System and Control Theory - Exercitation - December 7, 2015
Some of the proposed problems could be unsolvable. If so, explain why.

1. **Matrix equation for eigenvalue assignment**. Consider the reachable pair (A, B), where A is $n \times n$ and B $n \times m$. Let F be an assigned square matrix $n \times n$ having the desired eigenvalues (U.C.C.). Consider the matrix equation

$$AT + BR = TF$$

where T $n \times n$ and R, $m \times n$ are unknown matrices. Assume that there exists a solution (T, R) and assume that T is invertible. Then show that there exists K such that $(A + BK)$ has the same eigenvalues of F, namely $\sigma(A + BK) = \sigma(F)$.

2. **Find the Lyapunov function**. *Please, no question on this exercise.* Consider the system $\dot{x}(t) = Ax(t)$ with matrix

$$A = \begin{bmatrix} -1 & 1 \\ -1 & 2 \end{bmatrix}$$

Find a Lyapunov function (i.e. $V \in \mathcal{C}^1$, $V(0) = 0$, $V(x) > 0$, $\dot{V}(x) < 0$ $x \neq 0$).

3. **Stabilization.** Consider the system (A, B, C) whose transfer function is

$$\frac{1}{s^2 + 1}$$

Find a regulator based on an observer of the state such that the eigenvalues are -1, -2 for the estimated state feedback and $-2 \pm j2$, for the observer.

4. **Realization.** Consider the transfer function matrix

$$W(s) = \begin{bmatrix} \frac{1}{s^2 + 3s + 2} & \frac{s + 3}{s^2 + 3s + 2} \end{bmatrix}$$

$m = 2$ and $p = 1$. Find a minimal realization. (Hint. use the dual).

5. **Choose sensors and actuators.** Given the third order system with

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \quad C = \begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix}$$

You must place **one** actuator and **one** sensor only. Therefore assume that **only one** of the b_i can be different from 0 and that only one of the c_i can be different from 0. Select the non–zero entries in B and in C in such a way that the system can be asymptotically stabilized (no need to find the regulator).

6. **Equilibrum inputs.** Consider the chemical system

$$\begin{aligned} \dot{x} &= -\kappa xy + u_1 \\ \dot{y} &= -\kappa xy - \mu y + u_2 \\ \dot{z} &= +\kappa xy - \nu z \end{aligned}$$

with κ, μ, ν constant. Find the set of all constant $(\bar{u}_1, \bar{u}_2)$ for which the equilibrium state variables are positive $(\bar{x} > 0, \bar{y} > 0, \bar{z} > 0)$. $\mathcal{U} = \{\bar{u}_1, \bar{u}_2 : \ldots\}$.

7. Find the digital realization of the tranfer function

$$F(s) = \frac{s + b}{s + a}$$

(discrete–time equivalent system) $a \neq b$.