System and Control Theory - Exercitation - January 15, 2018 Some of the proposed problems could be unsolvable. If so, explain why.

1. **Wave**. Consider the parameterized curve $f(k) = \alpha\cos(\theta k) + \beta\sin(\theta k)$, $\theta > 0$, which should approximate the measured signal $y(k)$, $k = 1, \dots, N$. Write the resolving system to determine the constants α and β, in least square sense.

2. Given the system $\dot{x} = Ax + Bu$, $z = Cx$, with

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} \sqrt{3} & 0 \end{bmatrix}$$

Find the optimal control according to $\int_0^\infty (z^2(t) + u^2(t))dt$ $(Q = \mathrm{diag}\{3, 0\},\quad R = 1)$.

3. **Lyapunov forever**. Let $\dot{x} = Ax$ be asymptotically stable with initial state x_0. Prove that

$$\int_0^\infty x^\top(t)Qx(t)dt = x_0^\top P x_0$$

if Q is positive definite, where P solves the Lyapunov equation $A^\top P + PA = -Q$.

4. **Crane**. The output equations for the position of a crane cart are $x = \rho\cos(\theta)$, $y = \rho\sin(\theta)$. The state equations are $\dot{\rho}(t) = \xi(t)$ and $\dot{\theta}(t) = \omega(t)$, with $\xi(t)$ and $\omega(t)$ control signals. Find a control law which drives the system in the target point $\bar{x} > 0$, $\bar{y} > 0$.

5. **Robust stabilizing state feedback matrices** Let

$$A = \begin{bmatrix} \alpha & 1 \\ -1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad |\alpha| \le 1$$

α uncertain and **constant**. Determine the set of all linear state feedback $u = Kx$ which robustly stabilize the system $A + BK$ (the system is of the second order ...).

6. **Variable slope track** Consider the equations

$$\begin{aligned} \dot{\theta}(t) &= \omega(t) \\ \ddot{\xi}(t) &= -\tan(\theta(t)) + \phi(t) \end{aligned}$$

Write the state equations and linearize in the equilibrium with $\bar{\theta} = 0$ and $\bar{\xi} = 0$. You can chose *only one input* ω or ϕ and choose *only one sensor*, namely measure only one state variable. Which are the appropriate choices to stabilize (asymptotically) the system?

7. **Standard**. Solve the previous problem assigning the eigenvalues via state observer.

8. **Cruise control** The speed v and the direction θ of a boat satisfy the equations

$$\begin{aligned} \dot{\theta}(t) &= v(t)\sin(u_1(t)) \\ \dot{v}(t) &= -\alpha v^2(t) + u_2(t) \end{aligned}$$

(approximately) where $u_1(t)$ is the rudder angle and $u_2(t)$ is the engine propulsion (α is a positive parameter). Find a control which steers the boat in the direction $\bar{\theta}$ with speed $\bar{v} > 0$.