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Dynamics of the long jump

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Abstract

A mechanical model is proposed which quantitatively describes the dynamics of the centre of gravity (c.g.) during the take-off phase of the long jump. The model entails a minimal but necessary number of components: a linear leg spring with the ability of lengthening to describe the active peak of the force time curve and a distal mass coupled with nonlinear visco-elastic elements to describe the passive peak. The influence of the positions and velocities of the supported body and the jumper's leg as well as of systemic parameters such as leg stiffness and mass distribution on the jumping distance were investigated. Techniques for optimum operation are identified: (1) There is a minimum stiffness for optimum performance. Further increase of the stiffness does not lead to longer jumps. (2) For any given stiffness there is always an optimum angle of attack. (3) The same distance can be achieved by different techniques. (4) The losses due to deceleration of the supporting leg do not result in reduced jumping distance as this deceleration results in a higher vertical momentum. (5) Thus, increasing the touch-down velocity of the jumper's supporting leg increases jumping distance.

Keywords: Sports; Spring–mass model; Swing–mass; Leg lengthening; Angle of attack

1. Introduction

Running and jumping are two types of fast saltatoric movements, characterised by a series of alternating aerial and contact phases. The impact occurring during each contact phase serves to negate the vertical momentum. The flight phase is determined by the initial velocity vector of the centre of gravity at take-off and the gravitational acceleration.

The function of the leg in repetitive ground contacts at a constant energy level like in hopping or running is comparable to a spring as shown e.g. by Blickhan (1989), Alexander et al. (1986), McMahon and Cheng (1990) and Farley et al. (1993). Modelling the leg as a spring is suited to describe the landing if the body mass, the leg stiffness, and the initial conditions are known.

The spring–mass model is suitable to describe conservative systems. During the human long jump energy is in fact largely conserved. Nevertheless, due to the high running speed, the first so-called passive impact immediately after touch-down strongly influences the system dynamics. In the long jump this contribution accounts to about 25% of the total momentum and cannot be neglected.

Alexander (1990) proposed a two-segment model with an Hill-type extensor to predict optimum take-off techniques of the jumpers stance leg in high and long jumping. However, to cope with observed jumping distances unrealistic muscle properties had to be chosen. Even a detailed musculo-skeletal system with 17 segments including all important muscles (Hatze, 1981) does not describe the complete ground reaction force pattern in sufficient detail.

The understanding of body dynamics during landing or falling was significantly improved by the concept of wobbling masses introduced by Gruber (Gruber, 1987; Gruber et al., 1998). She showed that the different responses of soft tissues and hard skeleton to impacts are essential for predicting dynamical loads. In long jumping high impacts occur with forces up to 10 times body weight.

Our approach to long jumping is to describe the mechanics of the centre of mass and the mechanical function of the supporting leg using a 2D lumped parameter model with a minimum number of mechanical components. The action of the leg is described by a spring, the effect of soft

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Nomenclature

α	angle of the leg to the x-axis
c, d	constants in the nonlinear visco-elastic force function
ε	leg lengthening constant $\varepsilon = r_+/(\alpha_E - \alpha_0)$
F_G	ground reaction force (GRF)
g	gravitational acceleration
k	leg stiffness
k_{dyn}	generalised dynamic leg stiffness
ℓ	relaxed length of the leg (varies from r_0 to r_E)
λ	positional relationship, $\lambda = r_2(t_0)/r_1(t_0)$
m	total body mass
μ	mass ratio $\mu = m_2/m_1$
ν	exponent of the visco-elastic element
ω	natural frequency $\omega^2 = \mathrm{k\ m^{-1}}$
Δq	displacement of swing mass m_2 along r
r	leg length (distance between the c.g. and the ball of the foot)
r_+	leg lengthening $r_+ = r_E - r_0$
Δr	leg shortening $\Delta r(t) = \ell(\alpha) - r(t)$
Δs	tangential displacement of swing mass (m_2)
v	velocity
x	horizontal coordinate
y	vertical coordinate
Δy	displacement in y

Subscripts

0	refers to the instant of touch-down
1	refers to the proximal mass m_1
2	refers to the distal swing mass m_2
E	refers to the instant of take-off
MAX	refers to the instant of maximal leg shortening
q	refers to the displacement of the swing mass m_2 along r
r	refers to the orientation of the leg
s	tangential displacement of the swing mass (m_2)

tissues by the introduction of a visco-elastically coupled mass. Thereby, the influence of either initial conditions such as running speed and angle of attack (measured by video analysis) or model properties (like leg stiffness) on the jumping performance are investigated. The quality of the mechanical approach is judged by comparing the experimental force records with the results of the simulation.

2. Methods

2.1. Experiments

In training competitions in 1995 and 1996, 30 long jumps (distance: [5.49 ± 0.86 SD] m) of 18 male and female sport students (m = [75.1 ± 5.13 SD] kg, body height: [1.81 ± 0.06 SD] m) were filmed for later analysis with a VHS camera (50 half-frames/s). The vertical and horizontal ground reaction forces were recorded with a 3D force plate (IAT, Leipzig). Kinematic input parameters for the dynamic models were obtained by digitising the video sequences (APAS, Ariel) (Fig. 1).

2.2. The concept of leg stiffness

The leg length r is defined as the distance of the c.g. to the ball of the foot as the rotational centre of the system during the stance phase. The initial leg length r_0 and the final leg length r_E are generally not identical. Therefore, we introduce the leg lengthening parameter r_+ as the difference between both leg lengths:

$$r_+ = r_E - r_0. \tag{1}$$

The actual length of the relaxed leg $\ell(\alpha)$ during the contact phase is then defined in a linear approach by

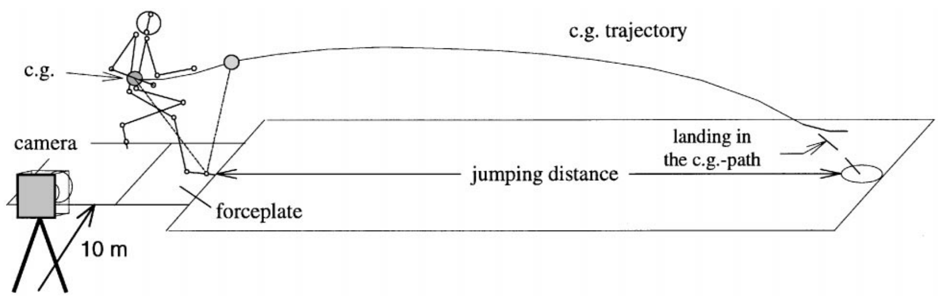

Fig. 1. Experimental set-up for the analysis of the last ground contact in long jumping. The body configuration defined by the positions of the joint markers was used to calculate the c.g. trajectory during the last ground contact and the flight phase. The jumping distance is estimated as the intersection point of the elongated ballistic curve (dashed line) and the ground.

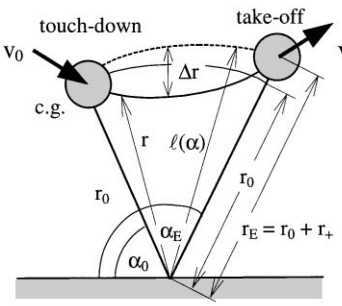

Fig. 2. Different leg lengths at touch-down and take-off can be described by the leg lengthening r_+. The actual shortening of the leg is Δr, $\ell(\alpha)$ denotes the length of the relaxed leg which increases with α.

(Blickhan et al., 1995):

$$\begin{aligned}\ell(\alpha) &= r_0 + r_+(\alpha - \alpha_0)/(\alpha_E - \alpha_0)\\ &= r_0 + \varepsilon(\alpha - \alpha_0)\end{aligned} \tag{2}$$

with ε constant, leg angle α at touch-down α_0, at take-off α_E, initial leg length r_0, change in r by r_+ during contact. Leg shortening $\Delta r(t) = \ell(\alpha(t)) - r(t)$ is zero at the instances of touch-down and take-off (Fig. 2).

The force exerted by the leg is related by the stiffness to the shortening of the leg Δr. The leg stiffness is defined by the ratio of the ground reaction force to the leg shortening Δr at maximum leg shortening:

$$k = F_{G,\,MAX}/\Delta r_{MAX}. \tag{3}$$

We generalise the instantaneous ratio in Eq. (3) as the dynamic leg stiffness:

$$k_{dyn}(t) = F_G(t)/\Delta r(t). \tag{4}$$

This definition is equal to Eq. (3) for the instant of maximum shortening of the leg and corresponds to the understanding in the literature (Farley and González, 1996).

2.3. Numerical methods

The mechanical models were built using standard software packages for dynamic simulations (ALASKA, Institut für Mechatronik; ADAMS, Mechanical Dynamics Inc.). Using given initial conditions, the parameter set was estimated which fulfils the least-square criterion between measured and calculated ground reaction forces.

For further parameter studies the models were translated into the equations of motion using the Lagrangian formalism, and solved by a numerical integration procedure using a fourth order Runge–Kutta algorithm (IDL, Creaso). The influence of initial and model specific parameters on the jumping result were investigated by varying parameter values. The model parameters were first adjusted visually and then calculated using a genetic optimization algorithm.

3. Model description and verification

A simple spring–mass system already predicts optimum strategies for the maximum jumping distance. For quantitative descriptions leg lengthening and mass distributions must be taken into account.

3.1. The leg as a linear spring

In a first approach to long jumping we will consider a model in which the leg operates as a spring. This gives basic insights into the influence of geometric parameters and the role of leg stiffness.

It is typical that the ground reaction force during the take-off phase shows a passive and an active peak (Fig. 3). The derived dynamic leg stiffness $k_{dyn}(t)$ has a first peak during the passive phase followed by a relatively constant stiffness during the active phase up to the last 30 ms before take-off.

Neglecting the passive peak, we can use a simple spring–mass system (Fig. 4) to describe the functionality of the contacting leg during flexion under the assumption of energy conservation. The equations of motion are (Blickhan, 1989):

$$\ddot{x} = x\omega^2\left(\frac{\ell}{\sqrt{x^2 + y^2}} - 1\right), \tag{5a}$$

$$\ddot{y} = y\omega^2\left(\frac{\ell}{\sqrt{x^2 + y^2}} - 1\right) - g, \tag{5b}$$

where ω is the natural frequency of the system with $\omega^2 = \mathrm{k\ m^{-1}}$. The relaxed spring length ℓ corresponds to the initial leg length r_0 which is in this first approach equal to final leg length r_E.

Since the vector of the landing velocity in long jumping has usually only a small vertical component ($|v_{0,Y}| < 1\ \mathrm{m\ s^{-1}}$), it is sufficient to consider the horizontal approach speed $v_0 = v_{0,X}$. For a given speed the

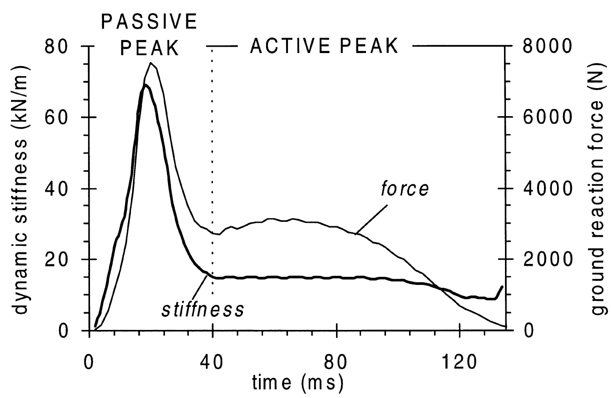

Fig. 3. Experimental result for the ground reaction force F_g and the instantaneous leg stiffness k_{dyn} as a function of the time.

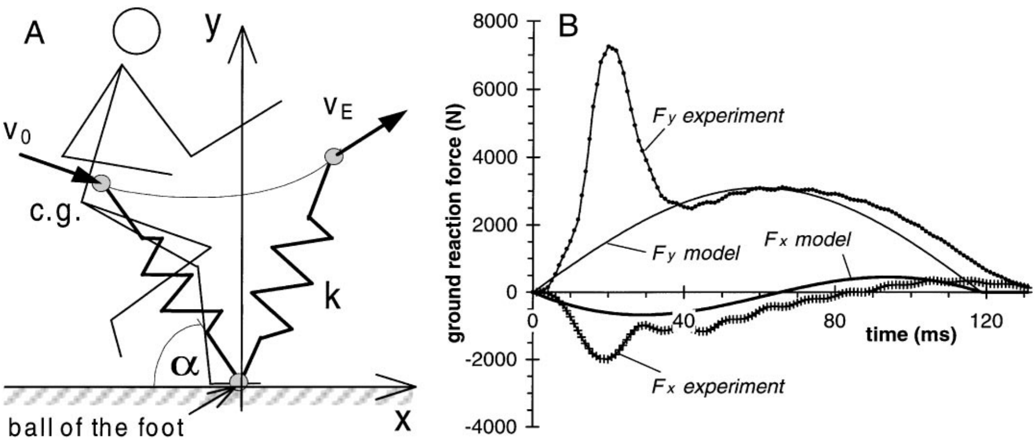

Fig. 4. (A) Schematic drawing showing the planar spring–mass model. The leg spring is defined by the stiffness k. The angle α describes the orientation of the leg with respect to the ground. (B) The model reflects a part of the measured ground reaction forces. The passive peak is missing and the active peak is either to short or to high.

influence of the angle of attack α_0 and the leg stiffness k on the jumping distance can be studied (Fig. 5A).

There is an optimum in jumping distance for a proper angle of attack and the appropriate leg stiffness. At a lower angle of attack the loss in horizontal velocity will prevail the influence of a higher vertical velocity and the jumping distance decreases. A steeper angle leads to over running with a smaller vertical impact. This is a general feature observed in all models.

The influence of leg stiffness is comparable to that of the angle of attack: A stiffer leg leads to faster repulsion and thus at a lower angle of attack to a loss in horizontal velocity and jumping distance. In contrast, a softer leg cannot produce the necessary vertical impact.

A high vertical impact requires a sufficiently high product of the mean vertical ground reaction force and contact time. This is only possible if the leg stiffness achieves a certain minimum value. With a higher stiffness and a corresponding optimal angle of attack (that is steeper angles and shorter contact times) the jumping distance remains nearly constant. The better the jump the closer the values come to the range where almost maximum jumping distance can be achieved (Fig. 5A). These features have been observed in all models.

An increase in running speed (not shown in Fig. 5) leads to a shift of the predicted optimum angle of attack to flatter (smaller) angles but has almost no influence on the minimal leg stiffness necessary to obtain the optimal jumping distance.

3.2. Considering leg lengthening

The simple spring–mass model predicts a significantly shorter active peak than has been measured (Fig. 4). Extending the model by considering lengthening of the relaxed length (i.e., leg length when leg force is zero) during ground contact improves the predictions (Eq. (2), Blickhan et al., 1995). Leg lengthening results on average in a more compliant spring and thus in longer contact times. Note that in order to obtain a similar change in momentum leg lengthening calculated from the active peak force pattern must be less than the cinematographic estimates as long as the passive peak and the corresponding momentum is excluded in the model (Fig. 6B).

Introduction of the leg lengthening shifts the range of close to optimum jumps to larger angles of attack (Fig. 5D). The optimum itself becomes more pronounced and shifts to low stiffness. In general very long jumps require higher active forces (Fig. 5F) and moderate leg shortenings (Fig. 5E). Even elite jumpers are not able to produce the forces and leg compressions to achieve the predicted range of close to optimum operation.

3.3. Mechanical model for the passive peak

The passive peak in jumping occurs directly after touch-down of the foot. The measured force pattern can be described accurately when a representative mass is coupled with a nonlinear viscosity to the rigid frame of the spring leg. This mass represents the rigid skeleton and its deceleration during touch-down as well as the relative movement of the soft tissues (muscle etc.) with respect to the rigid frame. We use the following dependency to describe the coupling between soft and hard tissues in one direction (here Δy):

$$F = -(c\,\mathrm{sgn}(\Delta y) + d \cdot v_y) \cdot |\Delta y|^{v} \tag{6}$$

where c and d are constants, the exponent v is about 2.5–4.5, and sgn describes the Signum function

$$\mathrm{sgn}(\Delta y) = \begin{cases} 1 & \text{for } \Delta y > 0, \\ 0 & \text{for } \Delta y = 0, \\ -1 & \text{for } \Delta y < 0. \end{cases} \tag{7}$$

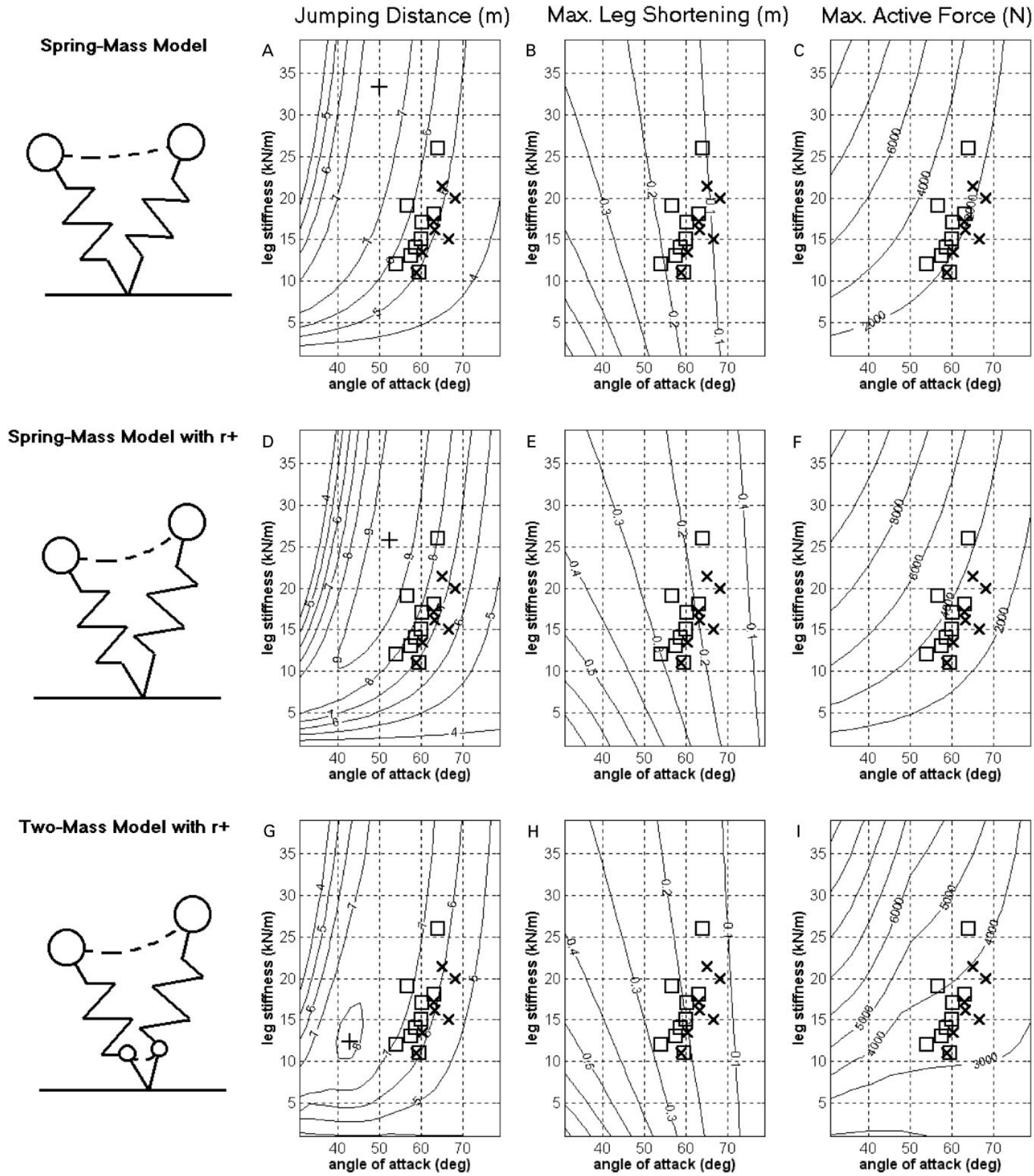

Fig. 5. Influence of angle of attack α_0 and leg stiffness k on jumping distance x_{JUMP} (A, D, G), maximum leg shortening Δr_{MAX} (B, E, H), and maximum active force $F_{\mathrm{MAX},r}$ (C, F, I) predicted using the simple spring–mass model (A, B, C), the spring–mass model with leg lengthening (D, E, F), and the two-mass model for the long jump (G, H, I). The remaining parameters have been chosen according to the mean values for the analysed jumps ($m = 75$ kg, $r_0 = 1.19$ m, $v_0 = 8.2$ m s^{-1}, see Table 1). The contour lines mark values of constant jumping distance x_{JUMP} in meters (A, D, G), maximum leg shortening Δr_{MAX} in meters (B, E, H), and maximum active forces $F_{\mathrm{MAX},r}$ in Newton (C, F, I).

The general dependencies are similar for the three models. The jumpers do not reach the optimum because their inability to generate high forces at large leg deflections. The spring–mass model with leg lengthening predicts longer jumps due to the absence of the passive peak (Fig. 6), + the predicted optimum for jumping distance, × jumps according to their angle of attack and calculated leg stiffness with $x_{\mathrm{JUMP}} < 5$ m, and □ jumps with $x_{\mathrm{JUMP}} > 6$ m.

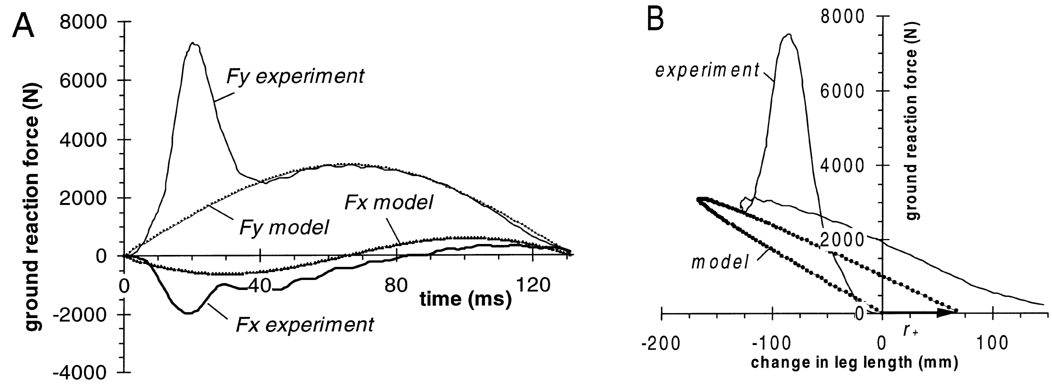

Fig. 6. (A) Ground reaction forces as predicted by the spring–mass model with leg lengthening. (B) Force-leg length relationship of the jumping leg. The measured leg lengthening r_+ cannot be reproduced with a spring–mass model with lacking passive peak when the active peak forces should be correct.

The selected visco-elastic coupling fulfils the following requirements

1. due to the nonlinearity the ground reaction force increases gradually within the first 10 ms,
2. the first peak is symmetric with time, and
3. the active and passive peak are clearly separated.

3.4. Assembling with the spring–mass system

A one-dimensional description of the vertical component of the ground reaction force during the long jump can now be obtained by combining the linear spring–mass model with the nonlinear visco-elastic system described above. The two force peaks are described by two systems in parallel with different dynamics.

A stack of two masses representing the body and the foot, respectively (Alexander et al., 1986; Özgüven and Berme, 1988) does not result in realistic dependencies. Nonlinear coupling is necessary. Both masses are effective masses taking the vertical projection and the bending of the leg into account. Depending on the orientation of the leg segments the masses of the leg and the body contribute. The mass of the foot is not sufficient to explain the transferred momentum during the passive impact.

We can separate the leg mass into the masses of the rigid bones, the foot, and of the soft tissues. If the coupling to the ground and the skeleton differs strongly, we would see several damped force oscillations during touch-down. This is not the case during the long jump.

Our experimental data can be described accurately with one distal mass and only one type of coupling. In this final model m_2 entails the foot, the skeleton and the wobbling masses distributed all over the body especially in the stance leg. Descriptions with realistic masses are only possible within a planar model.

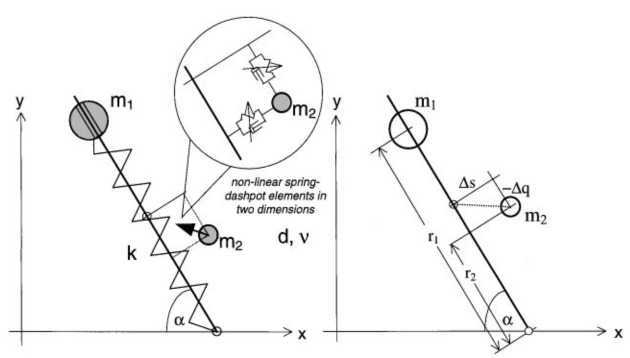

Fig. 7. The planar model for the long jump (schematic drawing with geometric parameters).

3.5. The planar model for the long jump

By taking planar movements of two distributed masses into account the model is able to describe the relationship between the horizontal and vertical force. Strategies of impact generation or avoidance can now be investigated. By actively hitting the supporting leg onto the board jumpers increase the passive peak and thereby vertical momentum and jumping distance.

In a simple planar spring–mass model the ground reaction force points always in the direction of the spring. During the actual long jump, however, significant deviations in the force direction can be observed within the first 40 ms. These can be attributed to the movement of the distal mass.

In our model the body mass (m_1) is supposed to glide on a massless rod. The orientation of this rod is defined by the position of the ball of the foot and the centre of the body mass. Similar to the simple spring–mass model, the body is coupled to the ground via a linear spring, representing the spring-like operation of the human leg (active peak). At a certain height, a second mass is fixed to the rod by nonlinear visco-elastic elements (Fig. 7).

The equations of motion are

$$\ddot{r}_1 = \dot{\alpha}^2 r_1 - \frac{k}{m_1}(r_1 - \ell(\alpha)) - g \cdot \sin\alpha \tag{8a}$$

$$\ddot{\alpha} = -\frac{1}{m_1 r_1^2}(r_2 \cdot F_s - \Delta s \cdot F_q) - \frac{1}{r_1}(2\dot{r}_1 \cdot \dot{\alpha} + g \cdot \cos\alpha) \tag{8b}$$

$$\Delta\ddot{q} = \Delta s \cdot \ddot{\alpha} + r_2 \cdot \dot{\alpha}^2 + 2\Delta\dot{s} \cdot \dot{\alpha} + (F_q/m_2) - g \cdot \sin\alpha \tag{8c}$$

$$\Delta\ddot{s} = -r_2 \cdot \ddot{\alpha} + \Delta s \cdot \dot{\alpha}^2 - 2\Delta\dot{q} \cdot \dot{\alpha} + (F_s/m_2) - g \cdot \cos\alpha \tag{8d}$$

with the nonlinear visco-elastic force functions:

$$F_q(\Delta q, \Delta\dot{q}) = -(c_q \cdot \mathrm{sgn}(\Delta q) + d_q \Delta\dot{q})\Delta q^{v_q} \tag{9a}$$

$$F_s(\Delta s, \Delta\dot{s}) = -(c_s \cdot \mathrm{sgn}(\Delta s) + d_s \Delta\dot{s})\Delta s^{v_s}. \tag{9b}$$

The properties of the element's coupling in radial and tangential direction are assumed to be the same ($c_q = c_s = c$, $d_q = d_s = d$, $v_q = v_s = v$). In addition to the parameters describing the mechanical properties of the simple spring-mass system (k, ε), the mass ratio $\mu = m_2/m_1$, the positional ratio $\lambda = r_2(t_0)/r_1(t_0)$, and the parameters describing the nonlinear visco-elastic elements must be identified (Eqs. (9a) and (9b)).

The simulations are calculated for given total mass, its touch-down velocity, given initial leg length and angle of attack. All other parameters including the initial conditions for the distal mass are estimated fitting the time course of the horizontal and vertical component of the ground reaction force (Figs. 8A, D and Table 1). Some of the parameters can be estimated independently using the experimental data: ε can be obtained from cinematographic data, k can be calculated by dividing the maximal force F_{MAX} during the active peak by the maximum leg shortening Δr_{MAX}.

Remaining systematic differences (Fig. 8B) occur because the point of centre of pressure shifts during the

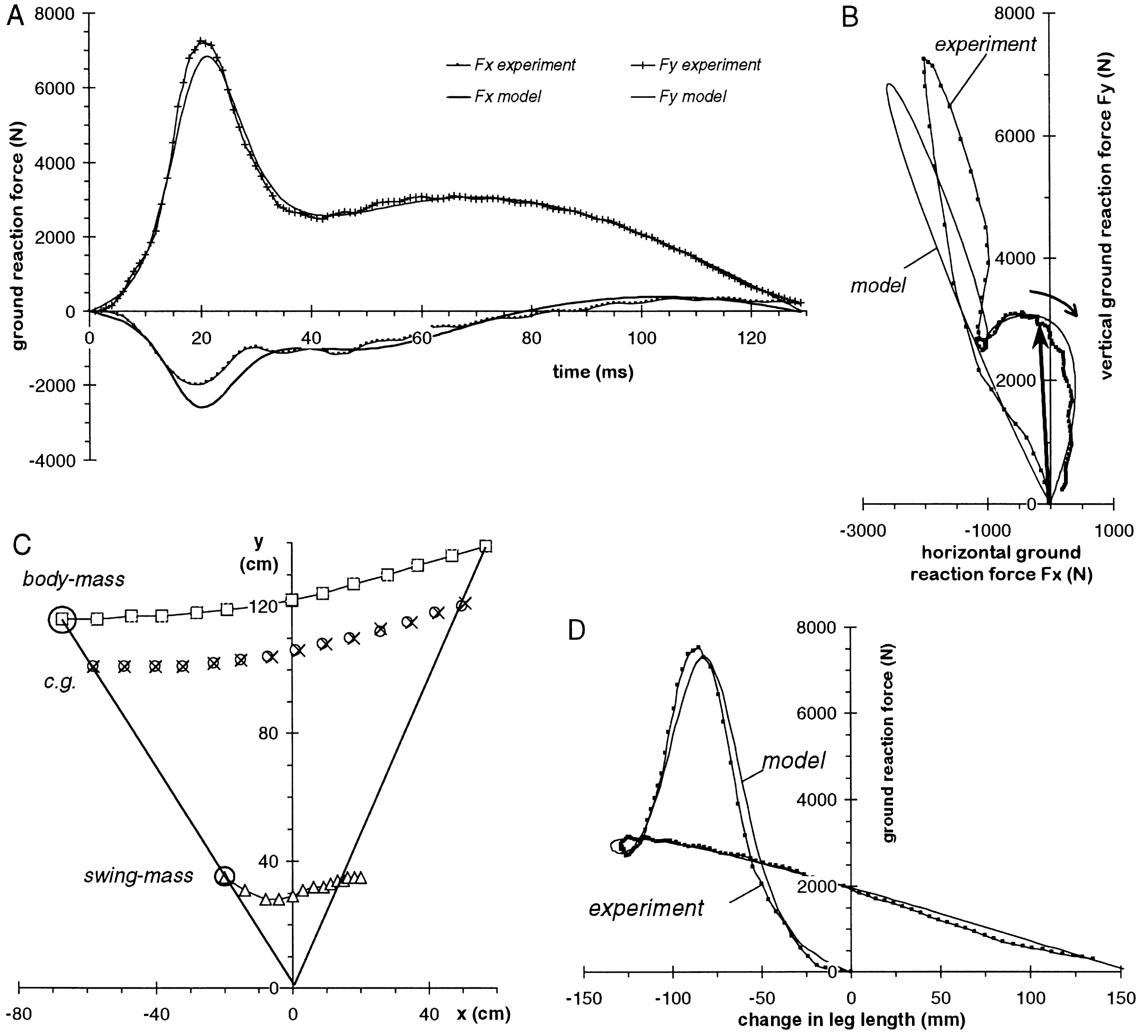

Fig. 8. Comparison between experimental and model results: (A) Ground reaction forces (GRF) in vertical F_y and horizontal F_x components as time series. (B) Tracings of the GRF in the $F_x - F_y$ plane. During heel strike (passive peak) the experimental GRF directs steeper than predicted by the model. (C) The positions of body-mass and swing–mass in the model defines the resulting c.g. (circles). Measured c.g.: crosses. (D) Force-leg length relationship of the jumping leg as simulated by the two-mass model and experimental result.

Table 1
System properties and initial conditions. Means and standard deviations (SD) are given for the experimental data of 30 trials and the corresponding numerical simulation

Symbol	Parameter	Model value (mean ± SD)	Experimental result (mean ± SD)	Units
k	Leg stiffness	14.6 ± 3.72	16.2 ± 3.80	kN m^{-1}
ε	Leg lengthening constant	3.36 ± 1.44	3.07 ± 1.28	10^{-3} m deg^{-1}
λ	Positional relation	0.252 ± 0.049	*No data available*	1
μ	Mass relation	0.269 ± 0.064	*No data available*	1
$\log d_2$	Non-linear spring-damper constant	7.45 ± 0.55	*No data available*	1
$v_2^{(0)}$	Initial velocity of swing mass	5.31 ± 0.59	*Foot*: 3.96 ± 1.40	m/s
$\alpha_{v2}^{(0)}$	Initial direction of v_2 (downwards)	32.7 ± 4.4	*Foot:* 30.05 ± 11.45	deg

ground contact which is not realised in the presented model.

The results are fairly stable with respect to the position and size of the second mass. The effective distal mass can be considered to be fixed at about 25% of the leg length from the ground (Fig. 8C) and amounts to approximately 27% of the body mass.

The parameters specifying the coupling to the skeleton are less sensitive as long as the basic properties described above are fulfilled. Interestingly, the predictions for the initial velocity of the distal mass are similar to the values obtained for the jumpers leg from video-graphic data (Table 1). The deviation can be explained by the fact that the average velocity of the leg is higher and less downward orientated than that of the foot.

Measured values for the leg stiffness come fairly close to the predicted optimum (Fig. 5G). The difference in general dependencies of the active force (Fig. 5I) is due to an increasing dominance of the passive peak for larger angles of attack at low leg stiffness.

4. Discussion

The presented mechanical model describes with a minimal set of parameters the dynamics of the long jump. As it is well known (e.g. Hay, 1993), the most influential factor for jumping distance is the running speed. The model predicts also that a certain angle of attack of the leg optimises jumping performance (Alexander, 1990). This optimum requires a relatively low minimal stiffness of the leg.

The controlled musculo-skeleton unit with its connective tissues behaves similarly to a spring with a certain stiffness. This stiffness and the leg shortening (Table 2) are not very different from that necessary for running (leg stiffness about 12–15 kN m^{-1}, leg shortening in running about 14 cm (Farley and González, 1996), in jumping: ca. 17 cm). To which extent this stiffness can be contributed to intrinsic properties of the participating tissues remains to be investigated.

For sufficiently high stiffness values many strategies with different angles of attack are possible to achieve distances which come close (up to 95%) to the theoretical maximum. Indeed, several techniques can result in the same jumping distance (Fig. 5). The proper strategy for an athlete depends on his ability to generate stiffness. Differences in stiffness can be compensated by changing the angle of attack of the leg. The kinetic energy of the runner dominates the energetics of the jump (Hay, 1993). To conserve this energy a quasi-elastic strategy is essential for a good performance. The leg largely redirects the movement.

Leg lengthening at take-off is mainly an active process. The runner places his leg with the knee slightly bent and takes off with a completely straight leg. This process — enhanced by lifting the arms and the swing leg — increases the distance over which acceleration takes place. It also compensates partly for the losses which necessarily occur during landing (passive peak).

It is impossible to avoid the impact during touch-down. Jumpers take, however, advantage of the passive peak generated during the impact by actively hitting the jumping leg onto the board. By this measure the passive peak, especially in the vertical component of the ground reaction force, is increased. Despite the fact that the generation of this peak clearly absorbs energy it enhances vertical momentum which is important to achieve long jumping distances. Thus, the new model describes quantitatively the dynamics and mechanisms of the most essential parts of the long jump and helps to understand jumping techniques. For individual jumpers detailed diagnostics are possible about techniques or conditional shortcomings.

4.1. General significance

Many models have been proposed to describe human jumping. As jumping in a less extreme form is part of standard locomotion, modelling of jumping is of general significance for human locomotion. Most studies so far have either been descriptive (Hay, 1993; Lees, 1994) or alternatively were based on very detailed modelling.

But even extremely detailed models using all major muscles (Hatze, 1981; Bobbert and Van Soest, 1994) fall short in describing the general dynamics of the process. The major reason is that the landing impact (contributing 25% of the total change in momentum) is not described adequately. The activation dynamics of the musculature precludes active generation of this peak, i.e. even if the musculature was activated and deactivated within 40 ms the muscle could not follow.

Force enhancement due to stretching of the activated muscle (Alexander, 1990) may contribute to the passive peak. Presently, we investigate the quantitative contribution of muscle forces. A major cause of the impact is the deceleration of distal masses. These masses consist of the skeleton and of soft tissues and are visco-elastically coupled to the ground or to each other.

The comparison between our results from the simulations and the experiments reveals that a large fraction of these masses can be identified as muscle masses. The type of coupling as measured for the heel (Gruber, 1987) proves to be necessary for adequate description of the time course of the event. The right damping is necessary to avoid injuries (stiff coupling) or elastic ringing (compliant coupling) making control at least difficult.

The spring-like behaviour of the leg could be replaced by a suitably activated musculo-skeletal system. Nevertheless, it is surprising to which extent the leg performs like a spring. It might be a strategy to simplify control (Bobbert et al., 1996). The shortening of the leg amounts to about 15% ($\Delta r_{MAX}/r_0$). A corresponding rotation of the knee results in lengthening of the quadriceps–patella tendon complex by about 35 mm. For the high loads observed the patellar tendon would be stretched by ca. 5 mm. The long aponeuroses of the musculus quadriceps may stretch elastically by about 20 mm. In this case the elastic properties of the passive tissues would largely determine the quasi-elastic operation of the leg and thus its stiffness. At higher knee flexion the conservative operation of the knee can probably not be kept up any longer due to the increasing demand of muscle force and the properties of the connecting tissues. Therefore the limited properties of the human leg do not allow to reach the theoretically possible maximum values. A higher take-off velocity angle will be accompanied by a smaller take-off velocity and thus a shorter jumping distance.

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