7

A Walking Pattern Generation Method for Humanoid robots using Least square method and Quartic polynomial

Seokmin Hong*, †, Yonghwan Oh†
Young-Hwan Chang† and Bum-Jae You†
* *University of Science and Technology(UST), Korea*
† *Korea Institute of Science and Technology(KIST), Korea*

1. Introduction

Humanoid robot has been developed for human's convenience in the human environment. Humanoid robots like ASIMO, HRP, WABIAN and Johnnie were successively developed (Hirai et al., 1998; Yamaguchi et al., 1999; Kajita et al., 2003; Löffler et al, 2003). Researches on humanoid have been done about balancing, walking pattern generation, motion generation, whole body cooperation and so on. In particular, many walking pattern generation methods have been developed to prevent a robot from tipping over. There are generally two groups for the walking pattern generation (Kajita et al., 2003; Hirukawa, 2006). The first group uses forward dynamics with multiple rigid-body (Hirai et al., 1998; Yamaguchi et al., 1999). This group demands precise information of the robot such as mass, inertia, and center of mass(CoM) of each link, and so on. In this group, the footprints of biped robot are changed for keeping the planned configuration of the robot (Hirukawa, 2006). On the other hand, the second group (Huang et al., 2001; Kajita et al., 2003; Löffler et al., 2003; Harada et al., 2004; Zhu et al., 2004; Oh et al., 2006) utilizes limited knowledge such as the total center of mass and total angular momentum. And this group makes use of the inverted pendulum model for the walking pattern generation and changes the configuration of the robot for keeping the planned footprints (Hirukawa, 2006). Our walking pattern method belongs to the second group. The inverted pendulum model is transferred the complex dynamic equation of humanoid robot into second order differential equation with some assumptions. Most researches on walking pattern generation dealt with periodic walking. However it is difficult to implement various gaits by periodic walking generator. Harada et al. introduced the analytical walking pattern method on real-time generation for coping with change of gait. Kajita et al. proposed the omni-direction walking pattern generation method using the preview control. Zhu et al. proposed walking pattern generation method with fixed ZMP and variable ZMP in order to make the biped walking pattern more human-like and more agile. This paper describes the omni-directional walking pattern method for the humanoid robots using the least square method with the quartic polynomials. And we design that the ZMP trajectory has the slope in single support phase. The CoM trajectory becomes effective in regard of velocity compared to the ZMP trajectory without the slope. We utilize a simple inverted pendulum model to represent the complex dynamics of the humanoid robot. This paper is organized as follows: It shows ZMP equation as the simplified model for the biped robot and arranges the relation of the ZMP and CoM in section 2. Three step modules are introduced for generating the ZMP and CoM trajectory in section 3 and we explain the procedure of generating the trajectory from footprints in section 4. Section 5,6 shows the results of simulations and experiment, and we conclude this paper in section 7.

2. Linear Inverted Pendulum Model

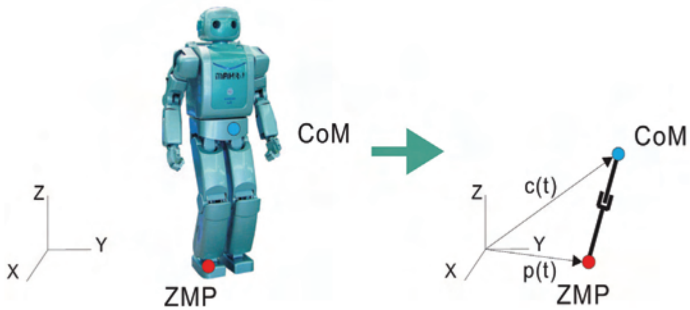


Fig. 1. Linear Inverted pendulum model

The humanoid robot is designed like human for doing various behaviors and adapting to human's environment. For this purpose, this robot consists of many links and joints. If the walking pattern took all dynamic properties of a number of links and joints into consideration, the waking pattern could make good performance. However, since it is difficult and complicated to calculate the walking pattern including all dynamic properties, a simplified model is required to control the biped robot. Fig. 1 shows the inverted pendulum model as the simplified biped robot. In Fig. 1, $p(t)$ and $c(t)$ denote the ZMP and CoM, respectively. The motion of the robot is designed by CoM trajectory instead of the full dynamics.

For simplicity, we use following assumptions:

1. The time derivative of the angular momentum about the CoM is zero.
2. The difference of CoM and ZMP at z-axis is constant.
3. The acceleration of ZMP at z-axis is zero.

Under these assumptions, the relation between ZMP and CoM can be represented as follows.

$$p(t) = c(t) - \frac{1}{\omega^2}\ddot{c}(t) \tag{1}$$

where ω is $\sqrt{g/(Z_{CoM} - Z_{ZMP})}$. Z_{CoM}, Z_{ZMP} and g are the values of the CoM and the ZMP at the z-axis and the gravity separately. The walking pattern of the ZMP and the CoM is made with Eq. (1). As mentioned above, the CoM is related to robot motion. Generally the robot motion has to keep the continuous jerk of the CoM for preventing the robot's motion from imposing a heavy burden to the robot. For the continuous jerk of CoM, more than cubic polynomials of the ZMP are sufficient to generate the ZMP and CoM trajectory. In case of the periodic walking pattern, it is proper to use cubic polynomial of the ZMP. But in case of various step length walking, these might take place large fluctuation of the ZMP trajectory. Since cubic polynomials constrain the shape of the ZMP, the ZMP trajectory makes the large fluctuation occur in order to satisfy Eq. (1) according to various steps. Therefore we choose quartic polynomials of the ZMP and then need an optimal method to obtain the solution owing to more numbers of unknowns than equations. In next section, the least square method will be explained as an optimal method in detail. Let us reconsider that the initial conditions $c(0)$, $\dot{c}(0)$ are known and the ZMP trajectory is given by quartic polynomials in Eq. (1). The general form of the ZMP and CoM could be found as follows.

$$p(t) = b_4t^4 + b_3t^3 + b_2t^2 + b_1t + b_0 \tag{2}$$

$$\begin{aligned} c(t) &= (c(0) - b_0 - \frac{2}{\omega^2}b_2 - \frac{24}{\omega^4}b_4)\cosh(\omega t) \\ &+ \frac{1}{\omega}(\dot{c}(0) - b_1 - \frac{6}{\omega^2}b_3)\sinh(\omega t) \\ &+ b_4t^4 + b_3t^3 + (b_2 + \frac{12}{\omega^2}b_4)t^2 + (b_1 + \frac{6}{\omega^2}b_3)t \\ &+ (b_0 + \frac{2}{\omega^2}b_2 + \frac{24}{\omega^4}b_4) \end{aligned} \tag{3}$$

$$\begin{aligned} \dot{c}(t) &= \omega(c(0) - b_0 - \frac{2}{\omega^2}b_2 - \frac{24}{\omega^4}b_4)\sinh(\omega t) \\ &+ (\dot{c}(0) - b_1 - \frac{6}{\omega^2}b_3)\cosh(\omega t) \\ &+ 4b_4t^3 + 3b_3t^2 + 2(b_2 + \frac{12}{\omega^2}b_4)t + (b_1 + \frac{6}{\omega^2}b_3) \end{aligned} \tag{4}$$

Equations (2), (3), and (4) present the fact that the coefficients of the CoM are presented by c(0), $\dot{c}(0)$ and those of the ZMP. Since c(0), $\dot{c}(0)$ are the provided values as the initial parameters, we focus on obtaining the ZMP coefficient. At the time interval of i-th support phase, Eq. (2), (3) and (4) are arranged by a matrix form as follows

$$\mathbf{X}_i = \mathbf{A}_i\mathbf{X}_{i-1} + \mathbf{B}_i\boldsymbol{\beta}_i \tag{5}$$

where

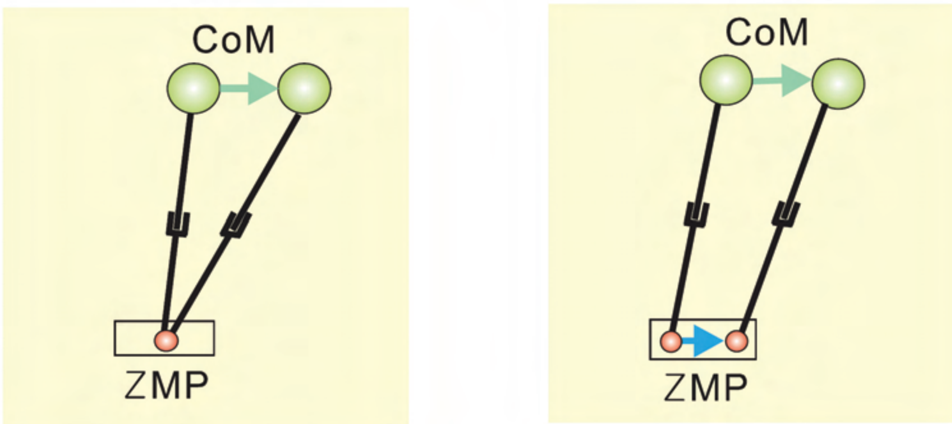
$$\mathbf{X}_i = \begin{Bmatrix} \omega(c(T_i) - p(T_i)) \\ \dot{c}(T_i) \end{Bmatrix}$$

$$\mathbf{A}_i = \begin{bmatrix} CT_i & ST_i \\ ST_i & CT_i \end{bmatrix}$$

$$\mathbf{B}_i = \begin{bmatrix} -\frac{24}{\omega^3}CT_i + \frac{12}{\omega}T_i^2 + \frac{24}{\omega^3} & -\frac{24}{\omega^3}ST_i + 4T_i^3 + \frac{24}{\omega^2}T_i \\ -\frac{6}{\omega^2}ST_i + \frac{6}{\omega}T_i & -\frac{6}{\omega^2}CT_i + 3T_i^2 + \frac{6}{\omega^2} \\ -\frac{2}{\omega}CT_i + \frac{2}{\omega} & -\frac{2}{\omega}ST_i + 2T_i \\ -ST_i & -CT_i + 1 \end{bmatrix}^T$$

$$\boldsymbol{\beta}_i = \begin{pmatrix} b_{i4} & b_{i3} & b_{i2} & b_{i1} \end{pmatrix}^T$$

CTi and STi denote cosh(ωt) and sinh(ωt), respectively. b_{in} (n = 1, . . . , 4) denotes the n-th order coefficient of the ZMP during the i-th support phase. Equation(5) will be used a basic equation of the ZMP and the CoM .



a) whitout the slope b) with the slope

Fig. 2. Inverted pendulum model without and with the slope

3. Step Modules

This section introduces three step modules which are proposed for making walking trajectory of the ZMP and the CoM: periodic step module, absorption step module, F-step module. Each step module can have the slope in single support phase. As shown in Fig. 2, the ZMP movement can be designed like the human walking in which the ZMP moves from heel to toe in single support phase. The CoM movement with a slope of the ZMP in single support phase becomes smaller than that with the flat line because the ZMP movement from heel to toe makes the CoM movement reduce. Section 4 shows the efficiency of the ZMP with the slope in single support phase through the simulation. The walking pattern method of the sagittal plane and the frontal plane is the same procedure. So we explain the walking pattern at the sagittal plane, hereafter.

3.1 Periodic Step Module

For generating the walking pattern, the initial values are needed. This step module provides the initial values as the seed of the walking pattern generation, because it makes the walking pattern independently with the position and velocity information of the ZMP at periodic steps. This consists of cubic polynomials (b_{i4} = 0) in double support phase and first order polynomials (b_{i4} = 0, b_{i3} = 0, b_{i2} = 0) in single support phase. Using the information of the ZMP, the solution can be obtained analytically. The obtained values by the periodic step module are used as the final values of position and velocity of the CoM at the first step. But the ZMP coefficients by the periodic step module are not used in making the trajectory because of not being proper to the arbitrary step. We only uses information of a particular position and velocity of the CoM obtained from this step module. It consists of 4 phases as shown in Fig. 3.

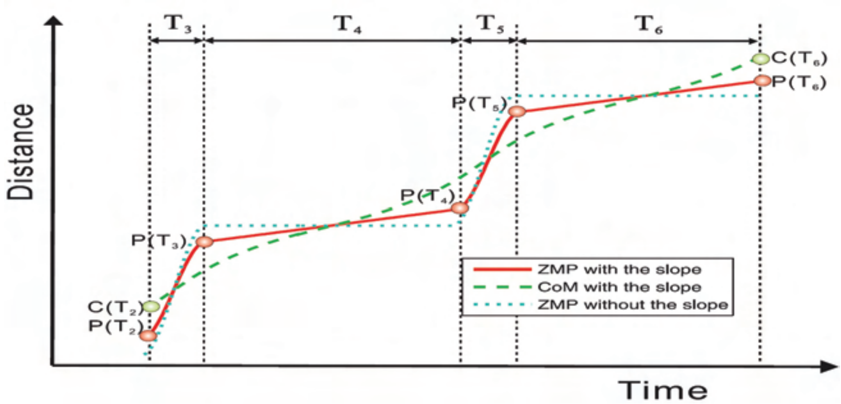


Fig. 3. Constitution of periodic step module

in Fig. 3., i.e. double support phase, single support phase, double support phase and single support phase. T_i means the interval time at the i-th support phase. The numbering is written down in Fig. 3 taking the first step into account. In Fig. 3, T_3 and T_5 are double support time, and T_4 and T_6 denote single support time. And it is known that the velocity of the ZMP is

$$b_{i1} = \begin{cases} \frac{p(T_6)-p(T_5)}{T_6} & \text{for } i = 3,6 \\ \frac{p(T_4)-p(T_3)}{T_4} & \text{for } i = 4,5 \end{cases}$$

at the meeting point of each phase. The values of $p(T_4)-p(T_3)$ and $p(T_6)-p(T_5)$ are the same value as the ZMP margin in single support phase. The value is selected less than a half of foot size considering keeping the stability margin. And as the term of the "periodic", it is assumed that this walking trajectory has the relation between initial and final condition of the ZMP and CoM as follows.

$$\omega(c(T_6) - p(T_6)) = \omega(c(T_2) - p(T_2)) \quad (6)$$

$$\dot{c}(T_6) = \dot{c}(T_2) \quad (7)$$

The relation of initial and final conditions as Eq. (6) and (7) causes the characteristics of periodicity. This relation also applies to the absorption step module as a constraint equation.The solutions of the periodic step module are as follows

$$\begin{Bmatrix} \mathbf{X}_2 \\ \boldsymbol{\beta}_3 \\ \boldsymbol{\beta}_5 \end{Bmatrix} = \mathbf{D}^{-1}\mathbf{E} \quad (8)$$

Where

$$\mathbf{D} = \begin{bmatrix} \mathbf{A}_6\mathbf{A}_5\mathbf{A}_4\mathbf{A}_3 - \mathbf{I} & \mathbf{A}_6\mathbf{A}_5\mathbf{A}_4\mathbf{B}_3^u & \mathbf{A}_6\mathbf{B}_5^u \\ \mathbf{0} & \mathbf{F}_3 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{F}_5 \\ \mathbf{0} & \mathbf{G}_3 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{G}_5 \end{bmatrix}$$

$$\mathbf{E} = \begin{Bmatrix} -\boldsymbol{\Gamma} \\ \alpha_3 T_3 - \alpha_6 T_3 \\ \alpha_5 T_5 - \alpha_4 T_5 \\ \alpha_4 - \alpha_6 \\ \alpha_6 - \alpha_4 \end{Bmatrix}, \quad \boldsymbol{\Gamma} = \begin{bmatrix} (\mathbf{A}_6\mathbf{A}_5\mathbf{A}_4\mathbf{B}_3^k)^T \\ (\mathbf{A}_6\mathbf{A}_5\mathbf{B}_4^k)^T \\ (\mathbf{A}_6\mathbf{B}_5^k)^T \\ (\mathbf{B}_6^k)^T \end{bmatrix}^T \begin{Bmatrix} b_{31} \\ b_{41} \\ b_{51} \\ b_{61} \end{Bmatrix}$$

$$\mathbf{F}_i = (T_i^3 \quad T_i^2), \quad \mathbf{G}_i = (3T_i^2 \quad 2T_i), \quad \boldsymbol{\beta}_i = (b_{i3} \quad b_{i2})^T$$

$\mathbf{B}_i$ in Eq. (5) is divided into B_i^u and B_i^k by the coefficients of the periodic step module.

$$\mathbf{B}_i^u = \begin{bmatrix} -\frac{6}{\omega^2}ST_i + \frac{6}{\omega}T_i & -\frac{6}{\omega^2}CT_i + 3T_i^2 + \frac{6}{\omega^2} \\ -\frac{2}{\omega}CT_i + \frac{2}{\omega} & -\frac{2}{\omega}ST_i + 2T_i \end{bmatrix}^T$$

$$\mathbf{B}_i^k = \begin{pmatrix} -ST_i & -CT_i + 1 \end{pmatrix}^T \quad (9)$$

and $\alpha_i = \frac{p(T_i) - p(T_{i-1})}{T_i}$ is the slope which is connected by two points $p\ (T_{i-1})$ and $p\ (T_i)$ in the i-th phase. This step module provides final values to F-step module for first step. The values are $c\ (T_3)$, $\dot{c}\ (T_3)$ obtained by $\mathbf{X}_3$ in order to increase the relation with first step.

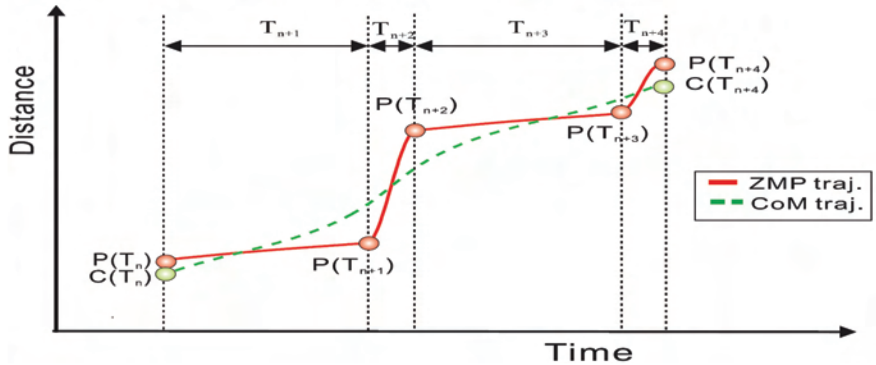


Fig. 4. Constitution of absorption step module

3.2 Absorption Step Module

The absorption step module generates the trajectory with positions of the ZMP from footprints and initial position and velocity of the ZMP and the CoM. This study defines a stride period as time consisted of 2 single support phases and 2 double support phases in Fig. 4. This step module makes the trajectory repetitively with the ZMP position information of the next stride period in every phase like Fig. 5.

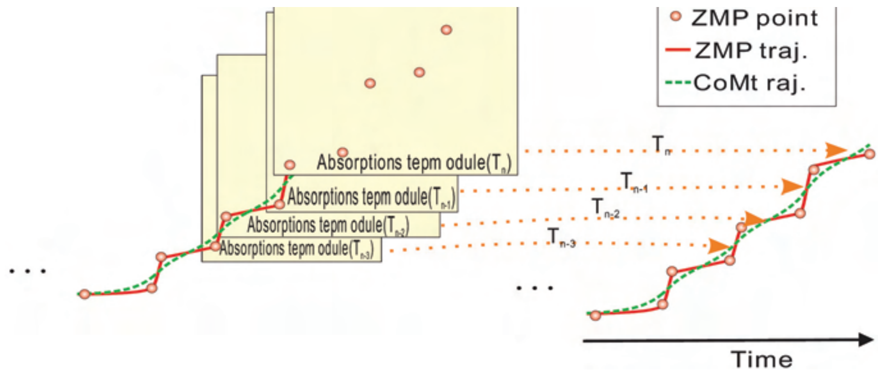


Fig. 5. Generation of ZMP trajectory using absorption step module

This is why it copes with variation of the ZMP trajectories when footprint is changed to walk with various steps and implements real-time walking pattern generation. This step module consists of 4 support phases: 2 single support phases and 2 double support phases. This uses the quartic polynomial of the ZMP trajectory. The numbers of unknown factors are more than the numbers of equations in this case. The redundant unknown factor plays an important role to reduce the fluctuation of the ZMP trajectory. Using Eq. (5), a matrix can be arranged from T_n to T_{n+4},

$$\boldsymbol{\kappa} = \boldsymbol{\gamma}^T \boldsymbol{\tau} \tag{10}$$

where

$$\boldsymbol{\kappa} = \mathbf{X}_{n+4} - \mathbf{A}_{n+4}\mathbf{A}_{n+3}\mathbf{A}_{n+2}\mathbf{A}_{n+1}\mathbf{X}_n$$
$$\boldsymbol{\gamma} = \begin{bmatrix} (\mathbf{A}_{n+4}\mathbf{A}_{n+3}\mathbf{A}_{n+2}\mathbf{B}_{n+1})^T \\ (\mathbf{A}_{n+4}\mathbf{A}_{n+3}\mathbf{B}_{n+2})^T \\ (\mathbf{A}_{n+4}\mathbf{B}_{n+3})^T \\ (\mathbf{B}_{n+4})^T \end{bmatrix}$$
$$\boldsymbol{\tau} = \begin{bmatrix} \boldsymbol{\beta}_{n+1}^T & \boldsymbol{\beta}_{n+2}^T & \boldsymbol{\beta}_{n+3}^T & \boldsymbol{\beta}_{n+4}^T \end{bmatrix}^T$$
$$\boldsymbol{\beta}_i = \begin{pmatrix} b_{i4} & b_{i3} & b_{i2} & b_{i1} \end{pmatrix}^T$$

β_i means the coefficient vector of the ZMP at the i-th phase. And there are 9 constraint equations for keeping the continuity of position and velocity of the ZMP trajectory on each phase. The equations are presented by a matrix as follows.

$$\boldsymbol{\varphi} = \boldsymbol{\Psi}\boldsymbol{\tau} \tag{11}$$

where

$$\boldsymbol{\varphi} = \begin{Bmatrix} p(T_{n+1}) - p(T_n) \\ 0 \\ dp(T_n) \\ p(T_{n+2}) - p(T_{n+1}) \\ 0 \\ p(T_{n+3}) - p(T_{n+2}) \\ 0 \\ p(T_{n+4}) - p(T_{n+3}) \\ dp(T_{n+4}) \end{Bmatrix}$$

$$\boldsymbol{\Psi} = \begin{bmatrix} \mathbf{F}_{n+1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{G}_{n+1} & -\mathbf{H} & \mathbf{0} & \mathbf{0} \\ \mathbf{H} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{F}_{n+2} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{G}_{n+2} & -\mathbf{H} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{F}_{n+3} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{G}_{n+3} & -\mathbf{H} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{F}_{n+4} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{G}_{n+4} \end{bmatrix}$$

$$\begin{aligned}
\mathbf{F}_i &= \begin{pmatrix} T_i^4 & T_i^3 & T_i^2 & T_i \end{pmatrix} \\
\mathbf{G}_i &= \begin{pmatrix} 4T_i^3 & 3T_i^2 & 2T_i & 1 \end{pmatrix} \\
\mathbf{H} &= \begin{pmatrix} 0 & 0 & 0 & 1 \end{pmatrix}
\end{aligned}$$

Equations (10) and (11) are changed into a matrix.

$$\boldsymbol{\xi} = \boldsymbol{\Phi}\boldsymbol{\tau} \tag{12}$$

where

$$\boldsymbol{\xi} = \begin{Bmatrix} \boldsymbol{\kappa} \\ \boldsymbol{\varphi} \end{Bmatrix}, \quad \boldsymbol{\Phi} = \begin{bmatrix} \boldsymbol{\gamma}^T \\ \boldsymbol{\Psi} \end{bmatrix}$$

The coefficients of each support phase were obtained with Eq. (12) and least square method. The least square method is used as the optimal method. The objective of using the method is that variation of the ZMP velocity is minimized. This prevents the ZMP trajectory from changing abruptly in order to satisfy Eq. (1) according to footprints. In order to do it, the cost function is defined as follows.

$$\begin{aligned}
\mathcal{J} &= \frac{1}{2}\sum_{i=1}^{4}\int_0^{T_{n+i}} w_{n+i}[\dot{p}(t) - \alpha_{n+i}]^2 dt \\
&= \frac{1}{2}\boldsymbol{\tau}^T\mathbf{W}\boldsymbol{\tau} - \mathbf{U}\boldsymbol{\tau} + c
\end{aligned} \tag{13}$$

where

$$\mathbf{W} = \begin{bmatrix} \mathbf{W}_{n+1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{W}_{n+2} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{W}_{n+3} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{W}_{n+4} \end{bmatrix}$$

$$\mathbf{W}_i = w_i \begin{bmatrix} \frac{16}{7}T_i^7 & 2T_i^6 & \frac{8}{5}T_i^5 & T_i^4 \\ 2T_i^6 & \frac{9}{5}T_i^5 & \frac{3}{2}T_i^4 & T_i^3 \\ \frac{8}{5}T_i^5 & \frac{3}{2}T_i^4 & \frac{4}{3}T_i^3 & T_i^2 \\ T_i^4 & T_i^3 & T_i^2 & T_i \end{bmatrix}$$

$$\mathbf{U} = \begin{Bmatrix} \alpha_{n+1} w_{n+1} \mathbf{F}_{n+1}^T \\ \alpha_{n+2} w_{n+2} \mathbf{F}_{n+2}^T \\ \alpha_{n+3} w_{n+3} \mathbf{F}_{n+3}^T \\ \alpha_{n+4} w_{n+4} \mathbf{F}_{n+4}^T \end{Bmatrix}$$

Using Lagrange multiplier,

$$\mathcal{L}(\boldsymbol{\tau}, \boldsymbol{\lambda}) = \frac{1}{2}\boldsymbol{\tau}^T \mathbf{W} \boldsymbol{\tau} - \mathbf{U}\boldsymbol{\tau} + c + \boldsymbol{\lambda}^T(\boldsymbol{\xi} - \boldsymbol{\Phi}\boldsymbol{\tau}) \tag{14}$$

The coefficients of each support phase are acquired.

$$\boldsymbol{\tau} = \boldsymbol{\Phi}_W^+ \boldsymbol{\xi} - \boldsymbol{\Phi}_W^+(\boldsymbol{\Phi}\mathbf{W}^{-1}\mathbf{U}^T) + \mathbf{W}^{-1}\mathbf{U}^T \tag{15}$$

where

$$\boldsymbol{\Phi}_W^+ = \mathbf{W}^{-1}\boldsymbol{\Phi}^T(\boldsymbol{\Phi}\mathbf{W}^{-1}\boldsymbol{\Phi}^{\mathbf{T}})^{-\mathbf{1}} \tag{16}$$

And w_i is the weighting factor. Generally, the stability of humanoid robot is whether the ZMP is located on the supporting zone which is shaped by supporting foot or not. The weighting factor suppresses the fluctuation of the ZMP trajectory for guaranteeing the stability of biped robot according to the various ZMP positions. The ZMP trajectory in single support phase is easy to diverge owing to small supporting zone. Therefore the weighting factor of single support phase is larger than that of double support phase. We find that the value of weighting factor w_n in single support phase is 100 times of the value in double support phase by simulation. The large fluctuation of the ZMP trajectory can occurs when more than the values of 3 ZMP positions are successively same at a stride period. That is why this phenomenon takes place due to Eq. (6) and (7). The ZMP trajectory moves abruptly to satisfy the constraint Eq. (6) and (7). There are three methods of absorbing the fluctuation of the ZMP trajectory. One is resetting the ZMP position, another is readjusting the weighting factor and other is readjusting the support phase time. In the first method, it is needed to set new extra ZMP positions to reduce the fluctuation irrespective of the footprints. Therefore, in this case, the double support time is increased and weighting factor is readjusted in a stride period with more than 3 same ZMP positions. The single support time don't need to change the value of phase time and weighting factor because the single support phase is already enough to be suppressed by weighting factor. The double support time is changed when the ZMP positions are more than 3 same values of the ZMP position in a stride period. The value of weighting factor at this double support phase is changed to the value of single support phase. But if the first phase time of a stride period was double support time, we did not change the support phase time and the weighting factor of the first double support phase for not having influence on total walking pattern time.

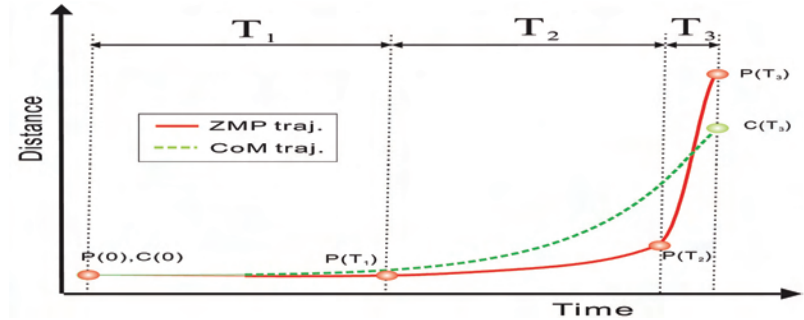


Fig. 6. Constitution of first step module

3.3 F-Step Module

This step module is for first step and final step. This module consists of 3 phases: double support phase, single support phase and double support phase. The difference of first step and final step is the procedure. The final step is in reverse order of the first step. The fluctuation of the ZMP trajectory might arise at the first phase time of the first step and the third phase time of the final step. That is why Eq. (1) has the non-minimum phase property. When the CoM moves forward at the starting point of walking, the ZMP moves naturally backward owing to the non-minimum phase property. We increase the phase time to suppress the non-minimum phase property at the first phase time of the first step and the third phase time of the final step. The changed phase time is selected within 2s.

F-step module is provided with the initial value of the ZMP and CoM which are set by the robot itself. Ideally the initial values of the ZMP and CoM are zero, but these values may not be zero because of the irregular ground condition and robot's kinematic error. It causes a problem that initial values are not zero when the trajectory is generated. Although this initial problem may be not critical effect on the robot, it could make the initial motion of the robot unnatural. We cope with the problem using least square method and the readjusted phase time in order that the more natural trajectory of the ZMP and CoM can be generated. The coefficients of each phases is decided with similar procedure which is introduced in the previous absorption step module section.

4. Implementation of Walking Pattern Generation

In this section, we show the procedure to generate the trajectory, which uses three step modules demonstrated in the previous section. The procedure is as follows.

Firstly, the walking parameters are selected. The parameters are step length, stride width, turning angle, single support time, double support time, user setting time, weighting factor values, initial value of the ZMP and CoM, and the ZMP positions according to footprints. Secondly, using periodic step module with the preset ZMP positions and velocities, final values of the first step are found. Thirdly, F-step module is used for the first step with initial values from user setting, final values from periodic step module and the preset ZMP positions according to the footprints in Fig.6. Fourthly, absorption step module is utilized in every phase. It makes the trajectory according to the ZMP positions of the next stride period.

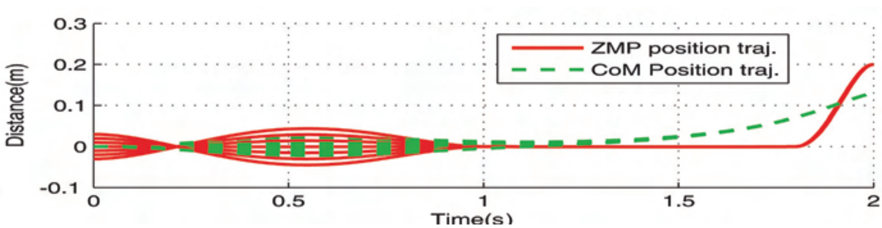
And for the final step generation, absorption step module finally has to finish single, double, single and double support phase before the final step. And if more than 3 successive ZMP positions are the same value in generating the trajectory by the absorption step module, the abrupt fluctuation of the ZMP trajectory arises for satisfying the periodic characteristic. Double support phase time and weight factors are readjusted except that the first support phase of a stride period is double support phase. And finally, F-step module is used for the final step with initial values from the absorption step module and final values from user.

5. Simulation

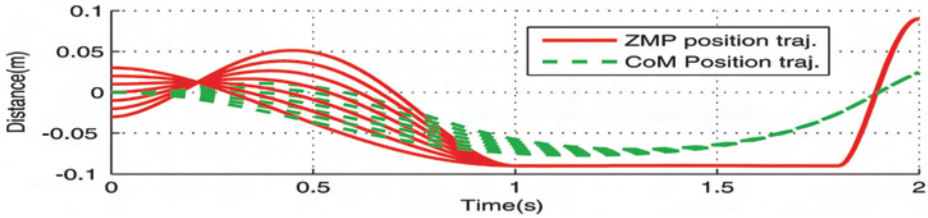
We show the advantages of the proposed method in this section. The proposed method has the characteristic of being insensitive to initial values of the ZMP and CoM. As shown in Fig. 7 and Fig. 8, the proposed method makes the effect of initial values be reduced, although the robot has the initial values. The ZMP trajectory is changed automatically to reduce the effect of initial value of the ZMP and CoM in the first phase of first step. The fluctuations in Fig. 7 and Fig. 8 look large. But this variation do not arise the unstable condition because both feet of the robot stay on the ground.
And the proposed method has the slope in single support phase as shown in Fig. 9 and Fig. 10. The ZMP trajectory with the slope in single support phase has an advantage of reducing the CoM velocity because of the ZMP movement. Therefore it helps more efficient robot motion having slope in single support phase. In Fig.10, the robot does the same motion with less CoM velocity. It means the efficiency of the joint angular velocity enhances. Therefore the robot can implement longer stride with the same joint angular velocity.
And finally to demonstrate the usefulness of the proposed method, we generate 'U' walking pattern in Fig. 11 and Fig. 12. This walking pattern is mixed with straight waling and turning walking.

6. Experiment

We did an experiment with real humanoid robot MAHRU-R which is improved mechanical part design of a humanoid robot platform MAHRU at KIST.
Let us Mahru-R introduce briefly. Fig. 13 shows MAHRU-R which has 1350mm height, 50 kg weight including batteries. It has 12-DOF in two legs and 20- DOF in two arms including hands with 4-DOF grippers. Also, it is actuated by DC servo motors through harmonic drive reduction gears. The body is equipped with an IMU (Inertial Measurement Unit) sensor which consists of 3-axis gyroscope and 3-axis G-force sensors. Each ankle and wrist is equipped with a force/torque sensor. A distributed system was built for humanoid using

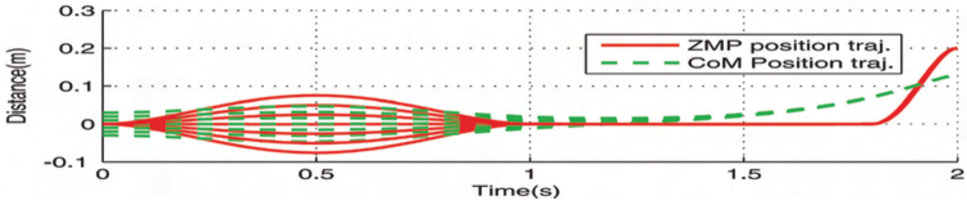


(a) ZMP& CoM trajectory at the sagittal plane

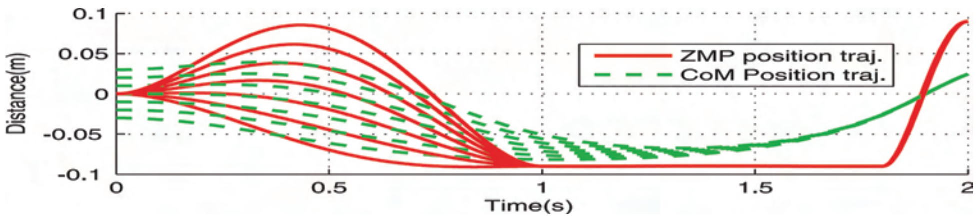


(b) ZMP& CoM trajectory at the lateral plane

Fig. 7. ZMP& CoM trajectory with initial ZMP position (from −0.04m to 0.04m), T_1 = 1.0s, T_2 = 0.8s, T_3 = 0.2s, step length = 0.2m, stride width = 0.18m



(a) ZMP& CoM trajectory at the sagittal plane



(b) ZMP& CoM trajectory at the lateral plane

Fig. 8. ZMP& CoM trajectory with initial CoM position (from −0.04m to 0.04m), T_1 = 1.0s, T_2 = 0.8s, T_3 = 0.2s, step length = 0.2m, stride width = 0.18m

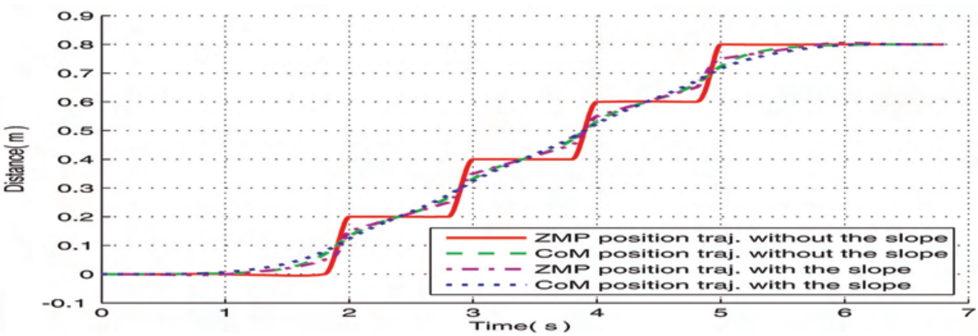


Fig. 9. ZMP& CoM position trajectory with T_s = 0.8s, T_d = 0.2s, step length=0.2m and slope = 0.25m/s

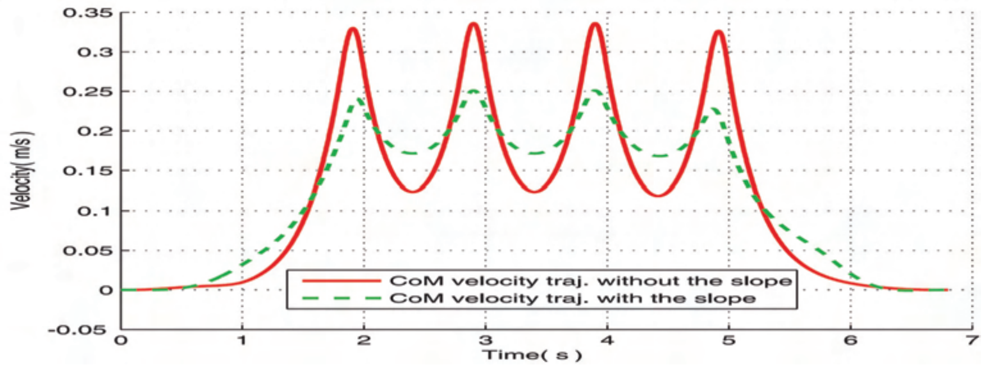


Fig. 10. ZMP& CoM velocity trajectory with T_s = 0.8s, T_d = 0.2s, step length = 0.2m and slope = 0.25 m/s

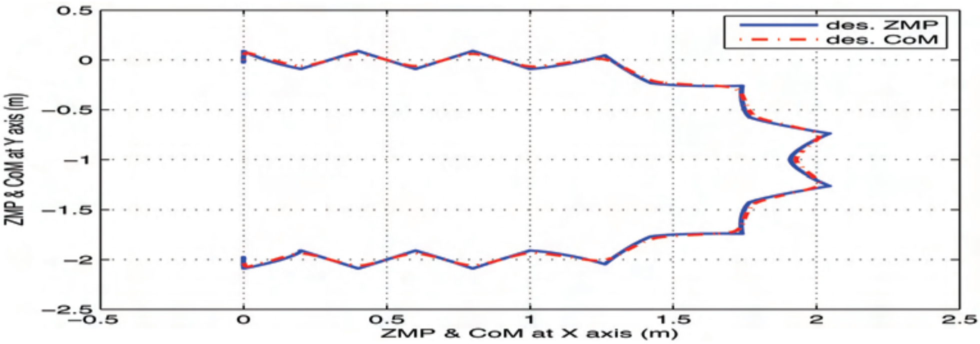


Fig. 11. 'U' walking pattern with T_s = 0.8s, T_d = 0.2s, step length = 0.2m, turn rate = 15 deg./stride and slope = 0

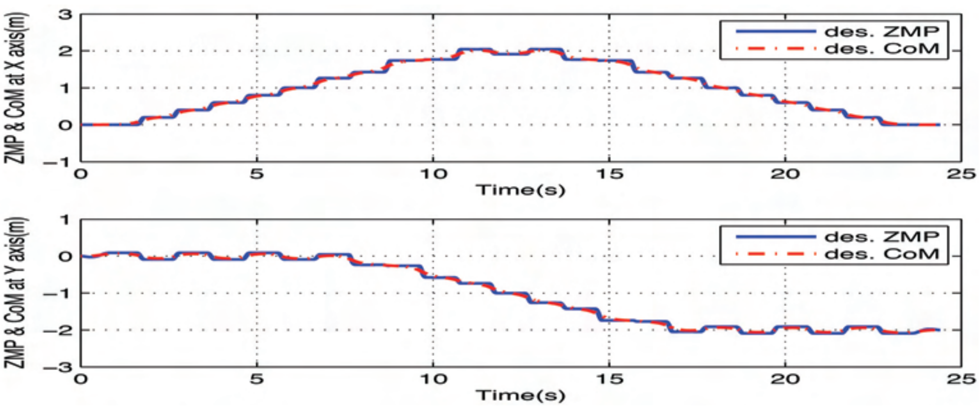


Fig. 12. 'U' walking pattern of ZMP and CoM at X and Y axis

Fig. 13. Humanoid robot MAHRU-R

sub-controllers and IEEE 1394 protocol communication lines between the main controller and sub-controllers. The main real-time control algorithm runs on a micro-ATX CPU board in the backpack of MAHRU-R, whose operating system is real-time Linux (RTAI/Xenomai). It allows user to make timer interrupt service routine with the highest priority to control the robot in real-time. To confirm the effectiveness of the proposed method, the experiment was done using MAHRU-R. As shown in Fig. 14, the walking pattern was zigzag walking which consisted of the straight walking and turning. Fig. 15 and Fig. 16 show the desired ZMP and CoM are useful to the real humanoid robot walking pattern. When the humanoid robot started turning after 6 second, we increased the step width from 0.17m to 0.19m in order to avoid the collision of both feet. So the end point of Fig. 16 is different to the starting point at Y axis. Fig. 17 shows the snapshot of zigzag walking.

7. Conclusion

This paper proposed a new walking pattern method for the humanoid robot. The propose method was based on the linear inverted pendulum model as the simple model of humanoid robot. And this method consisted of three step modules for generating walking pattern. Using these step modules, this paper addressed robustness of the proposed method against initial values of the ZMP and CoM, and validated the efficiency with the slope at the single support phase through simulations. And finally we showed zigzag walking to verify the usefulness of the proposed method using real humanoid robot MAHRU-R.

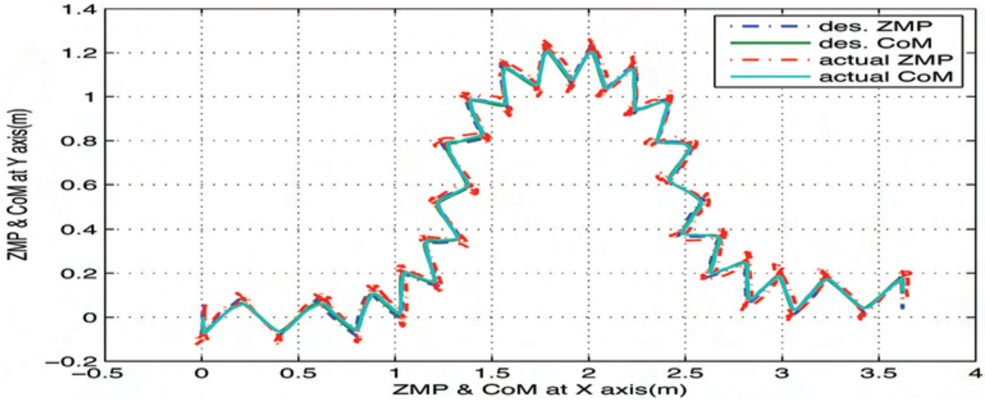


Fig. 14. Zigzag walking pattern with *Ts* = 0.95s, *Td* = 0.15s, step length = 0.2m, turn rate = 10 deg./stride and slope = 0

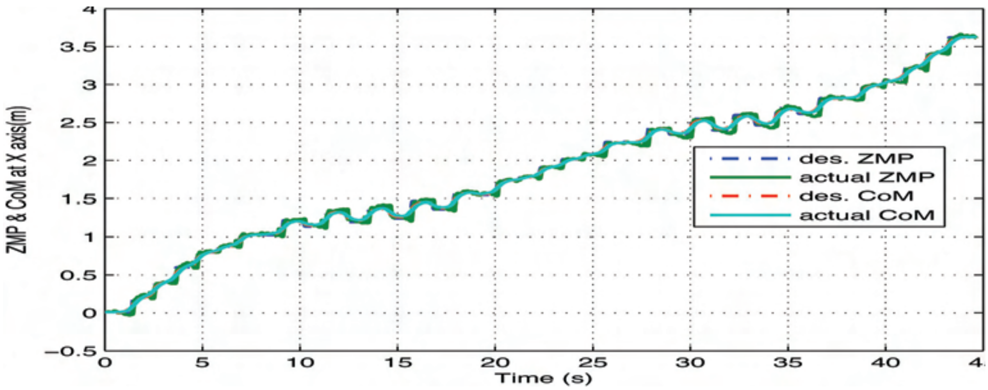


Fig. 15. ZMP and CoM of zig zag walking at X axis

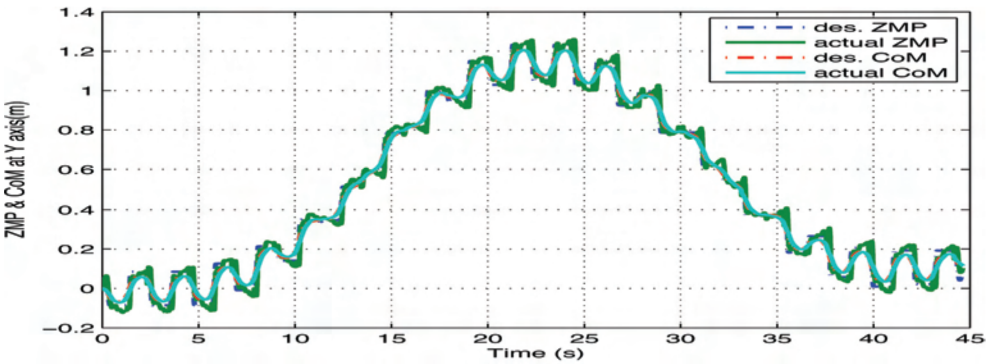


Fig. 16. ZMP and CoM of zig zag walking at Y axis

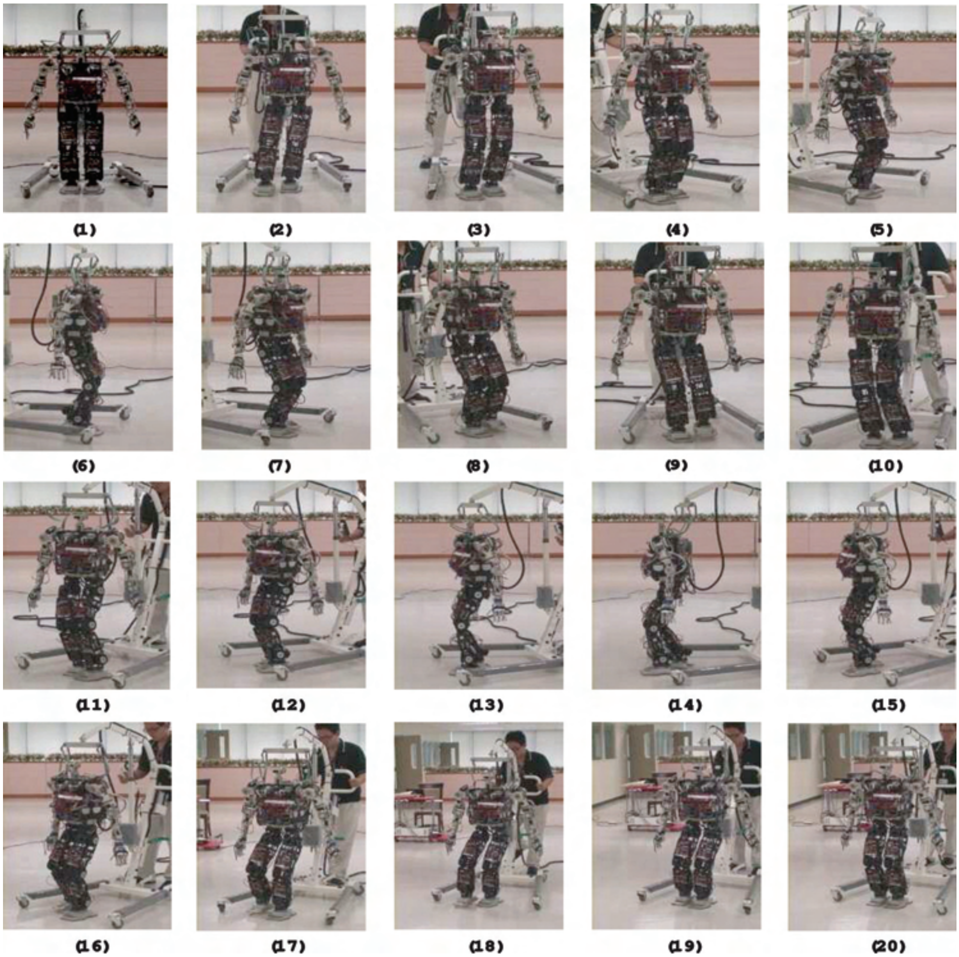


Fig. 17. Snapshots of zig zag walking

8. Acknowledgements

This work was supported in part by Ministry Knowledge Economy(MKE) & Institute for Information Technology Advancement(IITA) through IT Leading R&D support Project.

9. References

Harada, K.; Kajita, S.; Kaneko K. & Hirukawa, H. (2004). An Analytical Method on Real-time Gait Planning for a Humanoid Robot, *IEEE-RAS/RSJ Int. Conf. on Humanoid Robots*, Los Angeles, USA, pp. 640-655, November 2004.

Hirai, K.; Hirose, M.; Haikawa, Y. & Takenaka, T. (1998). Development of Honda Humanoid Robot, *Proc. of IEEE Int. Conf. on Robotics and Automation*, Leuven, Belgium, pp.

1321-1326, May 1998.
Hirukawa, H. (2006). On Humanoid Control, *Onassis Foundation Lecture Series 2006.*
Huang, Q.; Yokoi, K.; Kajita, S.; Kaneko, K.; Arai, H.; Koyachi N. & Tani, K. (2001) Panning Walking Patterns for a Biped Robot, *IEEE Trans. On Robotics and Automation*, pp. 280-289.
Kajita, S.; Kanehiro, F.; Kaneko, K.; Fujiwara, K.; Harada, K.; Yokoi K. & Hirukawa, H. (2003). Biped Walking Pattern Generation by using Preview Control of Zero-Moment Point, *Proc. of IEEE Int. Conf. on Robotics & Automation*, Taipei, Taiwan, pp. 1620-1626, September 2003.
Löffler, K.; Gienger, M.; & Pfeiffer, F. (2003). Sensor and Control Design of a Dynamically Stable Biped Robot, *Proc. of IEEE Int. Conf. on Robotics and Automation*, Taipei, Taiwan, pp.484-490, September 2003.
Oh, Y.; Ahn, K.; Kim, D. & Kim, C. (2006). An Analytical Method to Generate Walking Pattern of Humanoid Robot, *Pro. Of IEEE Int. Conf. On Industrial Electronics Society*, Paris, France, pp. 4159 - 4164, November 2006.
Yamaguchi, J.; Soga, E.; Inoue S. & Taknishi, A. (1999). Development of a Bipedal Humanoid Robot -control Method of Whole Body Cooperative Dynamic Biped Walking-, *Pro. of IEEE Int. Conf. on Robotics & Automation*, Detroit, USA, pp. 2299-2306, May 1999.
Zhu, C.; Tomizawa, Y.; Luo X. & Kawamura, A. (2004). Biped Walking with Variable ZMP, Frictional Constraint and Inverted Pendulum Model, *Pro. of IEEE Int. Conf on Robotics & Biomimetics*, Shenyang, China, pp. 425-430, August 2004.