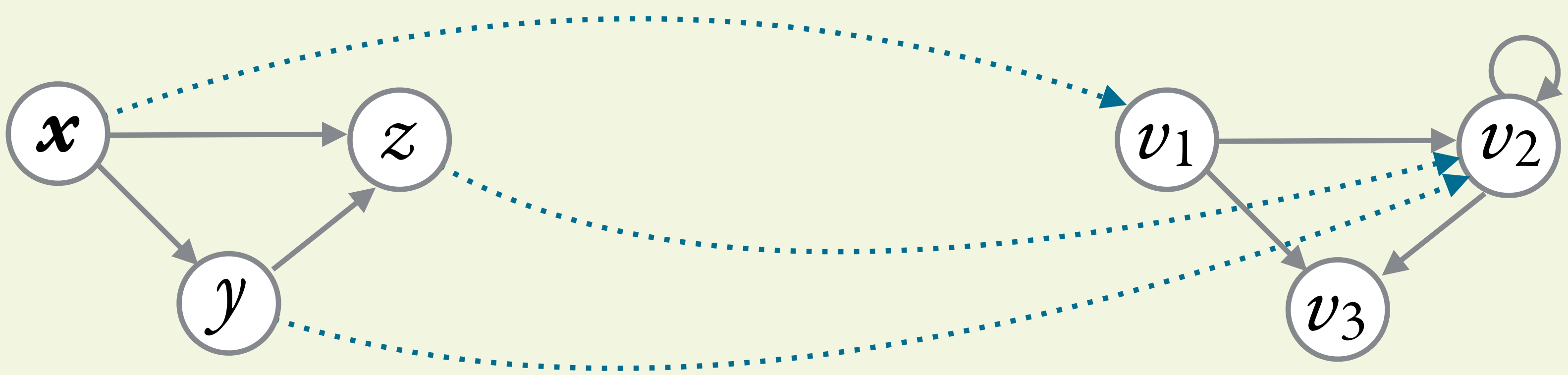

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Question: Which are the homomorphisms $G_\phi \longrightarrow G'$? What is the result of $\phi(G')$?

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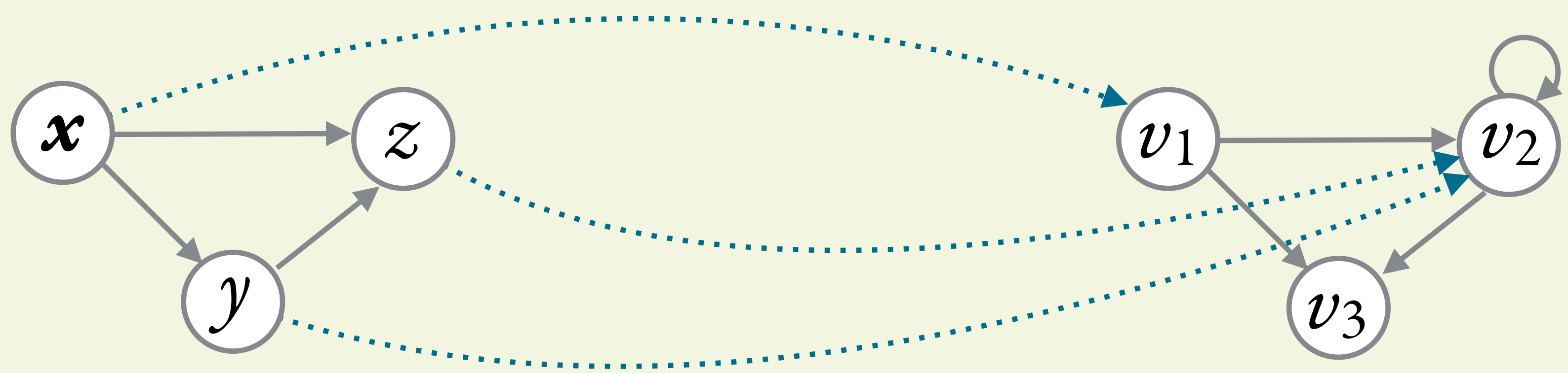

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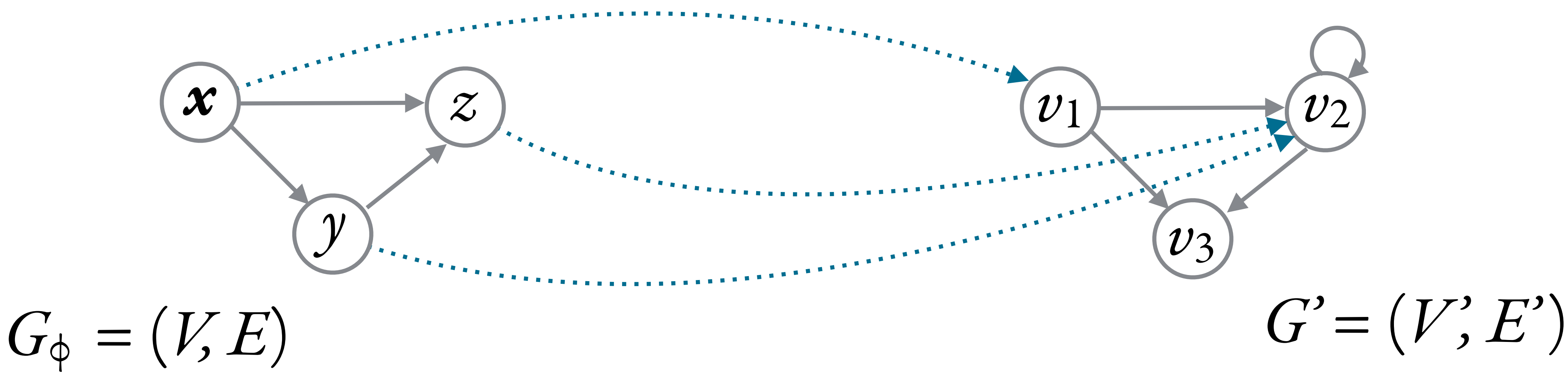
Evaluation via homomorphisms

Theorem. Evaluation of CQ is in NP (combined complexity)



Input: A CQ $\phi(x_1, \ldots, x_n)$, a graph G, a tuple $(a_1, \ldots, a_n)$
Output: Is $(a_1, \ldots, a_n) \in \phi(G)$?

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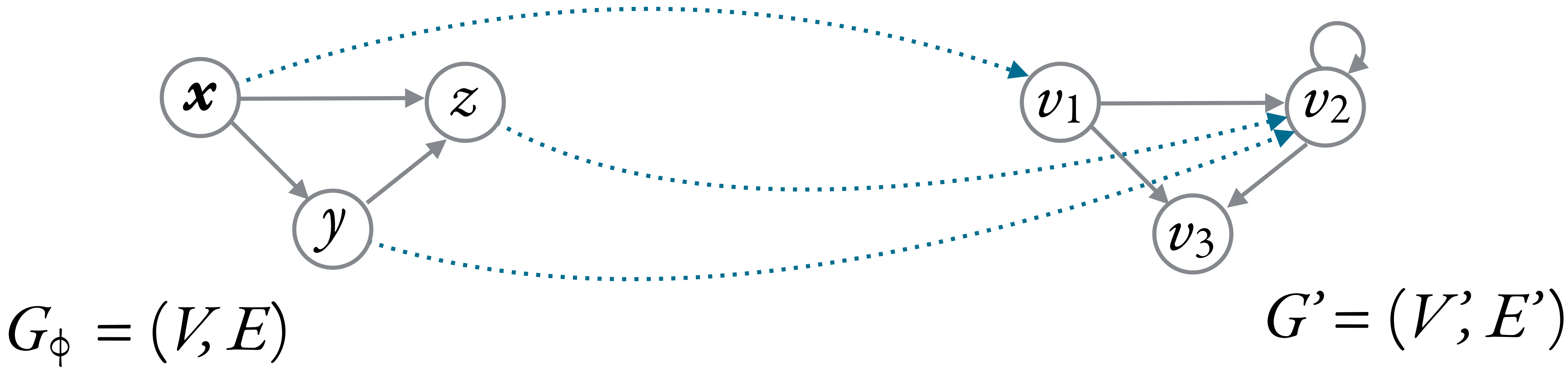

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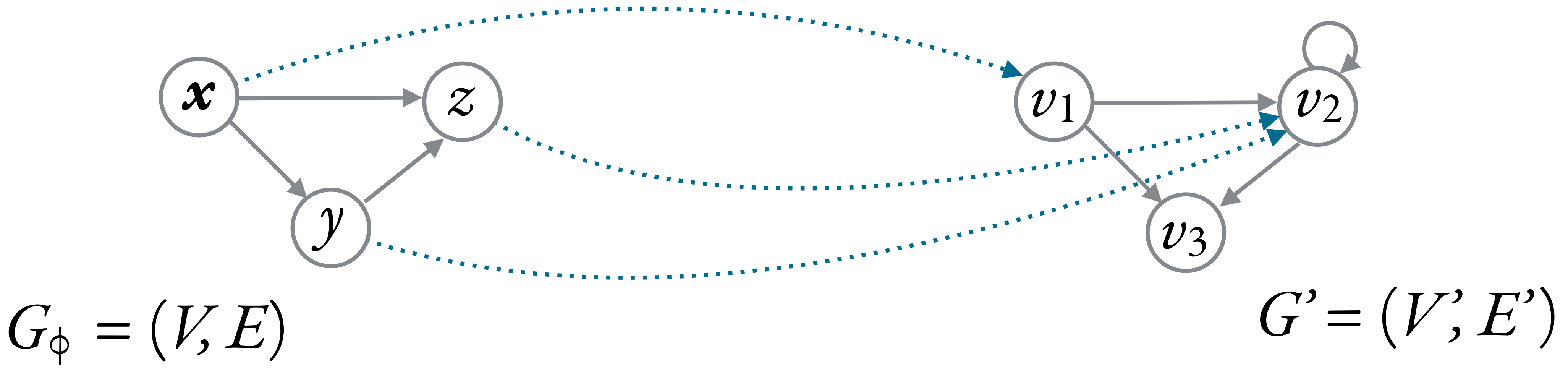

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Ideas?

1. Guess h: $G_\phi \longrightarrow G$
2. Check that it is a homomorphism
3. Output YES if $(h(x_1), \ldots, h(x_n)) = (a_1, \ldots, a_n)$; NO otherwise.

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NP-complete problem: **3-COLORABILITY**

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Output: Can we assign a colour from $\{R,G,B\}$ to each node so that adjacent nodes have always different colours ?

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Is there a *homomorphism* from G to K_3 ?

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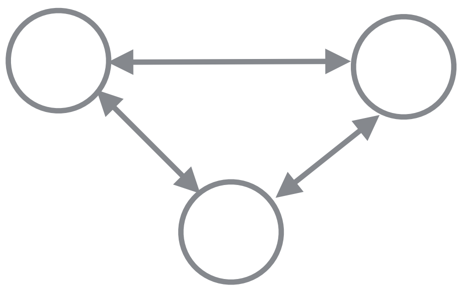

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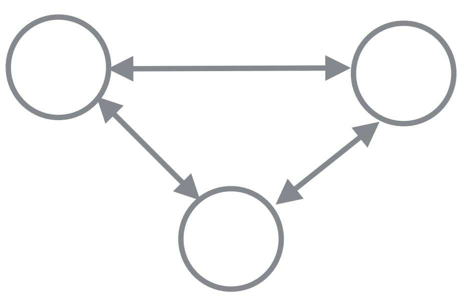

Reduction 3COL ⇝ CQ-EVAL: 1. Given G, build a CQ ϕ such that $G_\phi = G$.
2. Test if $() \in \phi(G)$.

Monotonicity and preservation theorems

Lemma. Every CQ is **monotone**:

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- "The relation R has at most 2 elements" $\notin$ CQ
- "There is a node connected to every other node" $\notin$ CQ
- "The radius of the graph is 5" $\notin$ CQ

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- One example of the few properties which still hold on finite structures.
- Proof in the finite is difficult and independent.

Static analysis with CQs

The satisfiability problem for CQ is decidable…

Question: What is the algorithm for CQ-SAT? What is the complexity?

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Answer: CQs are always satisfiable by their canonical structure!

$$G_\phi \models \phi$$

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problem: **CQ-CONTAINMENT**

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Output: Does $\phi(S) \subseteq \psi(S)$ holds for every structure S ?

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[⟸] Consider $(v_1,\dots,v_m) \in \phi(S)$

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Static analysis with CQs

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$h\,g$ is a homomorphism from S_ψ to S. Hence, $(v_1, \ldots, v_m) \in \psi(G)$.

Static analysis with CQs

problem: **CQ-EQUIVALENCE**

Input: Two CQs ϕ, ψ

Output: Does $\phi(S) = \psi(S)$ holds for every S ? (we write "$\phi \equiv \psi$")

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Amounts to testing if G_ϕ and G_ψ are **hom-equivalent**
(i.e. there are homomorphisms in both senses)

Static analysis with CQs

problem: **CQ-MINIMIZATION**

Input: A CQ ϕ

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Theorem. The minimization problem for CQ is NP-complete

Amounts to testing if there is a **non-injective endomorphism**

$g\colon G_\phi \longrightarrow G_\phi$

or, equally, if the **smallest graph hom-equivalent** to G_ϕ is G_ϕ itself
(we say that G_ϕ is a **core**)

Adding functional dependencies

A **functional dependency** is a sentence of the form

$$\boldsymbol{\gamma} = \forall \ldots\ R(x_1, \ldots, x_n) \wedge R(x_1', \ldots, x_n') \wedge \bigwedge_j (x_{ij} = x_{ij}') \Rightarrow (x_i = x_i')$$

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We often abbreviate this with

R: $1 \rightarrow 2$

Adding functional dependencies

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All the previous problems:

- CQ-CONTAINMENT
- CQ-EQUIVALENCE
- CQ-MINIMIZATION

remain in NP if we further restrict finite structures so as to satisfy any set of functional dependencies

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Modify the canonical structure S_ϕ ...

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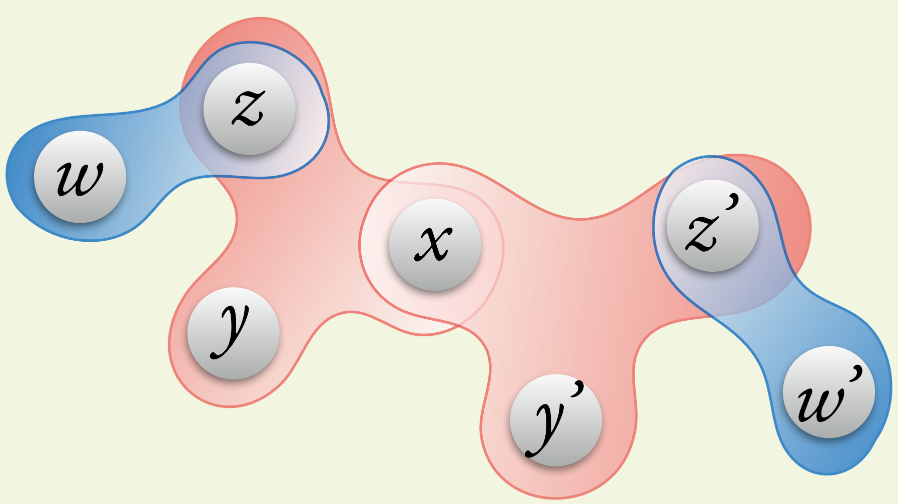
Adding functional dependencies

CQ $\phi = R_2(x, y, z) \wedge R_2(x, y', z') \wedge R_1(z, w) \wedge R_1(z', w')$

under functional dependencies $F = \{ R_1: 1 \rightarrow 2,\ R_2: 1 \rightarrow 3 \}$



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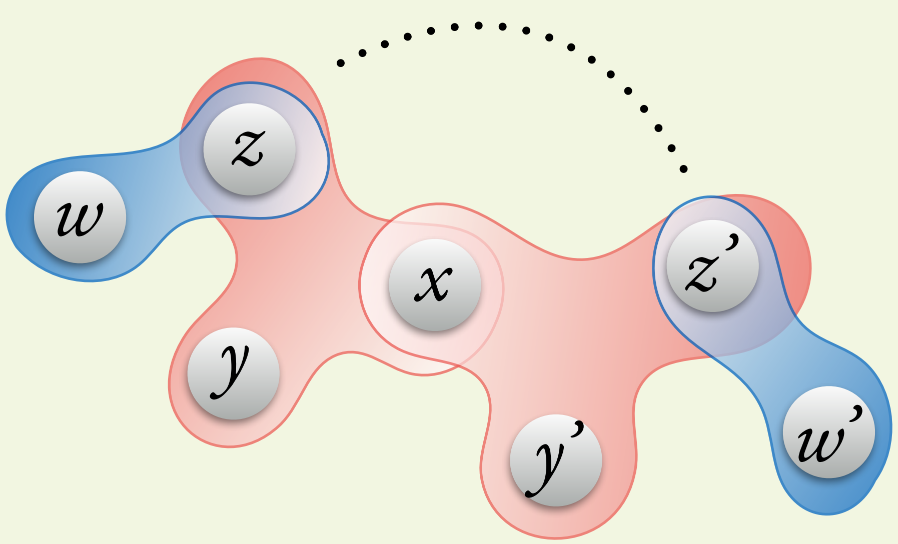

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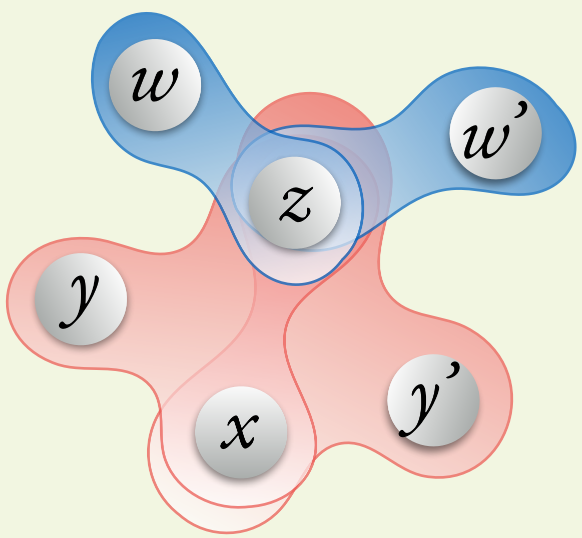

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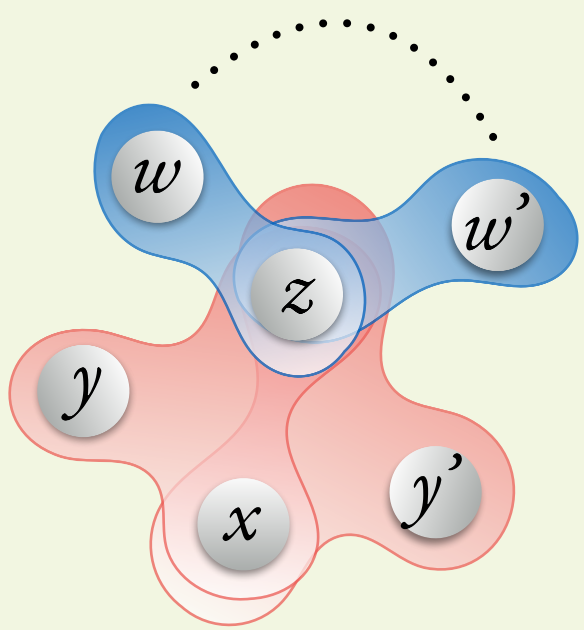

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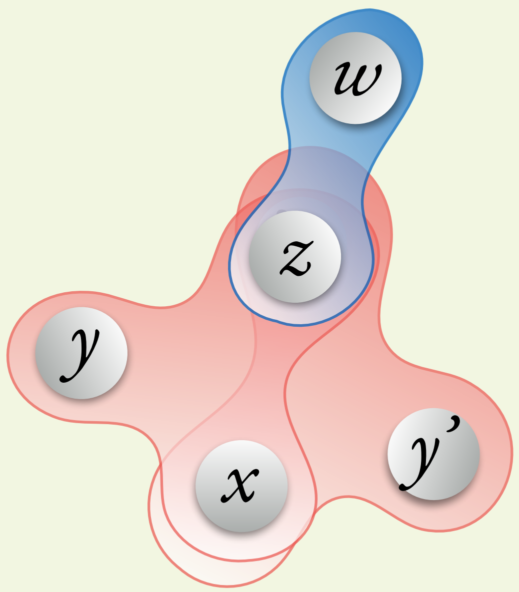

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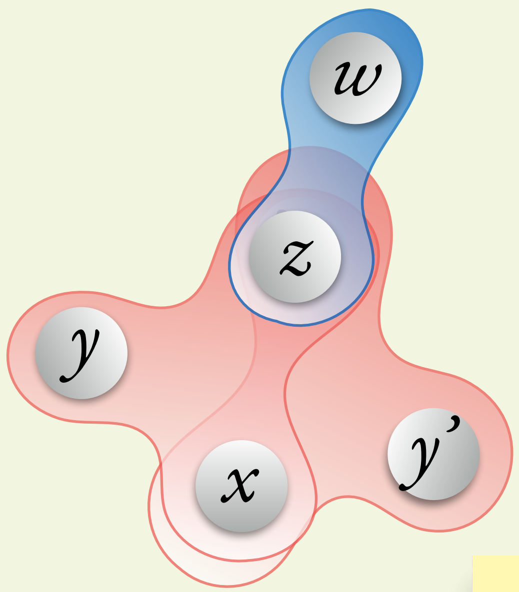

$= S_\phi$ (the **chased canonical structure**)

- S_ϕ is unique and can be constructed in polynomial time
- It is the "most general" constrained model of ϕ : $\phi(S) = h(\phi(S_\phi))$ for all S satisfying the funct. depend.

Adding functional dependencies

$\phi \in CQ$
FD's $F=\{fd_1, \ldots, fd_n\}$

chase →

$chase_F(\phi) \in CQ$

Adding functional dependencies

$\phi \in CQ$
FD's $F=\{fd_1, \ldots, fd_n\}$ —chase→ $chase_F(\phi) \in CQ$

The static analysis problems restricted to FD's can now be also shown in NP

- CQ-Containment $\phi \subseteq_F \psi$ iff $chase_F(\phi) \subseteq chase_F(\psi)$
- CQ-Equivalence $\phi \equiv_F \psi$ iff $chase_F(\phi) \equiv chase_F(\psi)$
- CQ-Minimization ϕ is minimal wrt structures verifying F iff $chase_F(\phi)$ is minimal