System and Control Theory - Exercises - November 26, 2018
Some of the proposed problems may have no solution. If so, explain why.

1. **Realization** Consider the system having impulse response

$$W(t) = e^{-2t} + 2e^{-t}$$

Find a minimal realization. Find the equivalent discrete–time system.

2. **Linearize and analyze.**

$$\begin{aligned} \dot{x}_1 &= -\alpha x_1 - \delta x_1 x_3 + u \\ \dot{x}_2 &= \alpha x_1 - \beta x_2 \\ \dot{x}_3 &= \beta x_2 - \delta x_1 x_3 \end{aligned}$$

Determine the equilibrium conditions taking $\bar{u} > 0$ as a parameter. Linearize the system. Then let $\alpha = \beta = \gamma = \delta = 1$, $\bar{u} = 2$ and analyze the stability of the linearized system.

3. **Magnetic suspension** The linearized model for a magnetic suspension is

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Design an observer based control, assigning poles -1, -1 to the controller and poles $-2 \pm j2$ to the observer.

4. **Unstable–bistable-stable**. Consider the system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \arctan(x_1) + \mu x_1 - x_2 \end{aligned}$$

Find the intervals of values of the real parameter μ for which: (u) there is a single unstable equilibrium; (b) there are two stable equilibria and one unstable equilibrium (the system is bistable); (s) there is a single stable equilibrium. Stable means that its linearization is asymptotically stable. (Hint: you need not compute the values ...).

5. **Permutation matrices.** A permutation matrix P has: a single nonzero element for each row equal to 1 and a single nonzero element for each column equal to 1. Consider the system

$$x(k+1) = Px(k)$$

with P permutation matrix. Analyze stability.

6. **Filtered derivator** Consider the filter

$$F(s) = \frac{2s}{s^2 + 2s + 2}$$

Write magnitude and phase as a function of the frequency ω.

7. **Implementation of the filter as observer.** Consider the actuator system

$$\dot{x}_1 = x_2 \quad \dot{x}_2 = u \quad y = x_1 + w$$

Can we design a Luenberger observer such that the joint actuator–observer system with inputs u (acceleration) and w (disturbance) and output $\hat{x}_2 = \hat{v}$ (estimated velocity) satisfies

$$\hat{v}(s) = F(s)w(s) + G(s)u(s)$$

where $F(s)$ is the previous filter transfer function? And $G(s) = ?$.