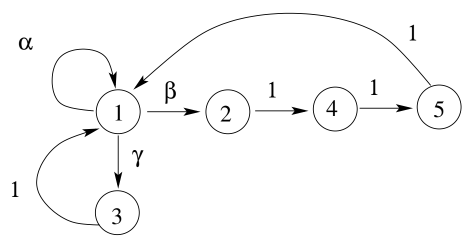
Written text of System and Control Theory - October 31st, 2014
Some of the proposed problems could be unsolvable. If so, explain why.

1. Consider the discrete–time system $x(k+1) = Ax(k) + Bu(k)$, with

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

   Determine an input sequence $u(0)$, $u(1)$, $u(2)\dots$ which drives the generic initial state $[\ x_1(0)\ x_2(0)\ x_3(0)\ x_4(0)\ ]^\top$ to the state $[\ \xi\ \xi\ \xi\ \xi\ ]^\top$.

2. Consider the Markov chain in figure, with $\alpha + \beta + \gamma = 1$ and $\alpha, \beta, \gamma \geq 0$. Write the corresponding matrix A and determine the asymptotic probability distribution.



3. Consider the continuous–time system $\dot{x}(t) = Ax(t)$, $y(t) = Cx(t)$, with

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} c_1 & c_2 \end{bmatrix}.$$

   Determine c_1 and c_2 so that the output $y(t)$ converges to 0 (for $t \to \infty$) for any initial state.

4. Compute the discrete–time system which is equivalent to the continuous–time system $\dot{x}(t) = Ax(t) + Bu(t)$, $y(t) = Cx(t)$, with

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

5. For which values of α and β the continuous–time system $\dot{x}(t) = Ax(t)$, with

$$A = \begin{bmatrix} -1 & -\alpha \\ 1 & -\beta \end{bmatrix},$$

   has periodic solutions? Which is the corresponding period?

6. The equations of a clock are $\dot{\tau}(t) = \omega$, $\dot{\omega}(t) = 0$, where τ is the time indication and ω is the clock speed. Based on the given definitions, analyse stability of the system.