System and Control Theory - Exercitation - November 06, 2017
Some of the proposed problems could be unsolvable. If so, explain why.

1. **Sudoku II.** Put the two positive numbers α and β in the matrices B and C, and set to 0 all the other entries, in order to have a reachable and observable system.

$$A = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & -3 \end{bmatrix} \quad B = \begin{bmatrix} x \\ x \\ x \end{bmatrix} \quad C = \begin{bmatrix} x & x & x \end{bmatrix}$$

2. **Orthonormal.** Consider the system $x(k+1) = Ax(k)$ with A orthonormal. Analyze stability (is the system unstable, stable, marginally stable or asymptotically stable?).

3. **Se non pensi tuo danno....** (I do not know how to translate this) Consider the continuous–time system with matrices A, B, C

$$A = \begin{bmatrix} -\alpha & \beta \\ \beta & -\gamma \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

Find α, β γ all positive in such a way that the impulse response of the system is $e^{-t}\cos(2t)$.

4. **Stability? reachability? observability?** Consider

$$A = \begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

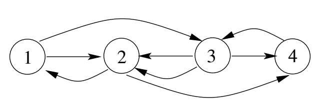
For which values of $\omega \geq 0$ the system is reachable and observable? For which values of $\omega \geq 0$ the system is asymptotically stable? For which values of $\omega \geq 0$ the system is stable?

5. **What about sampling?** In the previous exercise check that

$$e^{AT} = \begin{bmatrix} \cos(\omega T) & -\sin(\omega T) \\ \sin(\omega T) & \cos(\omega T) \end{bmatrix}$$

Assume $\omega > 0$. Find the equivalent discrete–time sampled–data system. Analyze for which values of $T > 0$ the systems is reachable. Analyze for which values of $T > 0$ the system is observable.

6. **Markov.** Consider the Markov chain whose graph is



Assume that, for all nodes, the leaving arcs have the same weights. Write the matrix A. Find the asymptotic probability distribution.

7. **Desired output** Consider a system having transfer function

$$\frac{1}{s+\alpha}$$

Find the input $u(t)$ such that the regime (long-term) output is $y(t) = p\cos(\omega t)$.

8. **Cruel professor.** Infinite–time reachability. We say that (A, B) is infinite–time reachable if there exists $u(t)$ such that $x(t) \to 0$ as $t \to \infty$. Under which conditions a system is infinite–time reachable?