System and Control Theory - Exercitation - January 20, 2017
Some of the proposed problems could be unsolvable. If so explain why.

1. **Undamped pendulum** Consider the system:

$$\ddot{y}(t) = -sin(y(t)) + u(t)$$

Write the system in the state variables $x_1 = y$ and $x_2 = \dot{y}$. Find an observer for the linearized system assuming that y is the output. Assign eigenvalues $\lambda_{1,2} = -2 \pm j2$.

2. **Undamped pendulum optimal control** Consider the previous system and find the state feedback optimal control for the linearized system according to the index

$$J = \int_0^\infty (3 * y(t)^2 + u(t)^2)dt$$

3. **Level measurements with noise** Consider the system

$$\begin{aligned} \dot{x}(t) &= d(t) \\ y(t) &= x(t) + w(t) \end{aligned}$$

where d and w are noises. Find the Kalman filter for this problem (take the weights in the Riccati equation as you like).

4. **Discrete-time model identification** Consider the system:

$$y(k+1) = ax(k) + bu(k)$$

where $y(0) = 0$, $y(1) = 1$, $y(2) = 2$ and $y(3) = 3$ are measured and $u(0) = u(1) = u(2) = 1$ are imposed. Find the least–square optimal value of a and b.

5. **Robust Lyapunov function** Consider the linear system $\dot{x} = Ax$

$$A = \begin{bmatrix} -1 & 1 \\ -1+\delta(t) & -1 \end{bmatrix} \quad \text{and} \quad P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Show that $x^\top Px$ is a Lyapunov function for the system with $\delta = 0$. Then find a bound p such that for $|\delta(t)| \leq p$ the system is robustly stable.

6. **Minimum time** Consider the linear system $\dot{x} = Ax + Bu$, $|u| \leq 1$

$$A = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

How can we find the control drives the state $x_1(0) = 1$ and $x_2(0) = 1$ to $\mathbf{0}$, $x_1(T) = 0$ and $x_2(T) = 0$ in minimum time T?

7. **Minimum effort control** Consider the problem for the system $x(k+1) = Ax(k) + Bu(k)$, with $m = 1$ (n states and single input). Consider the problem of minimizing $\sum_{k=0}^{N-1} u(k)^2$ under the constraint $x(N) = 0$ and $|u| \leq \omega$. Write this problem as an optimization problem

$$\begin{array}{rcl} \min & & f(\theta) \\ \Phi\theta & = & \Gamma \\ \theta^- \leq & \theta & \leq \theta^+ \end{array}$$

i.e. write the unknown vector θ, the function f, the matrices Φ and the vectors Γ, θ^- , θ^+.

8. ***LR* parameters** Consider the transfer function $F(s) = 1/(Ls+R)$. From two measurements we have $F(0) = \alpha$, $F(j\omega) = \beta - j\gamma$ (ω given). Find L and R (least square solution).