System and Control Theory - Exercitation - November 2nd, 2015
Some of the proposed problems could be unsolvable. If so, explain why.

1. Consider in $\mathcal{R}^3$ the subspace of all vectors such that
$$x_1 + x_2 + x_3 = 0$$
Determine a basis of this subspace and the basis of the orthogonal subspace.

2. Consider the system $\dot{x}(t) = Ax(t)$ with matrix
$$A = \begin{bmatrix} -\alpha & 1 & 0 \\ 0 & -1 & 1 \\ -\beta & 0 & -1 \end{bmatrix}$$
Determine the set of all real values, $\alpha > 0$ and $\beta > 0$, for which the system is asymptotically stable.

3. Consider the system $\dot{x}(t) = Ax(t)$ with matrix
$$A = \begin{bmatrix} -1 & -2 \\ 1 & \alpha \end{bmatrix}$$
Determine the set of all real values of α for which the system has periodic (including constant) solutions.

4. Consider a pair of matrices (A, B) of the form
$$A = \begin{bmatrix} x & \underline{x} & 0 & \dots & 0 \\ x & x & \underline{x} & 0 & : \\ : & : & : & : & : \\ x & x & \dots & \underline{x} & 0 \\ x & x & \dots & x & \underline{x} \\ x & x & \dots & x & x \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ : \\ 0 \\ 0 \\ \underline{x} \end{bmatrix}$$
where $\underline{x} \neq 0$, x is any number (Hessemberg form). Prove that this system is always reachable.

5. Consider the Markov chain whose equation is $x(k+1) = Ax(k)$ where
$$A = \begin{bmatrix} \frac{1}{2} & \beta & 0 \\ 0 & (1-\beta) & \gamma \\ \frac{1}{2} & 0 & (1-\gamma) \end{bmatrix}$$
The asymptotic distribution is $x_\infty = [\frac{1}{4} \ \ \frac{1}{2} \ \ \frac{1}{4}]^\top$. Determine β and γ.

6. Given a discrete–time system
$$x(k+1) = Ax(k) + Bu(k), \quad y(k) = Cx(k)$$
the impulse is the sequence $1, \ 0, \ 0, \ 0, \dots$ ($u(0) = 1$, $u(k) = 0$, $k > 1$). Assuming $x(0) = 0$, write the expression of the impulse response $y(k)$.