A dynamical mobility model

Beyond the *roboticle metaphor* : individual and collective autonomous robots moving around. How to monitor and control them.

Suggestions for a *Robotics Course*.

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A Control View of Robotics

- Robots are **intelligent machines** designed to help humans to satisfies their needs and desires.
- Their intelligence stems from

  their environmental monitoring ability to acquire useful information to control motion, object manipulation and all other activities needed to accomplish the assigned task.

Robot Design Issues

- what sensors gather and how effectors act
- which operating model is implied
- how the governor’s unit provides the observed behavior through its physical body
- how sensorimotor coordination should be properly designed

Sensorimotor Coordination

- behaviour-based design
- close connection between sensing and acting
- acquisition from the environment of all the relevant information to carry out the task

Sensor Grounding

- data operate at reactive level
- only a small number of them cross the deliberative fence
- it converts the signal nature of sensing into an appropriate symbolic representation
- question: *how long we can maintain sensory information at reactive level as opposite to the deliberative level*?

Subsymbolic Approaches

- Subsymbolic approaches usually are faced with
  - processing a large number of data, which
  - come from many different sources such as sensors or even instrumental devices and which
  - are required to feed effector apparati with a short latency time.

Role of Sensor Information

- sensors usually provide information about the environment in a form which depends on the physics of the interaction
- it is maintained at the physical level as a metaphor of the events observed in the environment
- Rationale: *identify some physical properties to be assigned to the objects recognized in the environment so that they are easily dealt with in order to drive the task execution and completion of the robot*.

Substratum Hypothesis

- *The close connection between the motor mechanism and the perceptual paradigm of a behaviour* means monitoring the physical objects involved in the task execution by
  - identifying their relevant properties
  - using an appropriate physical metaphor
  - grounding sensor data to a **substratum** *underlying both perception and action activities by driving the flow of information accordingly*

Substratum Definition

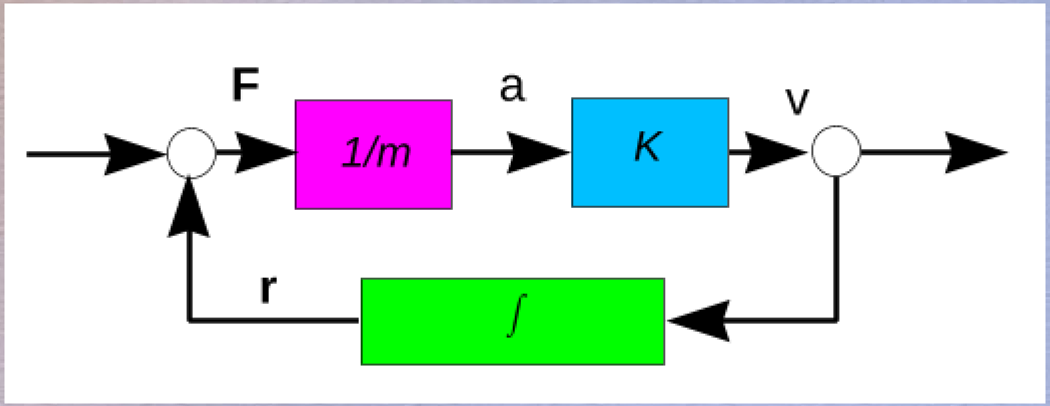
- Abstracting what sensor grounding operatively means, we can say
  - *the substratum is the semantic of the interaction*
- or, more evocatively,
  - it denotes the symbol grounding on which the robot governor's unit operates

Implementing Metaphors

- pure mechanics metaphor
- electrical metaphor
- fluid dynamic metaphor
- gas diffusion metahor
- kinetic theory of gas metaphor
- thermodynamic metaphor
- temperature metaphor

Pure Mechanics Metaphor

- It refers to the mechanical body interaction based on the well-known laws of *kinematics* and dynamics



- *The mass m acts as a perceptual schema* [in according to Arkin's terminology] *which senses a force* ***F*** *from the environment, translating it into the acceleration* ***a****. The motor schema K integrates it into the velocity* ***v*** *observed by the environment as movement* ***r***.
- Substratum: Hamiltonian or Lagrangian Mechanics

Electrical Metaphor

- it is very popular and it is commonly used in many motion planning applications [Latombe, La Valle]
  - *to implement behaviors as a couple of perceptual and motor schemas* [Arkin]
  - *to understand robot motion in an active fashion* by
- *assuming objects to be electrical charged particles as the robot is, but with opposite sign, so that the electrical field becomes the motion engine*.
- underline substratum: electrostatic field

Fluid Dynamic Metaphor

- within the navigation task
  - interpret the robot motion as it comes from the particle streaming in a fluid.
  - fulfill the environment disseminated of many obstacles with a fluid flowing from the home to the target position,
- so *the robot follows one of its streamlines, since they lap the obstacles without entering their contours*.
- underline substratum: velocity vector field

Gas Diffusion Metahor

- Used with the task of maintaining a group formation
  - *it forces each individual of the colony to adjust its relative position inside the group*
  - *it supports sensors to evaluate increasing/decreasing distance from near teammates (noise disturbance, entropy)*
- robots change directions and follow other robots by merely using the collision stimulus as an input signal
- underline substratum: gas diffusion process

Kinetic Theory of Gas Metaphor

- used in robot groups to
  - *analyze collision and implement sweep motion in a bounded region* [Jantz, Kerr]
  - *monitor suitable thermodynamic parameters to identify macroscopic states like random, periodic or deadlock motion* [Kinoshita].
- underline substratum: gas kinetic energy

Thermodynamic Metaphor

- use the thermodynamic approach to formulate the relationship, as it appears at macroscopic level, between
  - the effect of the behavior of a single robot and
  - both its diffusion and merging in a swarm
- *Each robot is interpreted as a gas particle, and the energy exchange is considered through thermodynamic equations* [D'Angelo, Funato]
- underline substratum: Gibbs' free energy

Temperature Metaphor

- thermodynamic extension of the fluid dynamic metaphor [D'Angelo, Pagello] which
  - *takes into account the role of the temperature gradient* as
  - *main source of the fluid streamlines*
  - *interprets stigmergy as a consequence of convective motion of the fluid*
- <u>underline substratum</u>: temperature gradient

Mechanics/Dynamics trade-off

- *Autonomous robots are physical machines which embody the intelligence of their movements. They can be interpreted as*
  - mechanical agents, namely, *pure physical devices which sense environmental forces to which they response accordingly to the physical laws*
  - dynamical agents, namely, *controlled devices equipped with suitable sensors and effectors to observe external events to which they properly response accordingly to the task to be executed*
- **Rationale**: *provide robots with an appropriate substratum as a result of a metaphor which better than others summarizes the environmental properties to be monitored to accomplish the given task*

Robot as Mechanical Agents

- They are mechanical devices governed by the laws of mechanics, such as,
  - *position and velocity binding* $\vec{v} = \frac{d\vec{r}}{dt}$
  - *Newton's second law* $\vec{F} = m\frac{d\vec{v}}{dt}$
  - *gradient based forces* $\vec{F} = -\,gradU$
- You should notice that the law of the conservation of energy is implied; in fact, from $\left(\vec{F} + gradU\right)\cdot d\vec{r} = 0$ it follows

$$\frac{1}{2}mv^2 + U = E$$

Roboticles

- Simplified model of a moving robot:
  - *implement the substratum by interpreting sensor signals as they come from a really observed fluid*
  - *immerse the robot so that its trajectory covering is driven by the streamline passing through its current position*
- From a mathematical point of view the fluid is completely determined by its velocity field, which is supposed to be continuous.

Roboticle Model

- Based on the fluid dynamic metaphor, the model assumes a robot to be an autonomous mobile platform moving on a plain surface, by
  - *collapsing the robot into a streamline point*, and
  - *specifying the action of the streamline by the means of a well specified dynamical law which takes the form of a set of differential equations*
- Thus, the so called mobility assumption, expresses *how the velocity field acts on the mobile platform*

$$\dot{x} = u(x, y, \lambda)$$
$$\dot{y} = v(x, y, \lambda)$$

Robots as Dynamical Agents

- Remember that a mechanical agent obeys to the law of the position velocity binding, which states $d\vec{r}=\vec{V}\,dt$, yielding to

$$\vec{\boldsymbol{V}}\cdot d\vec{\boldsymbol{r}}=V^2\,dt \qquad \vec{\boldsymbol{V}}\times d\vec{\boldsymbol{r}}=0$$

- On the contrary, dynamical agents obey to the so called embodied assumption

$$d\vec{\boldsymbol{r}}=\vec{\boldsymbol{V}}\,dt+\delta\vec{\boldsymbol{r}}$$

Roboticle-fluid Interaction

- We have referred the robot speed and steering, denoted with **V** for short, to the fluid velocity field, thus
  - the short displacement d**r** really observed is due to the superposition of the robot command **V**dt and the response δ**r** of the environment,
  - whose possible interactions are
    - $\vec{V} \cdot d\vec{r}$, which *characterizes the motion along a streamline*
    - $d\vec{r} \times \vec{V}$, which *provides the platform with the ability to change streamline*

Situated Dynamical Law

- *A robot is said situated if it acquires information about its environment only through its sensors while it is interacting with the environment*
- Notice that $\dot{x}$ and $\dot{y}$ are the velocity components of the robot as they were observed by the environment in opposite to the velocities u and v due to the fluid motion, directly applied by the robot governor's unit to its servos.

Environment Response

- If we denote with **θ** the robot current direction, the environmental response δr takes the form

$$\delta \vec{r} = \hat{\boldsymbol{\theta}} \times \delta \vec{r} \times \hat{\boldsymbol{\theta}} + (\delta \vec{r} \cdot \hat{\boldsymbol{\theta}}) \hat{\boldsymbol{\theta}}$$

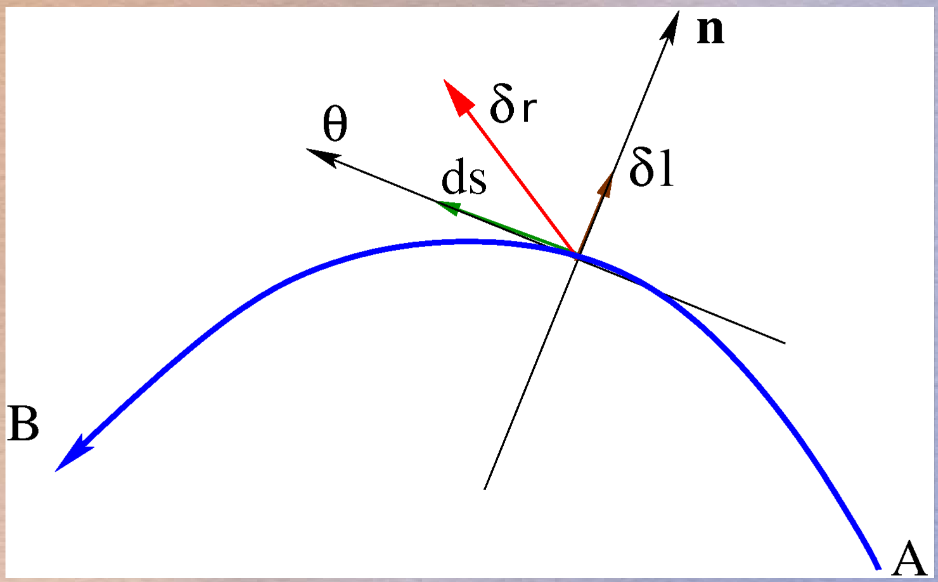
- that is, by introducing the versor **k** normal to the operating plain,

$$\delta r = \hat{\boldsymbol{\theta}} \times \hat{\boldsymbol{k}} \, \delta l + \delta s \, \hat{\boldsymbol{\theta}}$$

- where the latter component lies along the trajectory and the former is normally directed from left to right. Their definitions are

$$\delta l = (\delta \vec{r} \times \hat{\boldsymbol{\theta}}) \cdot \hat{\boldsymbol{k}} \qquad \delta s = \delta \vec{r} \cdot \hat{\boldsymbol{\theta}}$$

Trajectory Covering



- In the figure appears the versor **n** = **θ** x **k** so that the mobile frame of reference, centered on the roboticle, is identified by **n**, **θ**, **k**
- Notice that **k** = **n** x **θ**, namely, **n**, **θ**, **k** defines a *mobile frame of reference rotated of an angle θ with the respect to the fixed frame reference* which is easily inferable from the figure.

Robot Displacement

- Putting together the action of the robot control unit and the environmental response we can formalize the trajectory covering by

$$d\vec{\boldsymbol{r}} = ds\,\hat{\boldsymbol{\theta}} + (\hat{\boldsymbol{\theta}} \times \hat{\boldsymbol{k}})\,\delta l$$

- where the following identity has been used

$$ds = V dt + \delta s$$

Effort and Percept

- Since the are two possible robot-environment interactions, the trajectory covering is governed by the quantities
  - effort, *which stems from the trajectory covering due to the velocity* **V**, *provided by the robot control to its effectors and appearing as displacement* δ**r**

$$Vds = u(x,y,\lambda)dx + v(x,y,\lambda)dy$$

  - percept, namely, the amount of sensor data which results in moving the robot from a streamline to a near one

$$\delta P = v(x,y,\lambda)dx - u(x,y,\lambda)dy$$

Dissipative Function

- *If we require that the delivered effort doesn't depend on any particular covered trajectory, its differential form must be exact*; namely,
  - it must exist a scalar function *F*, termed dissipative function, such that

$$\vec{V} = \vec{V}_p = -grad F$$

  - and the velocity field $\mathbf{V}_p$ is said dissipative.
- It should be noticed that the cartesian components of $\mathbf{V}_p$ are

$$u_p = -\frac{dF}{dx} \quad v_p = -\frac{dF}{dy}$$

Internal Momentum

- *If we require that the perceptual information doesn't depend on any particular line connecting two different streamlines, it follows that*
  - there exists a scalar function *M*, termed internal momentum, such that

$$\vec{V} = \vec{V}_c = \hat{k} \times gradM = -rot\left(M\,\hat{k}\right)$$

  - and the velocity field $\mathbf{V}_c$, is said conservative.
- It should be noticed that the cartesian components of $\mathbf{V}_c$ are

$$u_c = -\frac{dM}{dy} \quad v_c = \frac{dM}{dx}$$

Fluid Dynamic Substratum

- *The most general velocity field, modeling the hypothetical fluid flowing through the environment as substratum, must be specified by the both components* so that we can write

$$\vec{V} = \vec{V}_p + \vec{V}_c = -gradF - rot\,\vec{M}$$

- where **M** = M **k** is a vector with just only one component, directed normally to the working plain *xy*.
- In such a case the velocity field is completely determined by two scalar functions, one working as a scalar potential and the other acting as a vector potential.
- Using the cartesian components we can write

$$u = u_c + u_p \qquad dM = v_c\,dx - u_c\,dy$$

$$v = v_c + v_p \qquad -dF = u_p\,dx + v_p\,dy$$

Perception and Action

- *By substituting the cartesian components of the roboticle velocity for the definitions of δP and Vds it yields to*
  - percept

$$
\begin{aligned}
\delta P &= vdx - udy = \left(v_c + v_p\right)dx - \left(u_c + u_p\right)dy \\
&= v_c dx - u_c dy + v_p dx - u_p dy = dM + \left[v_p dx - u_p dy\right]
\end{aligned}
$$

  - effort

$$
\begin{aligned}
Vds &= udx + vdy = \left(u_c + u_p\right)dx + \left(v_c + v_p\right)dy \\
&= u_p dx + v_p dy + u_c dx + v_c dy = -dF + \left[u_c dx + v_c dy\right]
\end{aligned}
$$

Committing Effort and Committed Perception

- *The committing effort δL defines how much effort can be used by effectors*

$$\delta L = v_p dx - u_p dy$$

- *the committed perception δQ quantifies the effective level of perceptual energy used by effectors*

$$\delta Q = u_c dx + v_c dy$$

Sensing and ...

- *The difference between any variation of perceptual flow and committing effort, which affects the dissipative part of the motion, is stored as agent internal momentum as a motion source*

$$\delta P - \delta L = dM$$

...and Acting

- *The effort Vds delivered to drive robot effectors is due to the attitude to convert the committed perception, which comes from the conservative part of the motion, partially compensated by decrementing the dissipative function*

$$Vds - \delta Q = - dF$$

Perception-Action Coupling

- The internal momentum and the dissipative function are general quantities by which the robot governor's unit stores information about the environment
- In this sense we can make explicit the coupling between perception and action

$$\delta L = g_{11} dM - g_{12} dF$$

$$\delta Q = g_{21} dM - g_{22} dF$$

Evaluation of Coefficients g_{ij}

- In fact, from the definition of *dM* and *dF*, by resolving with the respect to *dx* and *dy*, it yields to

$$dx = \frac{v_p\, dM - u_c\, dF}{u_p u_c + v_p v_c} \qquad dy = -\frac{u_p\, dM + v_c\, dF}{u_p u_c + v_p v_c}$$

- Thus, by substituting *dx* and *dy*, appearing in the definitions of δL and δQ, for the previously determined quantities, we obtain

$$\delta L = \frac{\left(u_p^2 + v_p^2\right) dM - \left(v_p u_c - u_p v_c\right) dF}{u_p u_c + v_p v_c} \qquad \delta Q = \frac{\left(v_p u_c - u_p v_c\right) dM - \left(u_c^2 + v_c^2\right) dF}{u_p u_c + v_p v_c}$$

Form of Coefficients g_{ij}

- From the preceding relations immediately follow

$$g_{11}=\frac{u_p^2+v_p^2}{u_p u_c+v_p v_c} \qquad g_{12}=g_{21}=\frac{v_p u_c-u_p v_c}{u_p u_c+v_p v_c} \qquad g_{22}=\frac{u_c^2+v_c^2}{u_p u_c+v_p v_c}$$

- Thus, if we define the dissipative $\mathbf{V}_p$ and conservative $\mathbf{V}_c$ velocities starting from their magnitudes, V_p and V_c, and directions, ψ and γ, respectively,

$$u_p=V_p\cos(\psi) \qquad u_c=V_c\cos(\gamma)$$
$$v_p=V_p\sin(\psi) \qquad v_c=V_c\sin(\gamma)$$

- we can rewrite the coefficients g_{ij} in a more useful form

$$g_{11}=\frac{V_p}{V_c\cos(\psi-\gamma)} \qquad g_{12}=g_{21}=\tan(\psi-\gamma) \qquad g_{22}=\frac{V_c}{V_p\cos(\psi-\gamma)}$$

Constraints on Coefficients gij

- The following properties stem from the preceding relations

$$g_{11}g_{22}-g_{12}g_{21}=1$$

$$g_{12}=g_{21}$$

Perception-Action Control

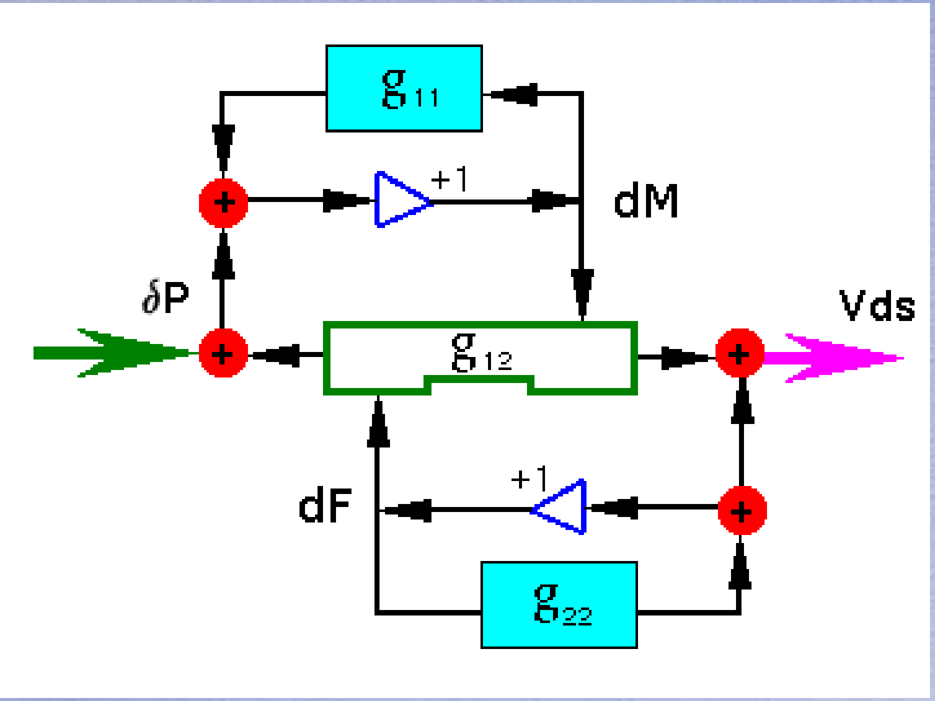
- The coupling between the percept δP and the effort Vds is given by substituting δL and δQ with their definitions

$$\delta P = \left(1 + g_{11}\right) dM - g_{12} dF$$

$$Vds = g_{12} dM - \left(1 + g_{22}\right) dF$$

Autopoietic Loop

- The graphical interpretation of the preceding relations is given by the following autopoietic loop
- *This model explains robot behavior by balancing perceptual information and motor commands with an appropriate amount of internal momentum and dissipation*



Functional View of Control

- A more attractive graphical interpretation can be easily obtained by rewriting the relations related to the perception-action control as follow

$$\frac{g_{21}}{1+g_{11}}\left[\delta P+g_{12}dF\right]=g_{21}dM \qquad \frac{g_{12}}{1+g_{22}}\left[g_{21}dM-Vds\right]=g_{12}dF$$

- Thus, by introducing the direct $\boldsymbol{K_f}$ and the inverse $\boldsymbol{K_b}$ gains with the definitions

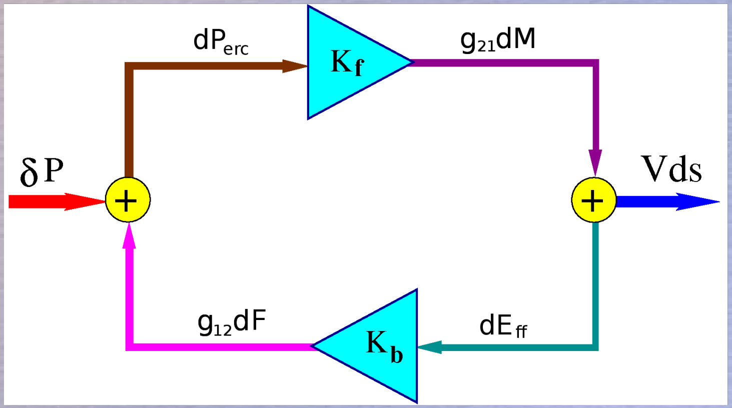
$$K_f=\frac{g_{21}}{1+g_{11}} \qquad K_b=\frac{g_{12}}{1+g_{22}}$$

- the preceding expressions take the form

$$K_f\left[\delta P+g_{12}dF\right]=g_{21}dM \qquad K_b\left[g_{21}dM-Vds\right]=g_{12}dF$$

Direct and Inverse Gain

- The last two equations give rise to the graphical interpretation of how roboticle control works sketched by the following figure



- where dP_{erc} and dE_{ff}, termed residual percept and effort, respectively, are given by

$$dP_{erc} = \delta P + g_{12}\, dF \qquad\qquad dE_{ff} = g_{21}\, dM - Vds$$

Autopoietic Loop Properties

- If we consider the definitions of residual percept and effort previously given, the functional relations expressing how the autopoietic loop works become

$$K_f\, dP_{erc} = dE_{ff} + Vds \qquad K_b\, dE_{ff} = dP_{erc} - \delta P$$

- which provide the amount of residual percept and effort flowing in the autopoietic loop

$$dP_{erc} = \frac{\delta P + K_b Vds}{K_f K_b - 1} \qquad dE_{ff} = \frac{Vds - K_f\, \delta P}{K_f K_b - 1}$$

- Thus, making the necessary substitutions, finally we obtain

$$g_{21}\, dM = \frac{K_f}{K_f K_b - 1}\left[\delta P + K_b Vds\right] \qquad g_{12}\, dF = \frac{K_b}{K_f K_b - 1}\left[Vds - K_f\, \delta P\right]$$

Loop Gain Evaluation

- A most useful computational method can be determined from the preceding relations. To this aim we need to substitute the coefficients g_{ij} appearing in the definitions of $\boldsymbol{K_f}$ and $\boldsymbol{K_b}$ with their trigonometric counterpart.
- Their final forms are very simple as it appears in the follow

$$K_f = \tan(\psi - \theta) \qquad K_b = \tan(\theta - \gamma)$$

- where θ is the current direction of the roboticle and, moreover,

$$g_{12} = g_{21} = \frac{K_f + K_b}{1 - K_f K_b}$$

Nominal trajectory

- *The dynamical law which drives the robot within its environment by properly interpreting all meaningful sensor information, can be viewed as an alternate method to specify the so called nominal trajectory*.
- The starting point is to consider the trajectory covered without any perceptual information. By so doing, we require $\delta P = 0$ along the whole trajectory. This condition implies

$$v(x,y,\lambda)\,dx - u(x,y,\lambda)\,dy = 0$$

- and the general solution depends on the ability to find out a scalar function which transform it into an exact differential form.

Total Energy

- Because the perceptual information δP is not an exact differential form we need to find out an integrating factor *1/E* such that

$$u\frac{dE}{dx}+v\frac{dE}{dy}=2\sigma E$$

- where

$$\sigma=\frac{1}{2}\left(\frac{du}{dx}+\frac{dv}{dy}\right)$$

Trajectory

- Thus, the *roboticle's dynamical law* can be also derived from the following relations

$$u = -E(x,y)\frac{dS}{dy} \qquad v = E(x,y)\frac{dS}{dx}$$

- The quantity *E(x,y)* is termed *total energy* whereas $S(x,y) = C$, with *C* some given constant, is the *implicit representation of the nominal trajectory* to be covered by the roboticle

Schema

- *Following the Arkin's point of view the perceptual space of a robot is interpreted by the means of potential fields*.
- The dissipative function *F* in the roboticle model plays the *same role as in Arkin's schema* because it represents *the attitude of a mobile robot to move towards an attractor or run away from a repulsor*.
- On the contrary, the internal momentum *M* provides the disegner triggering *robot control with a more adattive refinement of its moving*.

Implementing Schema

- The starting point is based on the relations appearing below

$$\dot{r} = -\frac{dF}{dr} = V\cos(\theta - \phi)$$

$$r\dot{\phi} = \frac{dM}{dr} = V\sin(\theta - \phi)$$

- expressing the velocity components in a polar frame of reference

Total Energy's Law

- The application of the total energy law yields to the relation

$$EdS = \frac{dM}{dr} + r\,\frac{dF}{dr}\,d\phi$$

Total Energy and Trajectory

- More specifically we can obtain the trajectory covered by the roboticle and also the total energy which appears *to depend on the changing rate of the dissipative function*

$$E = r\frac{dF}{dr}$$

$$dS = d\phi + \frac{M'(r)}{rF'(r)}$$

Move to Goal

- This motor schema assumes a linear increasing dissipation from the target

$$F = cr \qquad M = 0$$

- So, applying the roboticle's dynamical law

$$V\cos(\theta - \phi) = -c \qquad V\sin(\theta - \phi) = 0$$

- Arkin’s formulas follow immediately

$$V = c \qquad \theta = \phi + \pi$$

Avoid static Obstacles

- The dissipative function and the internal momentum take the following form

$$F = \frac{1}{2\tau}(r-a)^2 \qquad M = 0$$

- The roboticle dynamical law comes from the application of partial derivative operators

$$V\cos(\theta-\phi) = -\frac{r-a}{\tau} \qquad V\sin(\theta-\phi) = 0$$

- and, then,

$$V = \frac{a-r}{\tau} \qquad \theta = \phi$$

- The well-known Arkin's formulas stems from the introduction of the average speed

$$c = \frac{1}{a-r_c}\int_{r_c}^{a}\frac{(a-r)\,dr}{\tau} = \frac{a-r_c}{2\tau}$$

- So we have the *required relations*

$$V = 2c\,\frac{a-r}{a-r_c} \qquad \theta = \phi$$

Turn Around

- In this specific case we take a quadratic internal momentum and a null dissipative function

$$F=0 \qquad M=\frac{\omega_0}{2}r^2$$

- yielding to the following dynamical law

$$V\cos(\theta-\phi)=0 \qquad V\sin(\theta-\phi)=\omega_0 r$$

- *The Arkin's formulas for the motor schema are as they appear below*

$$V=\omega_0 r \qquad \theta=\phi+\frac{\pi}{2}$$

Example of Vehicle Control

- *Let us consider an autonomous vehicle moving along a straight line on a plane surface where some distinguishing objects can be easily recognized, such as pheromones delivered by the individuals of an ant system*.
- In the roboticle framework such a situation can be depicted by the following *F* and *M* functions

$$F = \frac{1}{2\tau}\left(x^2 + y^2\right) \qquad M = \frac{\omega_0}{2} x^2$$

Intuitive Meaning

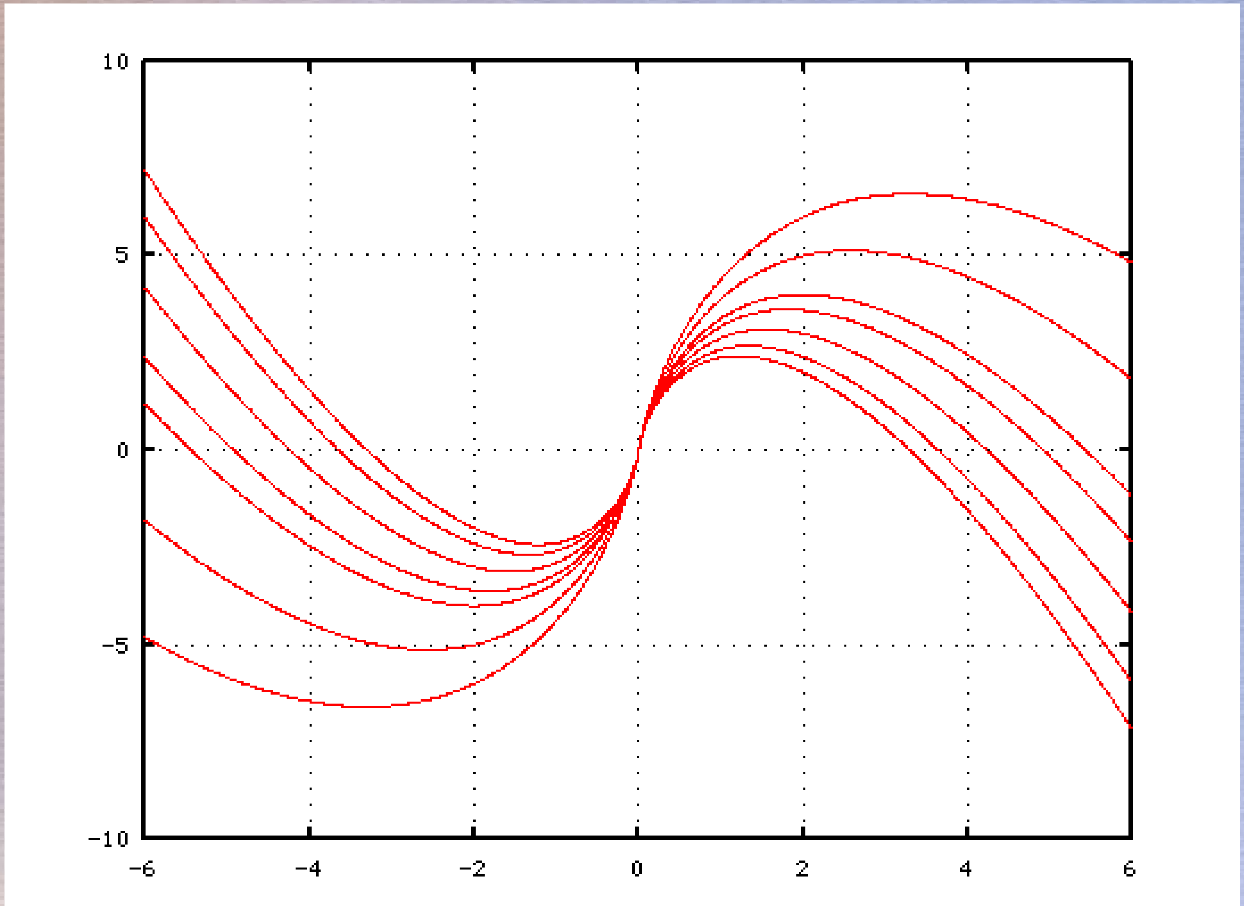
- *In the spirit of Arkin's schemas, their intuitive meaning is that the conservative component drives the vehicle along a straight line whereas the dissipative one causes the vehicle approaching the object*.
- The roboticle model yields to the following total energy *E* and implicit trajectory S(x,y) description

$$E = \frac{x^2}{\tau} \qquad S = \frac{y}{x} + m \log \frac{|x|}{a}$$

- where a is a given constant expressing the unit length as base of reference

Expected Behavior

- *The* *nominal trajectory* *covering of the assigned control is depicted by the following figure*



Making Components Explicit

- Robot control is built by driving speed and steering whose conservative and dissipative components are those appearing below,

$$u_p = -\frac{x}{\tau} \qquad v_p = -\frac{y}{\tau} \qquad u_c = 0 \qquad v_c = \omega_0 x$$

- using cartesian components; on the contrary, a polar frame of reference shows the following components for the same quantities

$$V_p = \frac{r}{\tau} \qquad \psi = \pi + \phi \qquad V_c = \omega_0 r \cos(\phi) \qquad \gamma = \frac{\pi}{2}$$

- *where r and ϕ are the distance and the direction of the robot with the respect of the frame centered on the object*

Direct and Inverse Gains

- *The autopoietic loop drives the robot by triggering its movement towards the observed object using the following direct and inverse gains*

$$K_b = \frac{1}{m - \tan(\phi)} \qquad K_f = \frac{m K_b}{K_b - \tan(\phi)}$$

- Remember that the current direction of the roboticle is given by

$$\tan(\theta) = \frac{v_p + v_c}{u_p + u_c} = \frac{y - mx}{x} = \tan(\phi) - m$$

How Autopoietic Loop works

- *The vehicle is moving under the control of a convenient amount of delivered commands*:
  - extract and convert part of them into information
  - filter by the inverse gain **K**$_b$.
  - sum up with the incoming *sensor data*
  - filter again through the direct gain **K**$_f$

How Autopoietic Loop works

- But the direct gain $\mathbf{K}_f$ depends on the inverse gain $\mathbf{K}_b$ accordingly to

$$K_f = \frac{m K_b^2}{K_b^2 - m K_b + m}$$

- *Thus the overall robot control only depends on the constant $m = \omega_0 \tau$ expressing the relative weight of the conservative part over the dissipative one*.

Filtering Control

- The following figure shows how the direct gain K_f (x-axis) should depend on the inverse gain K_b (y-axis) in order to implement the required control