System and Control Theory - Exercitation - October 28, 2016
Some of the proposed problems could be unsolvable. If so, explain why.

1. **Difference equation.** Consider the discrete–time system with input u and output y

$$y(k+2) = 2y(k+1) - y(k) + u(k)$$

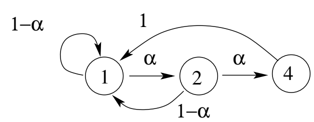
write a state–space representation A, B, C. Analyze stability.

2. **Transfer function.** Consider the continuous–time system with matrices A, B, C, and $W(s)$:

$$A = \begin{bmatrix} \alpha & 1 \\ 0 & \beta \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} \gamma & \delta \end{bmatrix}, \quad W(s) = \frac{1}{s^2+3s+2}$$

For which values of the parameters in A, B, C, the transfer function is $W(s)$?

3. **Markov chain.** Consider the following Markov chain



The asymptotic distribution is $x_\infty = [9/13\ 3/13\ 1/13]^\top$. Determine α.

4. **Bank account.** In a bank account with monthly interest i you deposit a fixed amount $\bar{u}$ each month, starting from the amount $x(0) = 0$. In T months you wish to reach the amount C. Determine $\bar{u}$.

5. **Sudoku** Put the positive numbers α and β in the matrix A and leave 0 elsewhere in order to have a reachable and observable system.

$$A = \left[\begin{array}{c|c|c} & & \\ \hline & & \\ \hline & & \end{array}\right] \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

6. **Switching system**. Consider the linear system $\dot{x}(t) = A_{\sigma(t)}x(t)$, $\sigma \in \{0,1\}$ with given matrices A_1 and A_2. Switching is periodic: $\sigma = 1$ for T_1 seconds then commutes and $\sigma = 2$ for T_2 seconds and so on. Initially at time $t = 0$, $A = A_1$. (A_1 on $[0, T_1)$, then A_2 $[T_1, T_2)$, then A_1 $[T_1 + T_2, T_1 + T_2 + T_1)$ ...). Given the initial state $x(0)$, determine the evolution of the state $x(t)$ at the switching instants $k(T_1 + T_2)$ and $k(T_1 + T_2) + T_1$.

7. **Find the mass values**. Consider the oscillating system with no damping and with mass and stiffness matrices

$$M = \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} \quad K = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$$

Assume that the proper frequencies ω_1 and ω_2 are measured experimentally. How can we determine the masses M_1 and M_2? (write proper equations). Solve numerically the exercise for $\omega_1 = 1$ and $\omega_2 = \sqrt{2}$

8. **Analysis** Consider the linear continuous–time system.

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \omega & -1 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

for which values of ω the systems is asymptotically stable? for which values of ω the systems is reachable? for which values of ω the systems is observable?