System and Control Theory - Exercitation - October 29, 2018
Some of the proposed problems might have no solution. If so, explain why.

1. **Stady-state amplitude** Consider the system with transfer function

$$W(s) = \frac{1}{s^2 + 2s + 5}$$

Consider the input $\cos(\omega t)$. At which frequency among $\omega = 0,\ 2,\ 4$, the steady state solution has the greatest amplitude? How much is it?

2. **Find the transfer function.** The frequency response amplification of an asymptotically stable system is

$$\frac{1}{\sqrt{(2-\omega^2)^2 + 9\omega^2}}$$

Find the transfer function.

3.

4. **Reachability-observability** Which are the conditions on the parameters such that the following systems is reachable and observable?

$$A = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \quad B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \quad C = \begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix}$$

5. **Zero transfer function**

$$A = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} \quad B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \quad C = \begin{bmatrix} c_1 & c_2 \end{bmatrix}$$

Do there exist **non-zero** vectors B and C for which the transfer function $W(s) \equiv 0$ identically?

6. **Limit vector.** Consider the matrices A and B and $u \equiv 1$.

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\kappa & -3 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

For which values of κ the state converges to a constant vector for any initial condition? which is the vector?

7. **Airplane.** Consider the approximate (linear) equation of an airplane

$$\ddot{h}(t) = \alpha\theta(t) - mg, \quad \ddot{\theta}(t) = \beta\phi(t)$$

where $\phi(t)$ is the control g is the gravity, α, β positive constants. Write the state equations and analyze stability.

8. **Markov.** Find a Markov chain matrix A of dimension 2 such that the asymptotic probability distribution is $\bar{x} = [1/3 \ \ 2/3]^\top$.

9. **Quick Water Pouring.** Given the first order system $\dot{x}(t) = u(t)$ and the initial state $x(0) = 0$ (empty glass) find an input $|u(t)| \leq 1$ such that $x(1) = 2$ (full glass).

10. **Physical units.** Given the systems (A, B, C), we chance physical units to the state variables (not to the input and output). Tell which of the following remain unchanged: (a) matrix A, (b) matrix B, (c) matrix C, (d) the transfer function, (e) the modes.