System and Control Theory - Exercitation - December 04, 2017
Some of the proposed problems could be unsolvable. If so, explain why.

1. **Stability**. Consider the system $\ddot{\theta}(t) = -\kappa\theta(t) + \alpha\sin(\theta(t)) - \beta\dot{\theta}(t)$. Write the state equations. Linearize the system and study the stability of the equilibrium with $\bar{\theta} = 0$ as a function of the parameter κ.

2. **Realization.** Consider the transfer function
$$W(s) = \frac{\gamma s}{(s+\alpha)(s+\beta)}$$
Find a realization. Write the discrete–time equivalent system.

3. **Observer.** Consider a linear continuous–time system with
$$A = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$
Let $u(t)$ be a given input. Write the equation of an observer assigning the eigenvalues $-1 \pm j$ to the error system.

4. **Conbustion.** Given the combustion system
$$\begin{aligned} \dot{x}_1 &= -2x_1^2 x_2 - x_1 + u \\ \dot{x}_2 &= -x_1^2 x_2 + d \\ \dot{x}_3 &= +x_1^2 x_2 - x_3 \end{aligned}$$
let d be fixed (methan flow). Find the equilibrium conditions for which $\bar{x}_1 = \bar{x}_3$ as a function of d. Analyze stability. Take $d = 1$ and find a state feedback for the oxygen flow $v = k(x - \bar{x}) + \bar{u}$, to assign the eigenvalues in $-2, -2, -2$.

5. **Oscillation** Consider the elastic system with mass matrix $M = I$ and stiffness matrix
$$K = \begin{bmatrix} \alpha+\beta & -\beta \\ -\beta & \alpha+\beta \end{bmatrix}$$
The resonance pulsations are $\omega_1 = 1$ and $\omega_2 = 3$. Find α and β.

6. **Bifurcation.** Consider the system
$$\ddot{y}(t) = -\kappa y(t) + \alpha y(t) - y^3(t)$$
where α is a real parameter. Determine the number of equilibria as a function of α. Analyze the stability of these equilibria.

7. **Can airplanes fly?** The approximated roll-height equations of a craft at a constant speed are
$$\ddot{\theta} = \alpha(u_1 - u_2), \quad \ddot{z} = \beta\cos(\theta)(u_1 + u_2) - p, \quad \ddot{x} = \gamma\sin(\theta)(u_1 + u_2),$$
Write the state equations. Find the equilibrium conditions for $\bar{\theta} = 0$. Can we assign arbitrary eigenvalues via state feedback? Can we assign arbitrary eigenvalues to an observer assuming $y_1 = \theta$, $y_2 = z$ and $y_3 = x$ as output variables?

8. **Catch the minimum.** Let $\phi(x)$ $x \in R^n$ be a continuously differentiable function which has a global isolated minimum in a point $\bar{x}$. Assume that $\nabla\phi(x) \neq 0$ for $x \neq \bar{x}$. Prove that the trajectories of the system
$$\dot{x}(t) = -\gamma\nabla^T\phi(x)$$
$\gamma > 0$ converge to $\bar{x}$.