System and Control Theory - Exercitation - December 2, 2016
Some of the proposed problems could be unsolvable. If so, explain why.

1. **Chemical reaction.** Consider the system:

$$\begin{aligned}\dot{x}_1(t) &= -x_1(t)x_2(t) + 1\\ \dot{x}_2(t) &= -x_1(t)x_2(t) + u(t)\\ \dot{x}_3(t) &= -x_1(t)x_2(t) - x_3(t)\end{aligned}$$

   Find a state feedback regulator for the linearized system which stabilizes the system in the point $\bar{x} = [1\ 1\ 1]^\top$. Assign eigenvalues $\Lambda_{SF} = \{-1, -1, -1\}$.

2. **Observer**. In the previous example (same conditions) with output $y(t) = x_1(t)$, find an observer assigning eigenvalues $\Lambda_{OB} = \{-2, -2, -2\}$.

3. **Digital implementation.** Consider the transfer function

$$W(s) = \frac{1}{(1+\frac{s}{a})(1+\frac{s}{b})(1+\frac{s}{c})}$$

   Find a realization and the discrete–time equivalent system. How do you choose the sampling time?

4. **Flow system** Consider the flow system

$$\dot{x}(t) = Bu(t) + f$$

   where $B = n \times m$ and $n < m$ and f is a constant vector. Assume that $rank(B) = n$. Show that any vector $\bar{x}$ can be an equilibrium state for some $\bar{u}$. Consider the feedback $u = \bar{u} - DB^\top(x(t) - \bar{x})$ where D is a diagonal matrix with positive diagonal entries. Prove closed–loop stability.

5. **Level control** Consider the system:

$$\begin{aligned}\dot{x}_1(t) &= x_2(t) - \sqrt{x_1(t)}\\ \dot{x}_2(t) &= -x_2^2(t) + x_3(t)\\ \dot{x}_2(t) &= u(t)\end{aligned}$$

   Find the equilibrium values with $\bar{x}_1 = \xi > 0$ free parameter. Let $u(t) = -\kappa(x_1 - \xi)$ and determine the values of κ ensuring asymptotic stability (IEP) corresponding to ξ.

6. **Lyapunov function?** Consider the linear system $\dot{x} = Ax$ and the quadratic function $V(x) = x^\top Px$ with $P = P(\alpha, \beta)$, where

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} \qquad P = \begin{bmatrix} 1 & \beta \\ \beta & \alpha \end{bmatrix}$$

   Determine the set of all values α and β, for which $V(x)$ is a Lyapunov function.

7. **Markov parameters.** Consider the linear system $\dot{x}(t) = Ax(t) + Bu(t)$ $y(t) = Cx(t)$ with one input, $m = 1$ and one output $p = 1$. Show that this is a minimal realization of $C(sI - A)^{-1}B$ if and only if the matrix of the Markov parameters $M_{ij} = CA^{i+j-2}B$

$$M = \begin{bmatrix} CB & CAB & CA^2B & \dots & CA^{n-1}B \\ CAB & CA^2B & CA^3B & \dots & CA^nB \\ CA^2B & CA^3B & CA^4B & \dots & CA^{n+1}B \\ \vdots & \vdots & \vdots & \dots & \vdots \\ CA^{n-1}B & CA^nB & CA^{n+1} & \dots & CA^{2n-2}B \end{bmatrix}$$

   has rank n (Hint. Write the matrix as a product ...).