THE COST OF REPAIRS

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LaBRI / CNRS

based on joint works with

Michael Benedikt,
Pierre Bourhis,
Cristian Riveros,
Slawek Staworko
What do you do when a computational object fails a specification?

Source

1000
10000
100000
...

Target

1050
10050
100050
...

Worst-case cost of repairing source into target:

\[
\max_{s \in S} \min_{t \in T} \text{dist}(s, t)
\]

Can be finite or infinite, depending on source and target... can we decide this?
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Can be **finite** or **infinite**, depending on source and target
...can we decide this?
Plan

A. Bounded repairability of regular word languages
   1) characterization
   2) streaming setting
   3) complexity

B. Bounded repairability of regular tree languages
   1) curry encodings, stepwise automata, contexts
   2) characterization
   3) complexity
Part A. Problem setting:

- Given two languages $S \subseteq \Sigma^*$ and $T \subseteq \Delta^*$ (represented by finite state automata)

- Decide whether $\max_{s \in S} \min_{t \in T} \text{dist}(s, t)$ is finite.
Part A. Problem setting:

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  (represented by finite state automata)

- Decide whether $\max_{s \in S} \min_{t \in T} \text{dist}(s, t)$ is finite.

Examples

- $10^* \text{ is bounded repairable into } 10^*50$
- $10^* \text{ is not bounded repairable into } (10)^*$
- $(1 + 0)^* \text{ is not bounded repairable into } (1 + 0^*5)^*$
Rule of thumb: “*If you need to edit, you’d better do it outside a loop!*”
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Source automaton

Target automaton

For any strategy that repairs traces of $X$ into traces of $Y$:
1. either $\text{traces}(X) \subseteq \text{traces}(Y)$
2. or the strategy has unbounded cost.
Rule of thumb: “If you need to edit, you’d better do it outside a loop!”

For any strategy that repairs traces of $X$ into traces of $Y$:

1. either $\text{traces}(X) \subseteq \text{traces}(Y)$
2. or the strategy has unbounded cost.
Characterization of bounded repairability of word languages

$S$ is repairable into $T$ with uniformly bounded cost

\[ \iff \]

Given some (trimmed) automata for $S$ and $T$ and the DAGs of strongly connected components...

Source DAG

Target DAG
Characterization of bounded repairability of word languages

$S$ is repairable into $T$ with uniformly bounded cost

\[ \uparrow \]

Given some (trimmed) automata for $S$ and $T$ and the DAGs of strongly connected components...

...every chain of components in the source is **covered** by a chain of components in the target.
Characterization of bounded repairability of word languages

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\[ \uparrow \]

Given some (trimmed) automata for $S$ and $T$ and the DAGs of strongly connected components...

...every chain of components in the source is covered by a chain of components in the target.

---

**Source DAG**

**Target DAG**

...every chain of components in the source is covered by a chain of components in the target.
An example

All chains of source DAG are covered by chains of target DAG
⇒ \( S \) is repairable into \( T \) with uniformly bounded cost.

\[
S = 30 \cdot 1^* + 30 \cdot 2^*
\]

\[
T = 10 \cdot 1^* + 20 \cdot 2^*
\]
An example

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⇒ $S$ is repairable into $T$ with uniformly bounded cost.

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All chains of source DAG are covered by chains of target DAG
⇒ \( S \) is repairable into \( T \) with uniformly bounded cost.

\[
S = 30*1^* + 30*2^* \\
\]

\[
T = 10*1^* + 20*2^* \\
\]

There is no covering relation compatible with prefixes
⇒ the repair strategy is not streaming
(i.e. implementable by a sequential transducer)
An example

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⇒ the repair strategy is not streaming (i.e. implementable by a sequential transducer)
Complexity of **non-streaming** bounded repairability problem:

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Part B. New tools for a more general setting...

<table>
<thead>
<tr>
<th>Languages of words:</th>
<th>Languages of unranked trees:</th>
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<tbody>
<tr>
<td>■ insersions / deletions</td>
<td>■ insertions / deletions</td>
</tr>
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<td>■ finite state automata</td>
<td>■ stepwise tree automata</td>
</tr>
<tr>
<td>■ components &amp; traces</td>
<td>■ components &amp; contexts</td>
</tr>
<tr>
<td>■ coverability of chains</td>
<td>■ coverability of synopsis trees</td>
</tr>
</tbody>
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Edit operations on unranked trees: deletions
Edit operations on unranked trees: deletions
Edit operations on unranked trees: insertions
Edit operations on unranked trees: insertions
An example

Source

```
r
  d
  |
  a  ...  a  b  ...  b
```

Target

```
r
  e
  |
  a  ...  a
  |
  b  ...  b
  |
  c  ...  c
```
An example
An example
An example
Bottom-up automata on ranked (binary) trees:

How to parse unranked trees?
Bottom-up automata on ranked (binary) trees:

How to parse unranked trees?

Encode them using binary trees!
The curry encoding

\[
\begin{array}{c}
1 \\
\end{array}
\xrightarrow{\cong}
\begin{array}{c}
0 \\
\end{array}
\]

\[
\begin{array}{c}
0 \\
\end{array}
\xrightarrow{\cong}
\begin{array}{c}
1 \\
\end{array}
\]

Stepwise automata = bottom-up on curry encodings
The curry encoding

\[
\begin{array}{c}
1 \\
\downarrow
\end{array} \quad \frac{0}{1} \quad \frac{2}{\cdots} \\
\frac{n}{\cdots} \quad \frac{0}{1} \quad \frac{2}{\cdots} \\
\frac{n}{\cdots}
\end{array}
\]

Stepwise automata = bottom-up on curry encodings
The curry encoding

\[
\begin{array}{c}
0 \\
1 \\
2 \\
\vdots \\
n
\end{array}
\]
The curry encoding

\[
\begin{array}{c}
\begin{array}{cccccc}
0 \\
1 \\
2 \\
\vdots \\
n \\
\end{array}
\end{array}
\]

\[
\begin{array}{c}
\begin{array}{cccccc}
\Rightarrow \\
0 \\
1 \\
2 \\
\vdots \\
n \\
\end{array}
\end{array}
\]

\[
\begin{array}{c}
\begin{array}{cccccc}
\Rightarrow \\
0 \\
1 \\
2 \\
\vdots \\
n \\
\end{array}
\end{array}
\]

Stepwise automata = bottom-up on curry encodings
The curry encoding

\[ 0 \rightarrow 1 \rightarrow 2 \rightarrow \cdots \rightarrow n \]

\[ 0 \rightarrow @ \rightarrow 1 \rightarrow @ \rightarrow 2 \rightarrow \cdots \rightarrow @ \]

Stepwise automata = bottom-up on curry encodings
An example

\[ \begin{array}{c}
  r \\
  \cdots \quad a \quad b \\
  \quad \cdots \\
  \quad b \\
  c
\end{array} \approx
\begin{array}{c}
  \@ \\
  \cdots \quad a \quad b \\
  \quad \cdots \\
  r \quad a \\
  b \quad c
\end{array} \]
An example

\[
\begin{align*}
\ast & \Rightarrow q_r \\
a & \Rightarrow q_a \\
b & \Rightarrow q_b \\
c & \Rightarrow q_c
\end{align*}
\]

\[
\begin{align*}
q_r & \ast q_a \Rightarrow q_r \\
q_b & \ast q_c \Rightarrow q_c \\
q_r & \ast q_c \Rightarrow q_{\text{final}}
\end{align*}
\]
An example

\[
\begin{align*}
  r & \rightarrow q_r \\
  a & \rightarrow q_a \\
  b & \rightarrow q_b \\
  c & \rightarrow q_c \\
  q_r \circ q_a & \rightarrow q_r \\
  q_b \circ q_c & \rightarrow q_c \\
  q_r \circ q_c & \rightarrow q_{\text{final}}
\end{align*}
\]
An example

\[ r \rightarrow q_r \]
\[ a \rightarrow q_a \]
\[ b \rightarrow q_b \]
\[ c \rightarrow q_c \]

\[ q_r \rightarrow q_a \rightarrow q_r \]
\[ q_b \rightarrow q_c \rightarrow q_c \]

\[ q_r \rightarrow q_{final} \]
An example

\[ r \rightarrow q_r \]
\[ a \rightarrow q_a \]
\[ b \rightarrow q_b \]
\[ c \rightarrow q_c \]

\[ q_r \rightarrow q_{r} q_{a} \rightarrow q_r \]
\[ q_{b} \rightarrow q_{c} \]
\[ q_{r} \rightarrow q_{c} \rightarrow q_{final} \]
Contexts = trees with holes

0 = 🔗 2

0 = 🔗 2

0 = 🔗 2

0 = 🔗 2
Contexts = trees with holes

Contexts can be parsed between two states: $p \xrightarrow{C} q$ (accessibility of states and components are defined accordingly).
Contexts = trees with holes

Contexts can be parsed between two states: $p \xrightarrow{c} q$

(accessibility of states and components are defined accordingly)
Recall: a run of a finite state automaton induces a chain of components...

Likewise, a run of a stepwise automaton induces a tree of components, called synopsis tree.
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Likewise, a run of a stepwise automaton induces a tree of components, called synopsis tree.
Characterization of bounded repairability of tree languages

$S$ is repairable into $T$ with uniformly bounded cost

\[ \uparrow \]

Given some (trimmed) stepwise automata for $S$ and $T$, all synopsis trees of $S$ are covered by synopsis trees of $T$

i.e. ...
Characterization of bounded repairability of tree languages

\( S \) is repairable into \( T \) with uniformly bounded cost

\[ \Downarrow \]

Given some (trimmed) stepwise automata for \( S \) and \( T \), all synopsis trees of \( S \) are covered by synopsis trees of \( T \)

\[ \exists \lambda : \text{cyclic components} \rightarrow \text{cyclic components} \]
Given some (trimmed) stepwise automata for $S$ and $T$, all synopsis trees of $S$ are covered by synopsis trees of $T$.

i.e.  $\exists \lambda : \text{cyclic components} \rightarrow \text{cyclic components}$

1. $\lambda$ preserves contexts:
   $\text{contexts}(X) \subseteq \text{contexts}(\lambda(X))$
Given some (trimmed) stepwise automata for $S$ and $T$, all synopsis trees of $S$ are covered by synopsis trees of $T$

i.e. $\exists \lambda : \text{cyclic components} \rightarrow \text{cyclic components}$

1. $\lambda$ preserves contexts:
   $\text{contexts}(X) \subseteq \text{contexts}(\lambda(X))$

2. $\lambda$ respects post-order of components:
   $X \leq_{\text{postorder}} Y \iff \lambda(X) \leq_{\text{postorder}} \lambda(Y)$

3. $\lambda$ preserves ancestorship of vertical components:
   whenever $\text{vertical-contexts}(X) \neq \emptyset$

\[\text{Source synopsis tree} \quad \rightarrow \quad \text{Target synopsis tree}\]
Given some (trimmed) stepwise automata for $S$ and $T$, all synopsis trees of $S$ are covered by synopsis trees of $T$, i.e.

$$\exists \lambda : \text{cyclic components} \rightarrow \text{cyclic components}$$

1. $\lambda$ preserves contexts:
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2. $\lambda$ respects post-order of components:
   $$X \leq_{\text{postorder}} Y \iff \lambda(X) \leq_{\text{postorder}} \lambda(Y)$$

3. $\lambda$ preserves ancestorship of vertical components:
   $$X \leq_{\text{ancestor}} Y \iff \lambda(X) \leq_{\text{ancestor}} \lambda(Y)$$
   whenever $$\text{vertical-contexts}(X) \neq \emptyset$$
delete $d$
delete $d$

$\{r\ldots a \atop \at \ldots d \at \ldots c \at \ldots\}$

$\ldots \at \ldots \at \ldots \at \ldots \at \ldots \at \ldots \at \ldots$
The diagram shows a tree structure before and after deletion. The tree is depicted with nodes labeled as $a$, $b$, $c$, and $d$. An arrow labeled "delete $d$" points from the original tree to the modified tree.

The tree structure is as follows:

Before deletion:
- $r$ is the root with $a$ and $d$ as children, $d$ has $b$ as a child.
- $c$ and $c$ are leaf nodes.
- $b$ and $b$ are leaf nodes.

After deletion:
- $r$ is the root with $a$ and $b$ as children, $b$ has $b$ as a child.
- $c$ and $c$ are leaf nodes.
delete $d$
Complexity of **non-streaming** bounded repairability problem:

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- **Bounded repairability for regular tree languages**
  Riveros, Staworko, P. – ICDT 2012

- **Which DTDs are streaming bounded repairable?**
  Bourhis, Riveros, Staworko, P. – ICDT 2013

...and other related topics

- normalized edit cost \( \sup_{s \in S} \min_{t \in T} \frac{\text{dist}(s, t)}{|s|} \)

- distance automata and limitedness problem

- energy games with perfect/imperfect information
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