Mechanical aspects of legged locomotion control

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Abstract

We review the mechanical components of an approach to motion science that enlists recent progress in neurophysiology, biomechanics, control systems engineering, and non-linear dynamical systems to explore the integration of muscular, skeletal, and neural mechanics that creates effective locomotor behavior. We use rapid arthropod terrestrial locomotion as the model system because of the wealth of experimental data available. With this foundation, we list a set of hypotheses for the control of movement, outline their mathematical underpinning and show how they have inspired the design of the hexapedal robot, RHex.
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1. Introduction: an integrative view of motion science

Motion science has not yet been established as a single clearly definable discipline, since the relevant knowledge base spans the range of biology (Alexander, 2003; Biewener, 2003; Daniel and Tu, 1999; Dickinson et al., 2000; Full, 1997; Grillner et al., 2000; Pearson, 1993), medicine (Winters and Crago, 2000), psychology (Haken et al., 1985), mathematics (Guckenheimer and Holmes, 1983) and engineering (Ayers et al., 2002). Locating the origin of control remains a substantial research challenge, because neural and mechanical systems are dynamically coupled to one another, and both play essential roles in control. While it is possible to deconstruct the mechanics of locomotion into a simple cascade—brain activates muscles, muscles move skeleton, skeleton performs work on external world—such a unidirectional framework fails to incorporate essential complex dynamic properties that emerge from feedback operating between and within levels. The major challenge is to discover the secrets of how they function collectively as an integrated whole. These systems possess functional properties that emerge only upon interaction with one another and the environment. Our goal is to uncover the control architectures that result in rapid arthropod runners being remarkably stable and possessing the same pattern of whole body mechanics as reptiles, birds and mammals (Blickhan and Full, 1993). Guided by experimental measurements, mathematical models and physical (robot) models, we postulate control architectures that necessarily include the constraints of the body's mechanics. We exploit the fact that body and limbs must obey inertia-dominated Newtonian mechanics to constrain possible control architectures.

This paper reviews the locomotion control hierarchy as a series of biologically inspired hypotheses that have given rise to a novel robot and that we are just beginning to translate into specific biologically refutable propositions. Here, we focus on the lowest end of this neuromechanical hierarchy where we hypothesize the primacy of mechanical feedback or 'preflexes'—neural clock excited tuned muscles acting through chosen skeletal postures (Brown and Loeb, 2000). Such notions are most succinctly expressed in the mathematical language of mechanics and dynamical systems theory. We view this paper, on one level, as a guide for the interested reader to the narrower technical literature within which these ideas have found their clearest

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(albeit incomplete, since the underlying mathematics is still far from worked out) expression. However, we intend as well that this presentation should be sufficiently explanatory as to stand alone for those outside the engineering and applied mathematics community, as an account of what we presently do and do not understand about the locomotion control hierarchy associated with the new machine, RHex.

Motivated by the view that synthesis is the final arbiter of understanding, we present the procession of inspiration, insight and implementation flowing from the biology toward engineering RHex, the most agile, autonomous legged robot yet built (Altendorfer et al., 2001; Buehler et al., 2002; Saranli et al., 2001), and the growing mathematical insight into locomotion arising in consequence. We organize the presentation of this flow from biology to engineering into three distinct conceptual areas as follows: (i) how the traditions of dynamical systems theory inform the overall framework and provide a point of departure for our work, addressed by hypothesis H_1; (ii) how a purely mechanical view of body morphology and materials design in conjunction with those dynamical systems theoretic ideas can begin to explain important features of animal locomotion, addressed by hypothesis H_2; and (iii) how the crucial and voluminous pre-existing data (arising from decades of painstaking work in neuroethology) about animals can offer hints on the manner in which a nervous system might be effectively coupled to the type of tunable mechanical system just described, addressed by hypothesis H_3.

Hypothesis H_1, introduced in Section 2, comprises a general orientation to the ideas of dynamical systems theory and their applicability to dynamical running. It asserts that the primary requirement of an animal's locomotion strategy is to *stabilize* its body around steady state periodic motions termed *limit cycles*. The section is concerned with elaborating the implications of this view as focused on patterns of mechanical response to perturbation, and reviewing the longstanding role dynamical systems thinking has had in the development of agile robot runners, including RHex.

We next introduce in Section 3 hypothesis H_2 proposing a specific solution to Bernstein's famous 'degrees of freedom' problem (Bernstein, 1967), representing a purely mechanical explanation for the appearance of synergies in animal locomotion. It posits the representation of a motor task via a low degree of freedom *template* dynamical system that is *anchored* via the selection of a preferred *posture*. The section underscores the intrinsic role that dynamical systems thinking plays in the development of this hypothesis, and explores some of its specific empirical concomitants through the illustrative example of the physical model, RHex.

On top of this physical layer, we introduce in Section 4 hypothesis H_3, a hypothetical architecture for its coordination via a tunable family of couplings to the nervous system. This proposed family of interconnection schemes between internal and mechanical oscillators is depicted summarily in Fig. 10, representing diagrammatically a plane of alternatives spanning on the one hand a range between pure feedback and pure feedforward control options, and, on the other, a range between completely centralized and completely decentralized computational options. We hypothesize a relationship between the 'noise' in the internal communication paths or internal computational world model, the time constants demanded by the physical task, and the preferred operating point to support its execution on this architectural plane. Once again, this predicted set of relationships is explored using the illustrative example of the physical model, RHex, and the very recent empirical relationships we have begun to observe between style of control or communication scheme and efficacy of behavior.

A brief conclusion reviews the nature of these hypotheses, and closes with the necessary humbling comparison of RHex to the wonderful, far more impressive locomotion capabilities of animals whose performance still far exceeds what we yet understand about, and even farther exceeds what we know how to build into legged running systems.

2. Stability-dynamical systems approach to motion science

Stability is essential to the performance of terrestrial locomotion. Arthropods are often viewed as the quintessential example of a statically stable design. Arthropod legs generally radiate outwards, providing a wide base of support. Their center of mass is often so low that their body nearly scrapes the ground. Their sprawled postures reduce over-turning moments. Hughes (1952) argued that the six legged condition is the 'end-product of evolution' because the animal can always be statically stable—at least three legs are planted on the ground with the center of mass within the triangle of support.

2.1. Dynamic stability in arthropod running

Statically stable design for slower arthropod locomotion does not preclude dynamic effects at faster speeds (Ting et al., 1994). Results from the study of six and eight-legged runners (Blickhan and Full, 1987; Full and Tu, 1990, 1991; Full et al., 1991) provide strong evidence that dynamic stability cannot be ignored in fast, multi-legged runners that are maneuverable. In running cockroaches, several locomotor metrics change in a direction that is consistent with an increase in the importance of dynamic stability as speed increases. Duty factors (i.e. the fraction of time a leg spends on the ground relative to the stride period) decrease to 0.5 and below as speed increases. Percent stability margin (i.e. the shortest distance from the center of gravity to the boundaries of support normalized to the maximum possible stability margin) decreases with increasing speed from 60% at 10 cm s^{-1} to values less than zero at speeds faster than
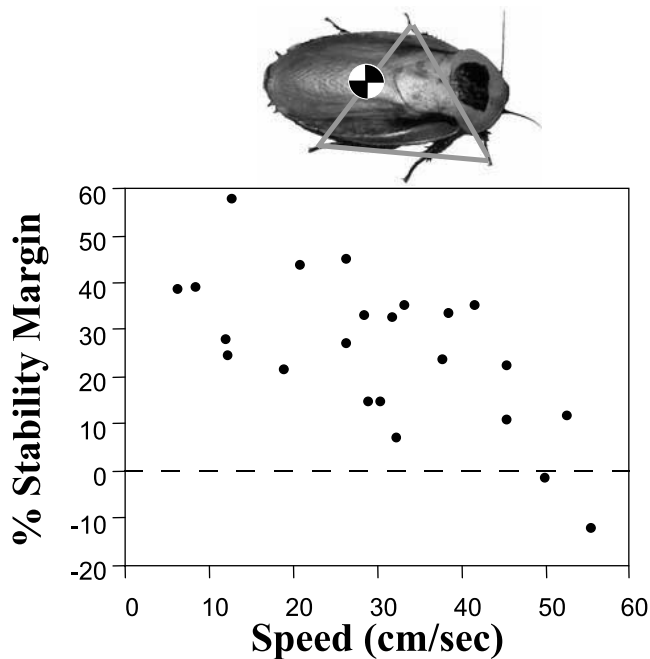
Fig. 1. Percent stability margin as a function of speed in running cockroaches. Percent stability margin is the shortest distance from the center of gravity to the boundaries of support normalized to the maximum possible stability margin. Percent stability margins of greater than zero indicate static stability. Values less than zero indicate static instability. Cockroaches show bouncing, spring-mass dynamics over 85% of their speed range. Modified from Ting et al. (1994).

$50 \, \text{cm s}^{-1}$ (Ting et al., 1994; Fig. 1). Negative percent stability margins indicate static instability. In cockroaches and crabs ground reaction forces create moments about the center of mass that cause pitching and rolling of the body. The resultant force of all legs or center of pressure is not directed through the center of mass throughout the stride. If the animal was stopped and characterized by static criteria, the resultant force vector would create a moment that could cause the animal to flip over. These polypedal runners remain dynamically stable because a force in one direction at one instant is later compensated by another force and distributed over time by the forces of inertia—the 'dynamics'.

At the fastest speeds, the importance of dynamics in arthropod legged locomotion is unambiguous. In cockroaches, the duration of double support (i.e. period when both tripods—or all six legs—are on the ground) decreases significantly with an increase in speed. The front leg, the shortest one, is lifted before the middle and rear leg so that only two legs of the tripod remain in contact with ground (Ting et al., 1994). At speeds greater than 1 m/s, the American cockroach runs quadrupedally and bipedally with aerial phases (Full and Tu, 1991). Even rapid running ants show aerial phases (Zollikofer, 1994). Ghost crabs propel themselves with two legs on the trailing side of the body as they leap into the air and landing on leading legs acting as skids (Blickhan and Full, 1987; Burrows and Hoyle, 1973).

These gaits demand dynamic stability using kinetic energy to bridge the gaps of static instability. In conjunction with a highly statically stable sprawled-posture, this ability to harness kinetic energy allows rapid and highly maneuverable locomotor performance. Terrestrial arthropods exploit the advantages of both static and dynamic stability.

2.2. Hypothesis H_1: dynamical stability

Dynamic behavior in nature's most statically stable designs argues for general hypotheses regarding function that view locomotion as a controlled exchange of energy. This notion is central to the formal understanding of stability at the foundations of dynamical systems theory.

We hypothesize that the primary requirement of an animal's locomotion control strategy is to stabilize its body around limit cycles. Stability denotes the tendency of a system at steady state to remain there, even in the presence of unexpected perturbations. Newtonian dynamics adds to each mechanical degree of freedom a velocity variable so that the dimension of the state space in question is double that of the purely kinematic 'configuration' space of joint variables. Thus, unlike purely kinematic models, dynamical models admit steady state motions that are not at rest, the most important for our hypothesis being limit cycles— periodic trajectories in state space in whose neighborhood there are no other periodic trajectories (Fig. 2). Perturbations shift the state onto those nearby trajectories which then either lead back toward the isolated limit cycle (stability) or away from it (instability). An *attractor* is a steady state motion in whose neighborhood every other motion leads back to it. Its *basin* is the complete set of states whose motions return back toward it.[3] We distinguish between perturbations to these state variables (positions and velocities), and parameters that represent both fixed characteristics, such as mass, and those altered volitionally such as leg stiffness. The latter appear as control variables. As is standard for dynamical systems models, this view predicts that perturbations to state variables will differ in rate of recovery, be coupled, and be subject to phase resetting.

2.3. RHex—an arthropod inspired dynamic robot

A recent comprehensive review of the growing insect inspired locomotion literature (Delcomyn, 2004) makes the useful distinction between biomimesis or 'biology-as-default' approaches to robot design (Ritzmann et al., 2000) and the bioinspired effort that we review in this paper. Rather than seeking to copy any specific morphological or even physiological detail, we hypothesize functional principles of biological design and test their validity in animal and physical models. In this paper, we

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[3] For a recent biomechanics oriented tutorial review of these ideas see Full et al., 2002, and for a complete technical introduction see Guckenheimer and Holmes, 1983.
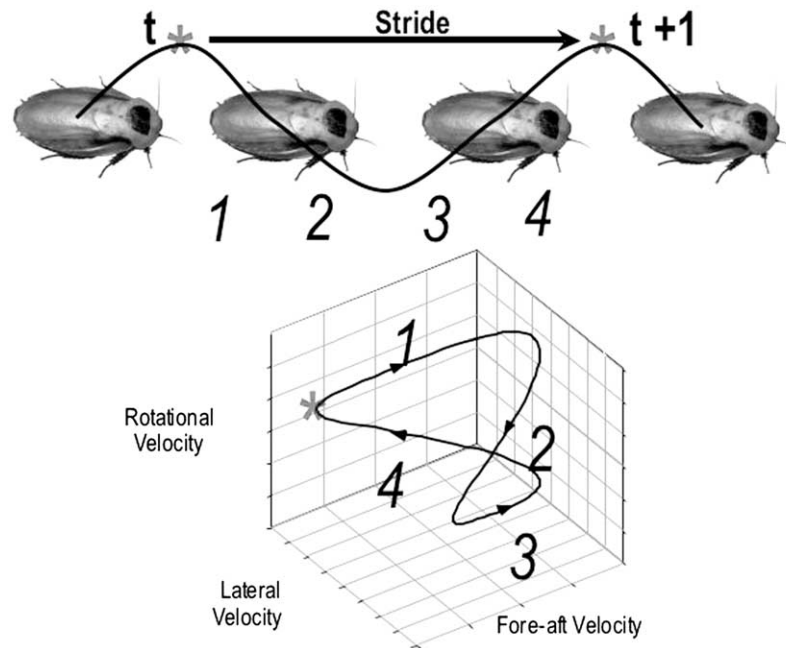
Fig. 2. Stable limit cycle for a running arthropod. The plot represents a limit cycle for rotational, fore-aft and lateral velocity of the animal's center of mass. The numbers show a time sequence through the stride (two steps). The stars show one complete cycle from t to $t + 1$. A limit cycle is a periodic trajectory in state space in whose neighborhood there are no other periodic trajectories. Perturbations shift the state onto those nearby trajectories which then either lead back toward the isolated limit cycle (stability) or away from it (instability). Modified from Full et al. (2002).

emphasize aspects of bioinspired mechanical design that confer dynamical stability.

The potential value of dynamically stable robotic locomotion was dramatically demonstrated two decades ago in Raibert's series of breakthrough mono-, bi- and quadrupedal hopping machines (Raibert, 1986). These first dynamically dexterous robots ushered in a new understanding that robot programming could be construed as managing the phase of energy expenditure in the working environment.

The role of tuned compliance in running has been explored in several legged robots since Raibert's work (Robinson et al., 1999). The central importance of under-actuated (i.e. there are fewer actuators than degrees of freedom and their limited power is explicitly accounted for) design for autonomous legged machines was demonstrated in the Scout class of quadrupeds (Buehler et al., 1998), which also pioneered the use of compliant sprawled posture in quadruped bounding with consequent self-stabilized roll (Papadopoulos and Buehler, 2000).

Integrating the virtues of these engineering insights with biological inspiration from dynamic legged locomotion in arthropods, we designed the hexapedal robot, RHex. RHex is the world's first autonomous legged machine capable of mobility in general terrain approaching that of an animal. RHex (Buehler et al., 2002) exhibits unprecedented mobility over badly broken terrain (Fig. 3). Its normalized speed is at least five times greater than that of any prior autonomous legged machine (Saranli et al., 2001). Its normalized efficiency (specific resistance of 0.6) again sets a new benchmark for autonomous legged machines,

approaching that of animals (Weingarten et al., 2004). Not coincidentally, RHex exhibits the mass center dynamics displayed by legged animals (Altendorfer et al., 2001).

The crucial new contribution RHex makes to legged locomotion lies in its ability to recruit a compliant sprawled posture (Saranli et al., 2001) for completely open loop stable dynamic operation (Altendorfer et al., 2003). Unlike prior legged machines that operate either only quasi-statically or only dynamically, RHex exhibits both capabilities. Its six legs and elongated body allow it to stand, creep, or walk with its center of pressure well contained within a tripod (or more) of support. However, as its speed moves into the regime of one body length per second and beyond, a well tuned RHex develops dynamic bouncing gaits (Altendorfer et al., 2001) characterized by regular periodic steady state, center of mass (COM) motions that resist severe and even adversarial perturbations (Saranli et al., 2001).[4] Recently, we have reported as well the introduction of stable and efficient bipedal gaits for RHex (Neville and Buehler, 2003). In view of this task open loop stability, RHex presents a physical model of the biological notion that 'preflex'

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[4] Intrigued by the utility of underactuated compliant physical models of locomotion, subsequent researchers (Quinn et al., 2001) have pursued literally the analogy RHex suggests to a rimless compliant spoked wheel (Coleman et al., 1997; McGeer, 1990) by adding additional 'spokes'. The limitation we originally noted in this design—the constrained range of achievable ground reaction force vectors (Saranli et al., 2001)—seems likely to effect rapid volitional maneuvers. The resulting constraint on spring loaded inverted pendulum bouncing mechanics can affect speed and efficiency.
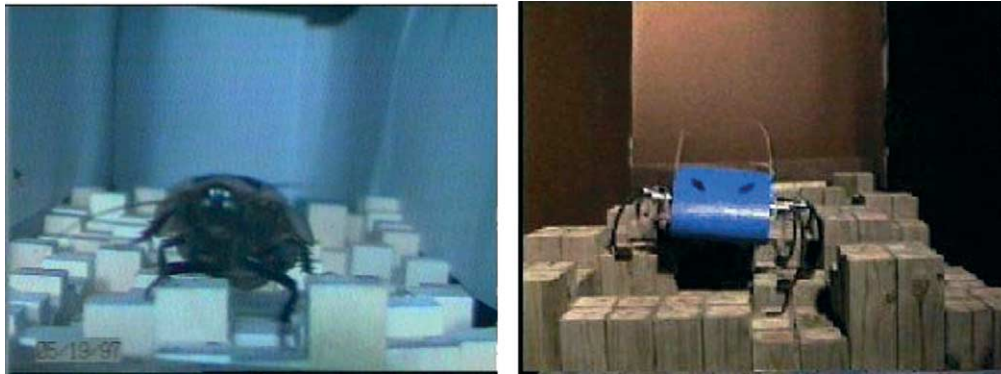
Fig. 3. Biologically inspired hexapod robot, RHex. A. A cockroach, *Blaberus discoidalis*, negotiating irregular terrain with obstacles as high as three times its 'hip' height without altering its preferred speed (Full et al., 1998b). B. RHex, a biologically inspired hexapod robot (Buehler et al., 2002) negotiating a scaled-up version of the same irregular terrain faced by the arthropod. Remarkably RHex completed the challenge without sensory information from the environment.

(Brown and Loeb, 2000) stabilization may represent a key advantage of sprawled posture running.

In summary, exemplifying hypothesis H_1, design for dynamic stability is the key to this new robot's performance. Rather than deliberately choosing its limb motions to place its mass center in a precisely planned manner, RHex expends its energy so as to create stable limit cycles. RHex's dynamical competence results from the stability of these limit cycles that exhibit a large enough basin to return the COM state back toward the steady state locomotion pattern in the face of recurring unanticipated perturbations.

3. Control-collapse of mechanical dimension

Although the notion of stable limit cycles and their basins introduced in hypothesis H_1 offers conceptual simplicity, it appears to ignore the vast disparities in shape, size and morphology that make animal locomotion seemingly so mysterious. The control of a dynamical system with many legs, joints, muscles and neurons seems hopelessly complex. Perhaps nowhere is Bernstein's 'degrees of freedom' problem (Bernstein, 1967) better exemplified than in arthropods with an assortment of multi-jointed legs. Our next hypothesis proposes a specific solution to this long-standing degrees of freedom problem.

3.1. Spring-mass dynamics of arthropod running

Surprisingly, the dynamics of the center of mass in arthropods is described by a simple model and appears to be common among diverse legged animals. In faster moving cockroaches and crabs, the mass center can be modeled as a bouncing ball or pogo-stick (Blickhan and Full, 1987; Full and Tu, 1990, 1991). Gravitational potential energy and forward kinetic energy fluctuate in phase. As the animal's body comes down on three or four legs, it is decelerated in the fore-aft direction while vertical force increases. Later in the step the body is accelerated forward and upward as vertical force decreases. The pattern is repeated for the next set of legs. The center of mass attains its lowest position at mid-stance much like we do when we run. In fact, the ground reaction force pattern for six- and eight-legged arthropods is fundamentally similar to two-, and four-legged vertebrates, despite the variation in morphology (Blickhan and Full, 1987; Cavagna et al., 1977; Full and Tu, 1990, 1991; and Heglund et al., 1982). All designs progress by bouncing. Running humans, trotting dogs, cockroaches and sideways running crabs can move their bodies by having legs work synergistically, as if they were one pogo-stick. Two legs in a trotting quadrupedal mammal, three legs in an insect and four legs in a crab can act as one leg does in a biped during ground contact. The center of mass of the animal undergoes repeated accelerations and decelerations with each step, even when traveling at a constant average velocity. Cockroaches and crabs do not necessarily show an aerial phase, but are clearly using a bouncing gait. These results suggest that a running gait should be redefined to include a complete dynamic description rather than depending on a single variable such as an aerial phase. McMahon et al. (1987) have shown that an aerial phase is not a requirement for the definition of a bouncing or running gait in humans. Gravitational potential energy and forward kinetic energy fluctuate in phase in humans running with bent knees and no aerial phase.

The simplest model that best explains the running motion is a mass (i.e. the body) sitting on top of a virtual spring (i.e. representing the legs) where the relative stiffness of all the legs acting as one virtual spring (k_{rel}) equals

$$k_{rel} = (F_{vert}/mg)/(\Delta l/l)$$

where F_{vert} is the vertical ground reaction force of the virtual spring at midstance, Δl is the compression of the leg spring, l is the length of the uncompressed leg spring and mg represents weight (Blickhan, 1998; McMahon and Cheng, 1990; Farley et al., 1993). Surprisingly, the relative, individual leg stiffness of a running cockroach and crab are remarkably similar to that found in trotting dogs,
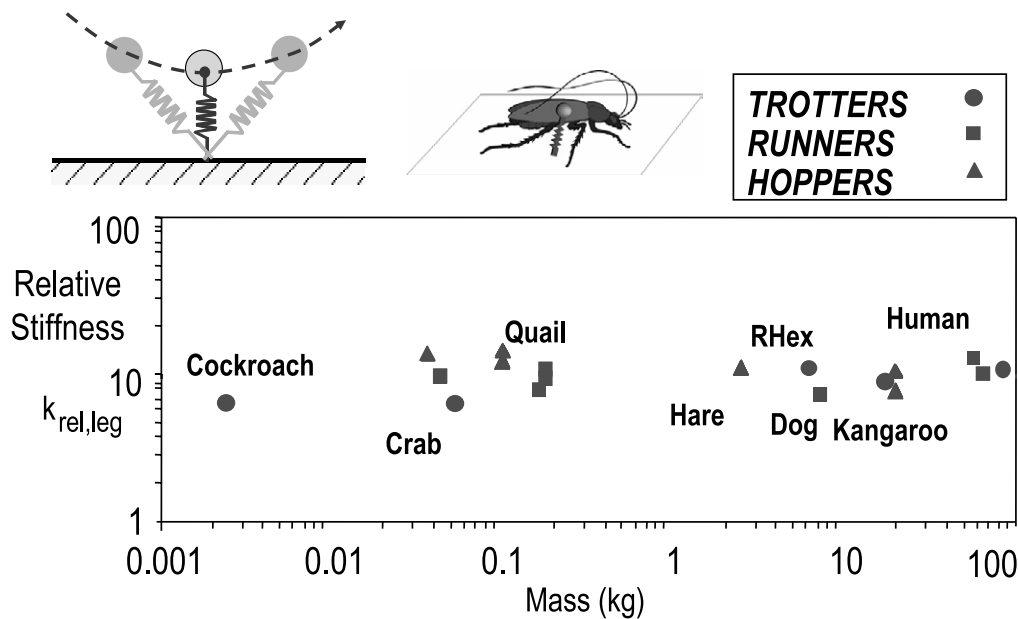
Fig. 4. Relative leg stiffness as a function of body mass for trotters, runners and hoppers. Relative individual leg stiffness is independent of leg number, skeletal type and body mass. Relative individual leg stiffness is a dimensionless number representing the ratio of normalized force to normalized compression. Normalized force is calculated by dividing the peak vertical ground reaction force by weight. Normalized compression is calculated by dividing leg spring compression by 'hip' height. Modified from Blickhan and Full (1993).

running birds and bouncing kangaroo rats (~ 10; Blickhan and Full, 1993; Fig. 4).

Further evidence that running arthropods are equivalent to a mass atop a tuned spring comes from the examination of stride frequency. In quadrupedal mammals, stride frequency increases with speed within a trot. At higher speeds, quadrupeds switch to a gallop where stride frequency remains constant, so longer strides are taken to go faster. Ghost crabs (Blickhan and Full, 1987) and cockroaches (Full and Tu, 1990) show this trot-gallop transition with respect to their stride frequency pattern. Speed and stride frequency at the trot-gallop transition scales with body mass such that a single function predicts values for four-, six- and eight-legged runners. A crab and a mouse of the same mass change from a trot to a gallop at the same speed and stride frequency, despite extreme differences in locomotor design (Full, 1989; Blickhan et al., 1993).

Spring-mass dynamics of the center of mass are not restricted to the sagittal plane. Sprawled-posture arthropod runners, such as insects, generate large lateral and opposing leg forces in the horizontal plane (Full and Tu, 1990). As in the sagittal plane, the three legs of the tripod appear to function synergistically as if they were one virtual, lateral leg spring (Schmitt and Holmes, 2000a, 2001). The lateral leg spring simply switches sides as the next tripod lands. This spring-mass model does remarkably well in reproducing the center of mass dynamics derived from measured leg force data.

Preliminary analysis of the kinematics of high-speed running in arthropods is consistent with the hypothesis that the complexity of control or degrees of freedom problem is solved by a controlled collapse of dimensions (Full et al., 2003). The cockroach, *Blaberus discoidalis*, has at least 42 degrees of freedom available. If these joint motions do not act synergistically (as if they were one) then many independent control signals might be required. A high degree of stereotypy and rhythmicity does not guarantee a reduced number of control signals. Multiple control signals could be required when the timing of joint-angle changes differ among legs or when one joint in a leg shows little movement while another undergoes large angle changes. Principle component analysis (PCA) on joint angle data from straight-ahead running revealed that three PC's could account for nearly all of the systematic variation of the limbs with a single component representing over 80% of the variation. PCA revealed strongly linear correlations between joint angles within and among all legs at all points in time. PCs generated from a reduced population of data were able to reconstruct data of different strides and other individuals. A preferred posture appeared common among individuals of the same species. At low speeds, more PCs were required to explain the variation. These results suggest that rapid running cockroaches operate within the same low dimensional subspace of the much higher possible available degrees of joint freedom. There appears to exist a *posture*, a targeted low dimensional set, toward which each animal's controller regulates transient perturbations. The simple posture suggests simple control.

Large animals derive a strongly favorable energetic consequence from pogo stick running (Alexander, 1988;

Fig. 5. Three generations of RHex legs. RHex's legs are designed to afford three degrees of passively compliant freedom so arranged that the radial 'spoke' direction is much more compliant than the relatively stiff lateral and tangential bending axes (Moore and Buehler, 2001). While the initial homogeneous (Delrin, left panel) legs and early succeeding generations of passively sprung four-bar (middle panel) leg constructions enforced point contact between toe and ground, the most successful designs are formed in 'half-circle' configurations (right panel) that promote a complicated rolling contact with the ground (Moore and Buehler, 2001).

Alexander and Vernon, 1975; Biewener and Baudinette, 1995; Cavagna et al., 1977) since springs store and return the kinetic energy of the mass center during stance. In our view, however, this agile pattern, characteristic of a pogo stick, exemplifies a general approach to solving Bernstein's 'degrees of freedom' problem (Bernstein, 1967) by representing in as few as possible degrees of freedom the task of translating the body's mass center (Fig. 5).

3.2. Hypothesis H_2: collapse of dimension

The spring-mass dynamics common to legged runners as diverse as arthropods strongly supports the proposal that simple models can characterize the task-relevant behavior of even the most complex systems. Multiple legs, joints and muscles operate synergistically to reduce the number of dimensions down to those of pogo-sticks (Fig. 6). We term
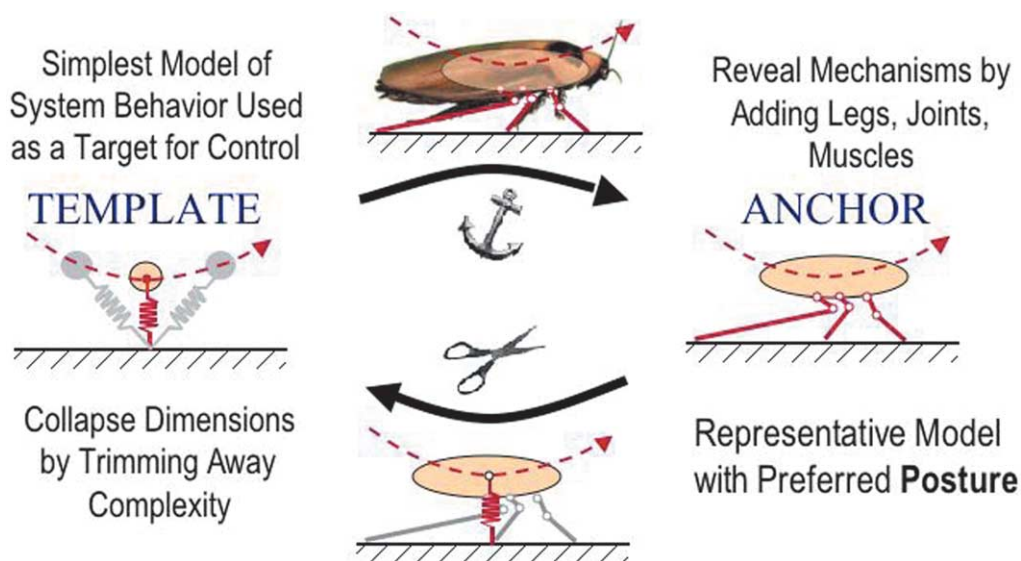

Fig. 6. Modeling locomotion—template and anchors. A *template* represents the simplest model of system behavior (fewest number of parameters) used as a target for control (Full and Koditschek, 1999). The most general template for locomotion is the spring-loaded inverted pendulum (SLIP). The simplest model is unable to reveal the mechanisms of interest producing locomotion. The template must be anchored to produce a representative model by adding legs, joints and/or muscles depending on the question asked. This representative model or *anchor* has a preferred posture. We hypothesize that for each placement of the body's mass center, there is a corresponding 'favorable' placement of leg angles and body attitude that trim away the controlled degrees of freedom down to that of the body. We term this the Posture Principle.

these simple models *templates* (Full and Koditschek, 1999). A template is the simplest model with the least number of variables and parameters that exhibits a targeted behavior of a system. The presence of a template tells us that a system can restrict itself to a low-dimensional subset of its high dimensional morphology in the space of possible motions. A template gives us the opportunity to hypothesize specific control principles that attain this collapse of dimensions.

Templates define the behavior of the body and serve as targets for control. However, they do not provide causal explanations of the detailed neuro-mechanical mechanisms that give rise to the template behavior. Minimal models must be grounded or anchored in sufficiently representative morphological and physiological details. *Anchors* are elaborated models with greater complexity than templates. Even the simplest anchors facilitate the creation of integrative hypotheses concerning the role of multiple legs, the joint torques that actuate them, muscle recruitments that produce those torques, and the neural circuits that activate the ensemble.

Templates are anchored by the introduction of a specific posture. For legged locomotion anchors represent the body segments, legs and muscles that are wrapped around the pogo-stick template in a *preferred posture* whose coordination mechanisms imply specific controls. We hypothesize that for each kinematic placement of the body's mass center, there is a corresponding 'favorable' placement of leg angles and body attitude that reduce the controlled leg degrees of freedom down to that of the body. We term this the Posture Principle.

Prior literature in motor science has been concerned with the identification of muscle synergies (Hogan et al., 1987; Saltiel et al., 2001). Task specification via such low dimensional virtual force fields has been explored in the robotics literature as well (Pratt et al., 2001). Alternatively, researchers focusing on the phenomenology of the kinematics rather than the forces associated with animal motion have discovered low degree of freedom patterns of movement in high dimensional limb traces (Lacquaniti et al., 1999), in some cases associated with a hypothesized underlying low dimensional reference trajectory (Domen et al., 1999). The new idea introduced in this hypothesis H_2 of posture anchored templates is that the underlying low dimensional motion and force patterns arise as physical solutions of low dimensional target dynamical systems that emerge mechanically from the properly shaped and tuned complex body.

Translating the empirical observations concerning animal runners reviewed in Section 3.1 into the more theoretical terms of this hypothesis affords, in turn, a mathematical framework for design and analysis that connects and unifies a number of independent prior threads running through the past two decades of dynamically dexterous robotics research. The notion of an anchor is biologically inescapable (animals, of course, are not literal pogo sticks) but can also be reinterpreted with respect to the dynamical systems theoretic idea of basins in the state space of the complex system leading down to a low dimensional surface that 'carries' the far simpler template dynamics—formally, an attracting invariant submanifold (Guckenheimer and Holmes, 1983). The notion of a posture is inherently zoomorphic, but also connects up to the long-standing idea of a pseudo-inverse for the resolution of kinematic redundancy (Murray et al., 1994). With this passage from empirical observation to geometric prescription we are now in a position to trace the prior threads of engineering research this hypothesis can bring together in the design and function of the robot RHex, a physical model of an anchored dynamical template (Altendorfer et al., 2001) engineered to prefer a specific posture.

3.3. RHex—using a spring-mass template anchored in an arthropod design

Mechanically, RHex has a rigid body with six compliant legs, each driven by their own servo-motor at the effective axle (Buehler et al., 2002). The robot uses an alternating tripod as do insects, with legs clocked to swing in parallel through stance, thereby mimicking in steady state (albeit generally not during transients) Raibert's quadruped whose paired telescoping legs swung through stance in parallel, using active control to enforce a literal pogo stick. The three legs of RHex's tripod sum to generate pogo-stick or spring mass template dynamics. Direct measurements of ground reaction forces at steady state in a well tuned gait reveal whole body dynamics that are remarkably similar to 2-, 4-, 6- and 8-legged runners (Fig. 7; Altendorfer et al., 2001). Kinetic and potential energy of the center of mass oscillate in phase as the robot bounces from step to step. Surprisingly, even estimates of relative individual leg stiffness are not significantly different from all legged animal runners, despite the radical difference in materials (Fig. 4).

RHex's legs (Fig. 5) are built from a carefully designed fiberglass composite that affords at least three degrees of passively compliant freedom so arranged that the radial 'spoke' direction is much more compliant than the relatively stiff lateral and tangential bending axes (Moore and Buehler, 2001). While the initial homogeneous (Delrin, Fig. 5 left) legs and early succeeding generations of passively sprung four-bar (Fig. 5, middle) leg constructions enforced point contact between toe and ground, the most successful designs are formed in 'half-circle' configurations (Fig. 5 right) that promote a complicated rolling contact with the ground (Moore and Buehler, 2001). These 'half-circle' fiberglass legs are much more robust and their resistance to breakage in repeated regimes of very high force permitted an aggressive cycle of empirical gait parameter tuning to be discussed below. As this review unfolds, we trust the reader will come to see that neither leg design nor algorithmic adjustment alone but, rather, their simultaneous coordination, has resulted in the significant performance increments over the original version of RHex (Saranli and Koditschek, 2003) documented in Weingarten et al. (2004).
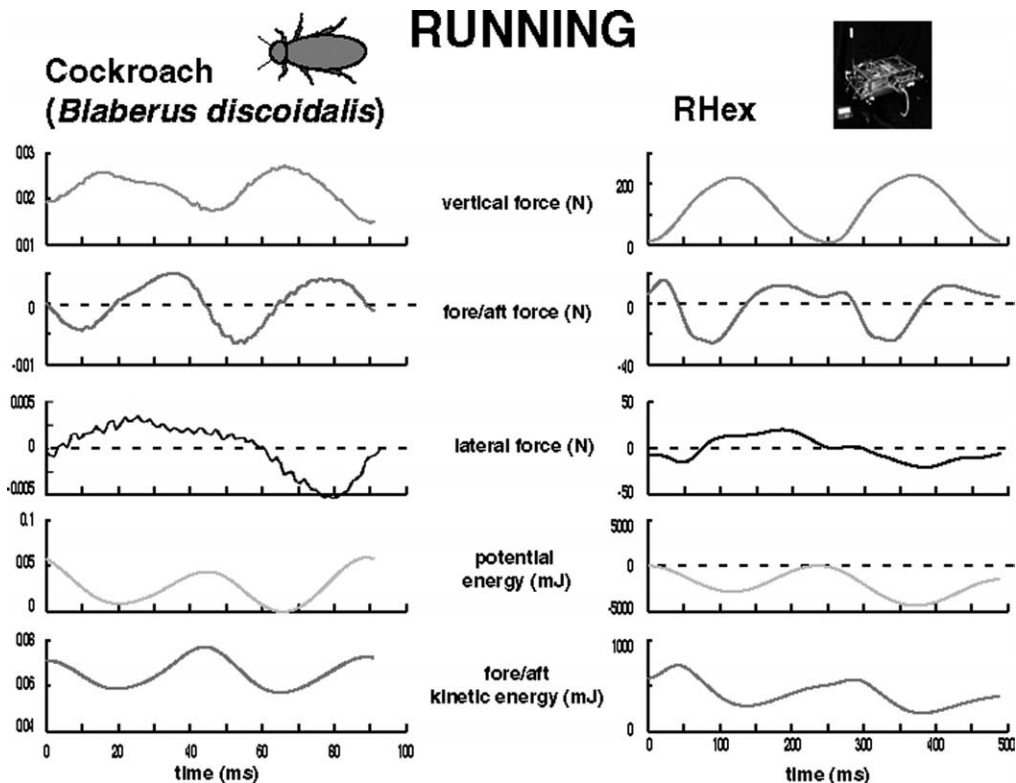
Fig. 7. The dynamics of the center of mass of a cockroach compared to the robot, RHex. Ground reaction force and energy of the center of mass during one stride (i.e. one complete leg cycle) for a 2.3 g cockroach, *Blaberus discoidalis*, (Full and Tu, 1990) and a 7 kg hexapedal robot, RHex (Altendorfer et al., 2001). Tracings represent the following (from top to bottom): Vertical, fore-aft and lateral ground reaction forces obtained from a force platform, gravitational potential energy and fore-aft kinetic energy fluctuations of the center of mass.

For all design generations, RHex's passive compliant legs introduce an effective posture principle. This may be most easily envisioned in the following thought experiment. When the hip motors are locked and a tripod of legs (two ipsilateral and one contralateral) is touching the ground at any point between 'toe' and 'elbow' there is enough passive leg compliance that the body's mass center can be readily moved around. For each center of mass position, the leg springs enforce a unique body attitude and set of leg configurations—the *posture* associated with that displacement. Throughout the stance phase of a dynamical gait, RHex's damped springy legs enforce this posture principle in a tireless and reliable manner (up to the limits of their materials' strengths) with no expenditure of energy (rather, in fact, a fair bit of dissipation) nor computation. Perturbations to the mass center, whether directly or from terrain variations communicated through the legs will be accommodated, and 'managed' with certain directions of energy dissipated and others promoted, in a purely 'preflexive' manner.

Choice of radial compliance represents one good example of the direct design influence of the anchored templates idea. The compliance properties of RHex's legs have been designed so that their combined stiffnesses contribute to the supporting tripod a net mechanical natural frequency in the sagittal spring-mass, radial direction commensurate with stride frequency governed by the zero-torque speed limit of a hip motor. However, in general, the selection of the mechanical posture principle remains largely a matter of empirical design constrained by our still very imperfect understanding of and implications for control over the materials properties that govern the legs' shape and compliance.[5] These crucial properties emerge from painstaking empirical iterations balancing the conflicting demands of robustness and ability to withstand very large peak forces, against ease of manufacture, driven by intuition concerning the desired posture principle. This struggle is leading to new hypotheses of design trade-offs, development and even evolution that can be tested on animals.

As we have suggested in introducing hypothesis H_2, above, the appearance of the spring-mass template in the presence of a carefully engineered posture forges an important conceptual link between the biological inspiration and a parallel line of prior theoretical ideas in robotics leading up to RHex. Notwithstanding the conceptual breakthrough Raibert's runners represented (Raibert, 1986), introducing to robotics the notions surrounding

[5] Even were we to take full advantage of the important advances in materials design and prototyping for robotics (Cham et al., 2002), our limited present mathematical insight into the nature of preflexive anchoring, would preclude comprehensive application in the present setting.

hypothesis H_1 reviewed in Section 2, it remained clear that a morphological copy of a literal pogo stick could not offer the foundation of a general purpose utilitarian platform. The ensuing decade witnessed a series of increasingly high dimensional dynamically dexterous machines for batting (Buehler et al., 1990, 1994; Rizzi et al., 1992; Rizzi and Koditschek, 1996), brachiating (Nakanishi et al., 2000), and even running (Westervelt et al., 2003) focused upon how to build controllers for usefully complex high degree of freedom morphologies resulting in low dimensional attracting invariant submanifolds carrying simple 'task worthy' dynamics. These formal geometric renditions of the posture anchored template hypothesis H_2 have been applied to simulation models of RHex and shown numerically to result in strongly stable highly maneuverable running (Saranli and Koditschek, 2003). However, they all rely upon sensory feedback and accurate internal dynamical models far beyond the resources presently available onboard RHex. Thus, we require an additional hypothesis addressing how the 'coordination' of multiple degrees of freedom might be accomplished over a range of control architectures presenting varying dependence upon sensory feedback and internal models. We turn to biology, once again, where the established notion of a central pattern generator offers a general perspective on coordination that we will rework in more specific terms as a family of parametrized architectures for coupling up internal neural 'clocks' to properly tuned physical 'mechanisms.'

4. Coordination: neural clocked mechanisms

The complexity of control may be solved by a collapse of dimensions down to stable templates with large basins as summarized in hypothesis H_1. These simple control targets, *templates*, appear to be *anchored* by a low dimensional posture, as postulated in hypothesis H_2. In turn, *simple postures* suggest *simple control*, such as a central pattern generator or clock-like signal that can excite the animal's tuned musculo-skeletal system. In this section, we introduce a third hypothesis proposing a plane of coordination architectures addressing the range of couplings between internal clocks and external mechanisms that can be observed in animal locomotion. Once again, this biological inspiration holds significant value for robotics.

4.1. Passive, dynamic self-stabilization in arthropods

Many-legged mobility systems negotiating rough terrain were hypothesized to use a follow-the-leader gait, precise foot placement and extensive tactile feedback. However, preliminary studies on rapid running cockroaches show that preferred speed is maintained during locomotion over rough terrain with barriers reaching three times the height of the animal's center of mass (Full et al., 1998a). Cockroaches use the same alternating tripod gait observed on flat terrain

and do not use a follow-the-leader gait. Simple feedforward motor output appears to be effective in the negotiation of rough terrain when used in concert with a mechanical system that stabilizes passively. These data lead to the hypothesis that dynamic stability and a conservative motor program may allow many-legged, sprawled posture animals to miss-step and collide with obstacles, but suffer little loss in performance. Rapid disturbance rejection appears to be an emergent property of the mechanical system. Following the empirical demonstration of mechanical self-stability in spoked 'rimless' wheels and associated physical (McGeer, 1990) and mathematical (Coleman et al., 1997) walking models, a plate-like foot was shown empirically and in simulation to confer mechanical self-stability in a spring-loaded hopping monoped (Ringrose, 1997), anticipating results concerning the self-stabilizing spring-loaded, inverted pendulum (SLIP) template that we now describe.

4.1.1. Predictions from models

To explore the role of the mechanical system in control, Kubow and Full (1999) designed a two-dimensional, feed-forward, dynamic model of a hexapedal runner. The model adopted a dorsal view, because sprawled posture animals operate more in the horizontal plane. More importantly, instability by spinning out of control was assumed to be more important than falling. The model was driven by a feed-forward signal with no equivalent of neural feedback among any of the components. The model's forward, lateral and rotational velocities were similar to that measured in the animal at its preferred velocity. Surprisingly, the model self-stabilized to velocity perturbations on a biologically relevant time scale. The rate of recovery depended on the type of perturbation. Recovery from rotational velocity perturbations occurred within one step, whereas recovery from lateral perturbations took multiple strides. Recovery from fore-aft velocity perturbations was the slowest. Perturbations were dynamically coupled where alterations in one velocity component necessarily perturbed the others. Perturbations altered the translation and/or rotation of the body that consequently provided mechanical feedback' by altering leg moment arms. The model supported the hypothesis that self-stabilization by the mechanical system could assist in making the neural contribution of control simpler.

A simpler three-degree of freedom mechanical model or template for the horizontal plane dynamics of rapidly running legged animals developed by Schmitt and Holmes (2000a,b) stands as an exemplar with regard to neuromechanical stability analysis (Fig. 8A). As mentioned above, the legs involved in each stance phase of an insect's tripod can be modeled by a single virtual or effective passive elastic member, the 'foot', which is set in contact with the ground according to a preset feedforward protocol. The body is free to rotate. The resulting lateral leg spring model exhibits asymptotically stable periodic gaits similar to those of insects over a range of forward speeds. The lateral leg spring
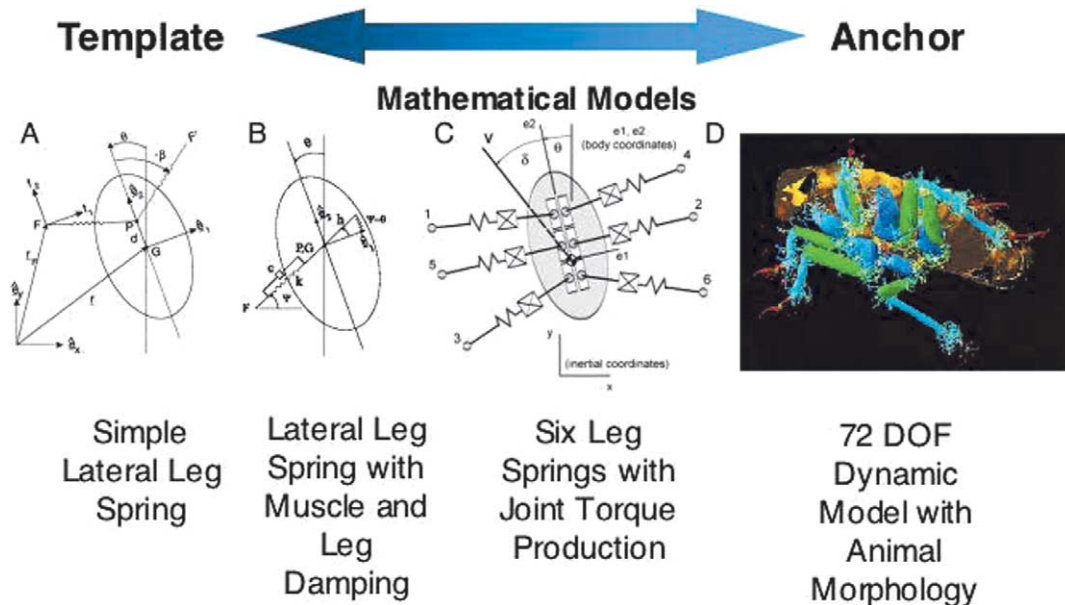
Fig. 8. Dynamic mathematical models of insect locomotion. A. Simple lateral leg spring model, LLS (Schmitt and Holmes, 2000a,b, 2001). The horizontal plane model bounces from side to side on a single virtual leg spring that switches sides as it progresses forward. The body is free to rotate. The single leg spring represents the summed behavior of three legs of an insect's tripod. Remarkably, the LLS self-stabilizes to perturbations without the equivalent of neural feedback from the environment. B. Lateral leg spring model with damping and a Hill-muscle model capable of generating torques (Schmitt and Holmes, 2003). C. Six-legged dynamic model that reproduces the individual leg ground reaction forces seen in insects during running. Each leg is modeled as a linear spring endowed with two inputs, force-free length and 'hip' position. These inputs allow legs to generate axial forces and hip torques (Full et al., 2004). D. Full dynamic model. The cockroach, *Blaberus discoidalis*, was sectioned and reconstructed in 3D. The ADAMS model shown here runs dynamically by applying torques only at the joints.

model belongs to a class of mechanical models for which neural or other detailed feedback is not necessary for stability. The purely mechanical effect of angular momentum transfer from step to step can produce strong asymptotic stability of body orientation and angular velocity and neutrally stable headings and forward speeds. The moments involved are primarily due to lateral forces generated at the feet. Lateral and yaw oscillations, which might seem inefficient, are actually necessary for passively stable gaits. Varying parameters of the model (mass, leg spring stiffness, leg angle, leg length and inertia) reveals that animals operate near or at the stability optimum for each parameter (Schmitt et al., 2002). These findings support the hypothesis that a tuned mechanical system is required for rapid passive recovery from perturbations.

Schmitt and Holmes (2003) elaborated the lateral leg spring model by adding damping to the leg and a Hill-type muscle capable of generating 'hip' torques (Fig. 8B). These additions preserved passive asymptotic stability for body orientation and rotational velocity, added stability in forward speed, but did not reproduce the moments observed in cockroaches. Full et al. (2004) anchored the model further by replacing the single virtual leg spring with six legs to examine the effects of the large lateral and opposing leg forces measured in sprawled-posture runners (Fig. 8C). Each leg was modeled as a linear spring endowed with two inputs, force-free length and 'hip' position. These inputs allowed legs to generate axial forces and hip torques. Inputs were determined from measured foot force and kinematic

body data from the cockroach, *Blaberus discoidalis*. The model predicted stable and unstable regions of stride frequencies, stride lengths and leg touchdown positions. The model was only stable when the animal's actual locomotor kinematics were used. This more anchored model argues that stability should be added to the morphological and neuro-muscular explanations that are hypothesized to limit locomotor behavior.

4.1.2. Experimental measurements

If the legs and muscles of arthropods contribute to passive dynamic control during rapid running as suggested by dynamic models, then motor signals should be simple, muscles should be more than actuators to manage energy, legs should be tuned spring-damper systems and bodies should recover rapidly from perturbations.

The strain pattern of three anatomical extensor muscles (177c, 178 and 179) acting across the coxa-trochanter-femur joint was determined by using video motion-analysis on free running cockroaches (Ahn, 2000; Ahn and Full, 2002; Full et al., 1998b). Simultaneous electromyographic recordings showed that all three muscles are activated by two or three muscle action potentials at the beginning of the stance period. Muscle power output was measured using these in vivo strain and activation patterns. Although all muscles were activated with a simple neural signal at nearly identical phases, muscle 177c generated mechanical energy like a motor, muscle 179 absorbed energy like a brake, and muscle 178 did both. Muscles receiving a simple clock-like neural

signal can manage energy. The multi-functional mechanical behavior of muscles supports the hypothesis that tuned mechanical feedback can simplify neural control.

To test the hypothesis that individual legs of arthropods can provide passive self-stabilization, Dudek and Full (2001) oscillated legs dynamically. The cockroach hind limb has the potential to act as a passive, exoskeletal leg spring-damper in the sagittal plane because of its more vertically oriented joint axes. Dynamic oscillations in the dorso-ventral direction (orthogonal to the plane of rotation for the joints) yielded stiffness, damping, and resilience values (Dudek and Full, 2002, 2004). Resilience of the limb ranged from 65 to 85% depending on whether the body-coxa joint was free to rotate or not, but was independent of oscillation frequency. Stiffness and damping coefficients allow estimation of a damping ratio (ζ), assuming the body is sitting on top of a support tripod during the stance phase of running. Estimates from individual legs predict that running cockroaches will be under-damped permitting energy storage and return. While the resilient legs are part of an under-damped system, they can store and return, at best, only 50% of the energy used to lift and accelerate the center of mass during a step.

Dynamic oscillations of individual legs focus attention on the extent of energy absorption and suggest that managing energy with respect to perturbations may be as important as energy storage and return. Dudek and Full (2004) measured the mechanical properties of the support tripod directly by dynamically oscillating cockroach bodies while they ran tethered atop a Styrofoam ball floating on an air bearing. A servomotor, driving a lever attached to the animal's body, imposed sinusoidal force oscillations in the sagittal plane and recorded the induced displacements. The measured dimensionless stiffness of the tripod (14) was remarkably similar to that estimated from force platform data (16, Blickhan and Full, 1993; Fig. 4). As predicted from individual legs, the support tripod was under-damped ($\zeta = 0.15$) when perturbed by force oscillations of less than half body mass and at the low frequencies corresponding to the stride frequencies used in running. Most engineering control systems have a damping ratio between $0.3–0.8$ permitting energy storage and return (McMahon and Greene, 1979). During high frequency force perturbations of more than body mass, the support tripod is under-damped ($\zeta = 0.75$), but easy to control with little overshoot. In short, the mechanical properties of the support tripod of running cockroaches appear well suited to allow spring-like bouncing, but passively reject rapid perturbations.

To test the self-stabilization hypothesis in freely running arthropods, Jindrich and Full (2002) perturbed rapidly running insects by designing a novel apparatus. The apparatus used chemical propellants to accelerate a small projectile that generated reaction force impulses of less than 10 ms in duration. The apparatus was mounted onto the thorax and positioned to propel the projectile laterally. The propellant was sufficient to produce a nearly ten-fold

increase in lateral velocity relative to maxima observed during unperturbed locomotion. Lateral velocity began to recover within 13 ms after the start of a perturbation. This duration is comparable to all but the fastest reflex responses measured in insects (Höltje and Hustert, 2003) and is likely shorter than a purely neurally mediated correction when the delays of the musculo-skeletal system response are included. Cockroaches recovered completely in 27 ms and did not even require step transitions to recover from lateral perturbations. Instead, they exhibited viscoelastic behavior in the lateral direction with spring constants similar to those observed during unperturbed locomotion. The rapid onset of recovery from lateral perturbations supports the hypothesis that preflexes augment or even dominate neural stabilization by reflexes during high-speed running.

4.2. Hypothesis H₃: tunable coordination control architecture

The need for coordination emerges, in general, from the inescapable presence of compartmentalized modular redundancy (Gerhart and Kirschner, 1997)—multiple copies of resources such as fingers, arms, legs—whose recruitment over time must be managed. It is simplest to introduce our formal hypothesis about the coordination of running by reference to the system of coupled oscillators depicted in Fig. 9, originally introduced in Klavins et al. (2002).

An animal's mechanical system—its mass supported by multiple limbs—undergoes in locomotion cyclic exchanges of energy through coupling with the environment. From the perspective of coordination, it is convenient to reinterpret the two dimensional nature of each mechanical degree of freedom mentioned in Section 2.2 using special polar coordinates whose angle corresponds to the mechanical phase of oscillation and whose magnitude represents the total mechanical energy.[6] The Newtonian constraint that position be altered only through the intermediary of a force changing its velocity, is interpreted in the new coordinates as allowing phase velocity (frequency) to be altered only through the intermediary of a power input changing its energy. In this view, a 'mechanism' is represented as a neutrally stable second order oscillator, affording a range of persistent frequencies (phase velocity), any one selectable by the choice of the total mechanical energy operating point. We represent this second order property of a mechanical degree of freedom by means of the double circle icon in Fig. 9. In contrast, the simplest rendering of the internal neural circuitry that might direct the coordination of these mechanisms is a pattern generating unit with properties of a frequency tuned clock—a first order oscillator whose

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[6] The existence of such 'convenient' action-angle coordinates is formally guaranteed if we presume that the balance of energy losses to the environment against energy injections from the internal power plant justify the adoption of Hamiltonian or 'lossless' mechanics models (Arnold et al., 1997).
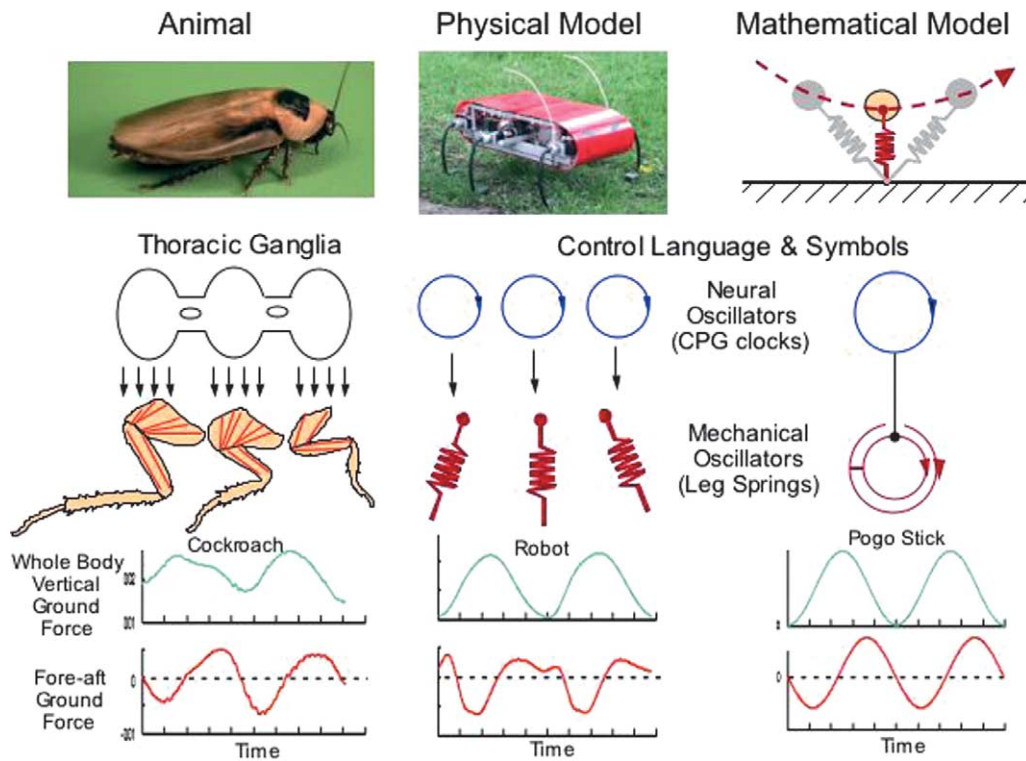
Fig. 9. Representation of insect, robot and template locomotion as coupled oscillators. Systems are modeled as using two different types of oscillators. The hypothesized thoracic ganglion central pattern generators in arthropods are represented by first order clocks or oscillators (single circles). A first order system cannot oscillate without some switching controller unless its state lies on a circle (Winfree, 1980). Because frequency is the control input to the system, we denote a first order oscillator by a single circle. The musculo-skeletal system is represented by mass-spring system or second order oscillator. A second order oscillator has a phase velocity (frequency) altered only through the intermediary of a power input changing its energy. In this view, a 'mechanism' is represented as a neutrally stable second order oscillator, affording a range of persistent frequencies (phase velocity), any one selectable by the choice of the total mechanical energy operating point. We represent this second order property of a mechanical degree of freedom by means of the double circle icon. The ground reaction forces for one stride of the animal and models are shown at the bottom of the figure.

phase velocity can be directly adjusted that we accordingly represent as a single circle in Fig. 9. This model arises from the widely accepted view of CPG as a tunable limit cycle (Cohen et al., 1982). While the number of mechanical oscillators is exactly prescribed by the mechanical degrees of freedom, the number of clocks required to model an animal's motor control system is of course a matter of speculation and, to some extent, convenience. It seems fair to assert that few neuroscientists would posit the clocks as being fewer in number than the mechanical degrees of freedom.

4.2.1. A plane of coordination architectures

Both clocks and mechanisms can oscillate in isolation. By their coupling, we get a more complex family of oscillatory systems that parametrizes two trade-offs in the evolution of this locomotor control architecture. One trade-off addresses the use of feedback vs. feedforward control strategies (the extent to which the clocks' frequencies are influenced by those of the mechanisms), and the other concerns the range between centralized vs. decentralized coordination schemes (the extent to which one clock's frequency is influenced by those of its neighbors'). We depict the resulting plane of coordination architectures in

Fig. 10, whereby one point on the plane represents a specific choice of control architecture—that is, a commitment to a particular choice of internal centralization and strength of influence between internal and mechanical components. Choosing within this two dimensional continuum of trade-offs largely determines the efficacy of a particular gait in a particular environment (Klavins et al., 2002; Klavins and Koditschek, 2002; Weingarten and Koditschek, 2004; Weingarten et al., 2004) and we hypothesize that each locomotion task within each variant environment has an associated preferred point of operation on this plane.

The plane of Fig. 10 represents a parametrization of various styles of hypothetical phase coupling between neural and mechanical system components (Klavins et al., 2002; Klavins and Koditschek, 2002). Weighing the costs and benefits of the information exchange required to realize a given architecture can be used to make specific predictions about how animals' coordination capabilities will change or even fail as internal noise (decrements in the available neural channel capacity) or external bandwidth requirements (increments in the speed and or precision of the required mechanical coordination) are varied. In the face of the highest bandwidth performance tasks, the neural communications channels may be too noisy to permit high
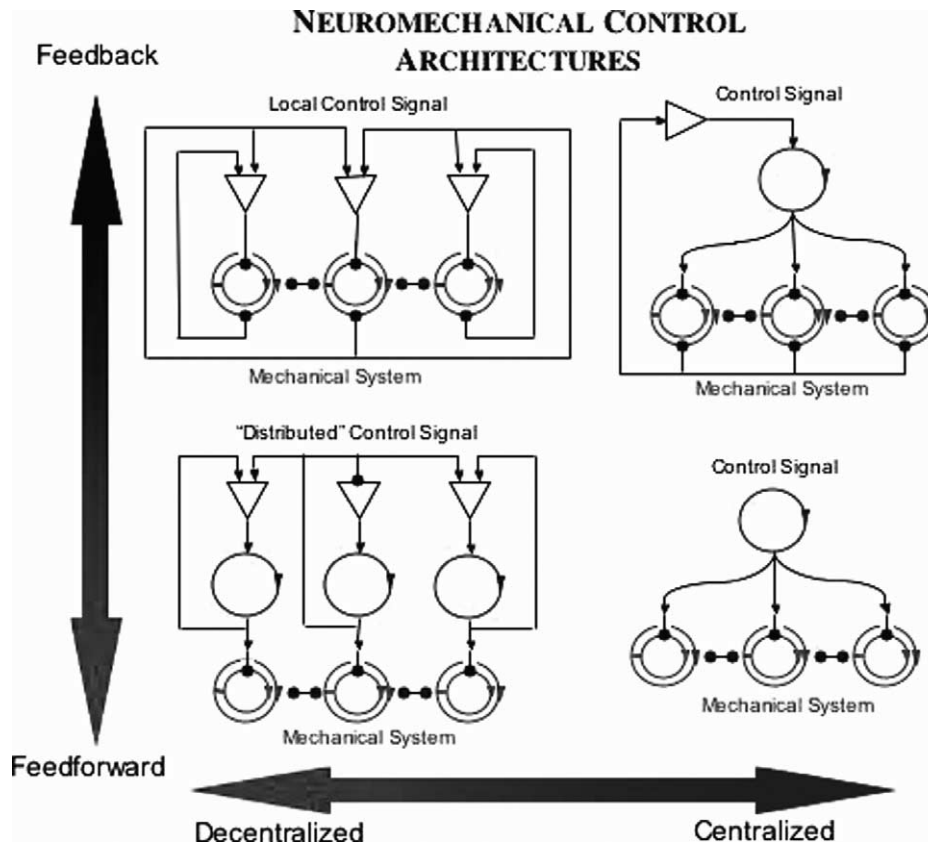
Fig. 10. Plane of neural control architectures.[8] A set of controllers is represented by a plane with two axes. One axis spans the range from purely feedback to solely feedforward control. A second axis ranges from decentralized to completely centralized control. Animals are modeled as two different types of oscillators. We again use the single and double circle conventions for internal and mechanical oscillators introduced in the caption to Fig. 9.

enough feedback or synchronization gains, and the animal may be forced to operate in a decentralized and feedforward manner, where coordination is achieved through mechanical coupling, and stability is achieved by preflex. As the bandwidth requirements of the task decrease relative to the available internal neural channel capacity, higher reflex and synchronization loop gains could be tolerated, increasing the efficacy of feedback and central authority.

4.2.2. Hypothesized control points on the plane

We hypothesize that when an animal runs fast, has noisy sensors or a musculoskeletal system tuned to its environment, it will operate more in a feedforward, decentralized fashion attaining stability through preflexes and coordination through mechanical coupling of springy legs (lower left corner of Fig. 10). When an animal runs more slowly, has accurate sensors or is in an uncertain environment, it will function in a predominantly feedback, centralized fashion via neural reflexes and synchronized oscillators (upper right corner of Fig. 10). Because we couple neural control to the mechanical system, these hypotheses can be parameterized in a mathematically tractable manner and tested experimentally.

Second, we hypothesize that diverse behavioral repertoires require animals to move within this two-dimensional

coordination space by 'tuning' coordination controls to adapt locomotion to different environments and to different operating regimes within any particular environment. Because these problems depart from mature linear systems theory, no clear mathematical prediction is yet available to tie given points in this architectural space to specific tasks in a given environment (Kuo, 2002). We presently rely upon robot experiments to provide an empirical guide for how much feedforward or centralization should be expected in the available environments (Fig. 9 center column).

4.3. RHex—experiments with and theory of a tunable coordination controller

There are two tightly interwoven but conceptually distinctive aspects of a locomotor CPG, as we have increasingly come to understand[7] the demands upon the RHex clock and its connection to the robot's body. They lead to and begin to corroborate in a manner we now suggest

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[7] This section represents a metaphorical account of SLIP mechanics based upon intuition arising from our first mathematical analysis of a primitive RHex-clock-driven 3 DOF virtual bipedal SLIP (Altendorfer et al., 2004) reviewed at the end of this section.

[8] We thank Eric Klavins for some of the ideas, discussions, and art work leading up to this figure.

the planar view of coordination architectures advanced in hypothesis H_3.

First, from the perspective of Hamiltonian mechanics, reviewed in the presentation of H_3 in Section 4.2, a mechanism is a system of coupled oscillators, the period of each a function of its (conserved) energy. In this perspective, the job of the internal clock is to entrain the coupled phases of the mechanism at the desired total energy operating point. Second, from the perspective of accomplishing useful work in an uncontrolled world, internal energy supplies need be metered into the environment at the right time, the right magnitude, and in the right direction to balance the inevitable countervailing influences, for example, damping, gravity, or rough terrain. Feedback is necessitated by the unpredictability of such perturbations. Decentralization is necessitated by their distributed and often independent occurrence (Weingarten and Koditschek, 2004).

Associated with these two views can be identified a matching pair of intuitive approaches to control. In the first, Hamiltonian, view, the internal clock can be used to adjust the mechanism's operating point by advancing or retarding the relative phase angle of leg touchdown via leg recirculation. Raibert (1986) discovered that the SLIP template experiences a fore-aft acceleration or deceleration in stance as a function of the touchdown angle: more horizontal approaches loose speed (in the extreme case that the body's velocity vector is exactly aligned with the leg angle at touchdown, it will simply bounce backward) while more vertical touchdowns gain speed. Since delayed leg touchdowns tend toward more horizontal approach angles, while quicker touchdowns tend to occur at more vertical angles, the timing of leg recirculation holds sway over running speed. In the second view of performing work upon the environment, Raibert observed that injecting energy into the leg spring during decompression could restore energy dissipated throughout the rest of the running cycle. In the absence of a leg spring with tunable compliance, a hip spring can be similarly excited during the appropriate phase interval to perform positive work on the COM. Interpreting an actuator's proportional derivative (PD) controller as a tunable damped spring suggests that relaxing or strengthening the proportional gains with the appropriate timing can achieve the same result.

Whereas the Raibert (1986) SLIP controllers required far less modeling and computational effort than the active SLIP anchoring and control algorithms reviewed at the end of Section 3.3, and are partially decentralized (by presuming a decoupled mechanism) in contrast, they incur essentially the same need for high quality sensory feedback. We now introduce the RHex internal clock as a complete implementation of hypothesis H_3 that affords as much or as little feedback or centralization as the designer specifies, parametrizing the full range of operating conditions on the architectural plane of Fig. 10 introduced by hypothesis H_3. As one moves around in this plane, the stabilizing influence of feedback and decentralization is vitiated by the limited communications, computation, and power budget available.

4.3.1. The RHex clock: centralized feedforward coordination

For reasons of cost and expediency and biological inspiration the initial versions of RHex had no sensors other than hip motor shaft angle measurement devices encoders at each of its (only six) motors. These were used to implement the proportional derivative (PD) hip angle tracking control to be discussed shortly. However, there was no possibility of sensing nor of reacting to the body's COM position or orientation at all. Thus, from the point of view of the COM control task, these locomotion controller versions have no task oriented sensing and are effectively open loop. We will use the terminology 'task open loop' to denote such a situation in a physical model, equivalent in an animal model to the absence of neural feedback from the environment.

Sensors are generally costly. This is not necessarily a consequence of their material properties (cost, weight, size, system complexity and reliability)—but almost inevitably a reflection of the substantial communications and computational burden they incur. Accordingly, as in biology, there is a long and important tradition within robotics that questions the need for and value of sensors in general manipulation tasks (Mason, 1993). It is clear that both robots and animals need sensory feedback for competence across the broad behavioral range, including locomotion. Indeed, subsequent generations of RHex incorporate a growing sensor suite whose use in feedback we will sketch after a careful exposition of the original task open loop version. Thus, we explore in this section the manner in which RHex's slowly growing array of proprioceptive and exteroceptive sensor modalities will allow careful research into the biological validity of our hypothesis H_3, concerning the choice of operating point on the architectural plane of Figure 9 (as well as offering insight for robotics designers concerning when and how sensory feedback may be most useful).

RHex's running gaits are organized around the standard arthropod alternating tripod by means of a centralized phase reference signal. (Fig. 11). This clock is deformed to reproduce a recurring slow and a fast period each cycle, the former representing the idealized leg stance interval, the latter representing leg recirculation. The shaped clock signal is split into a phase and anti-phase copy—presented to the opposing tripods of legs as a reference signal to be tracked by the corresponding hip motors. When a leg encounters the ground, its springs comply, transmitting a ground reaction force up to the rigid body that is integrated together with the corresponding influences of all other stance legs to produce motion.

The reader should resist the natural temptation upon first encounter to conceive of the RHex clock signal as a classical reference trajectory generator designed to dictate the details of limb kinematics at each instant as is generally postulated in the primate fine motion control literature (Kawato, 1999; Moran and Schwartz, 1999). Rather, this
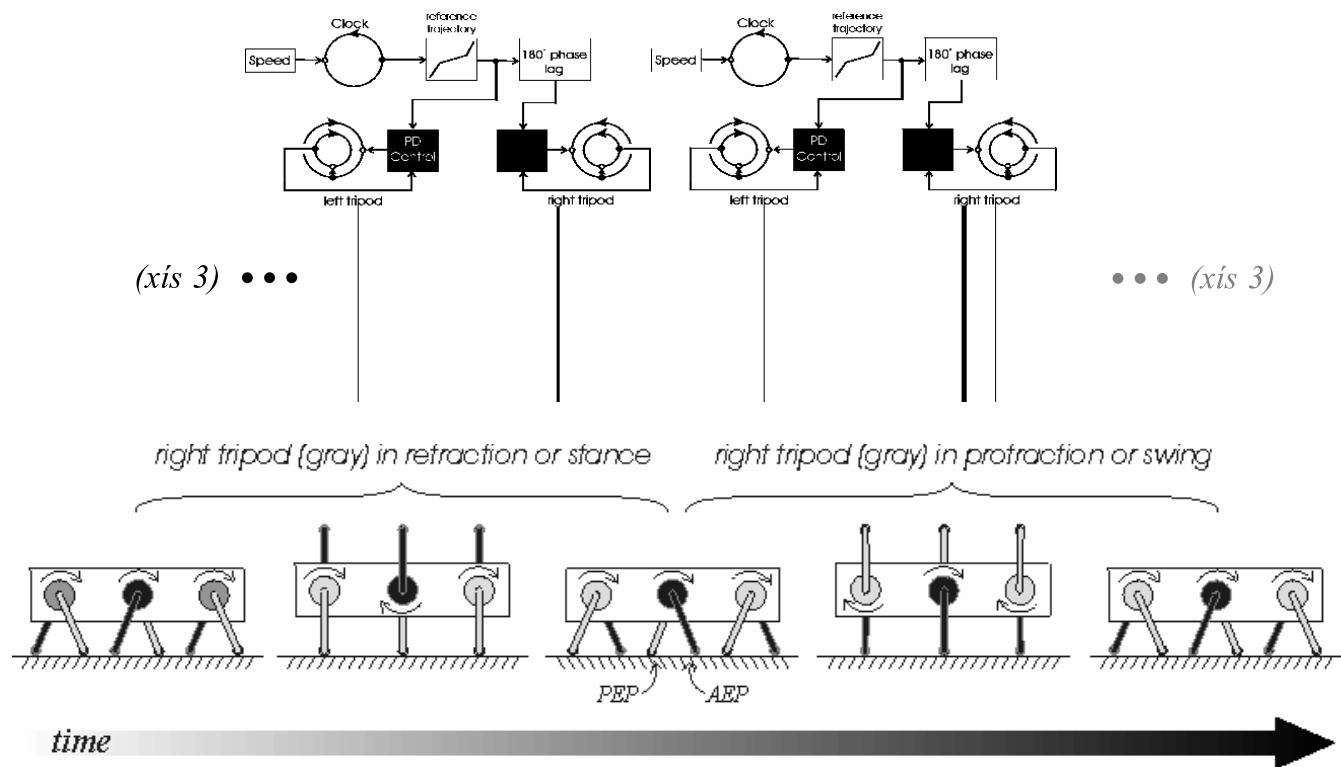
Fig. 11. A simple schematic of RHex's locomotor controller. Identical copies of the phase and anti-phase version of a two-stroke clock signal are tracked by decoupled proportional, derivative (PD) controllers at each of the hip motors comprising the respective tripods. During retraction, the leg moves slowly. While in protraction, it moves quickly, sweeping out a greater angle in the same amount of time. An animation sequence showing a full stride for the right tripod is shown in gray. The counter motion of the left tripod is shown in black. The anterior (AEP) and posterior (PEP) extreme leg positions are labeled. Modified from (Klavins et al., 2002).

biologically inspired 'clock' functions as an internal dynamical system—one that excites the appropriate dynamical response when properly coupled to the mechanical system. One way to appreciate the importance of getting this coupling right is to consider the very complex relationship between clock parameters and locomotion performance. For example, the clock period is not well correlated with running speed, nor, indeed can there be found any simple monotonic relationship between any pairing of clock and mechanism parameters. We proceed to review what actually has been learned empirically and theoretically about the relationship of the clock parameters to locomotion performance at the centralized feedforward point of operation on the coordination plane (lower right hand corner of Fig. 10). Then, we shall discuss some very early experience moving the coordination operating point around on the plane. Wishing to emphasize again the centrality of coupled internal and mechanical dynamics we reiterate the cautionary note introducing this paragraph: when sensory feedback is used to adjust the clock parameters, and particularly when the centralized internal clock is split into distinct individual elements coupled internally to each other as well as back and forward to the mechanism, as we shall explore in the last subsection, below, the view of the internal clock system as a mere reference trajectory generator holds little appeal.

4.3.2. Tuning the clock: empirically won performance using mechanical self-stability

Our present best understanding is that the centralized feedforward clock imposes a leg recirculation strategy that supports mechanical self-stabilization in the sagittal plane analogously to the horizontal plane stabilization induced by the simple lateral leg spring (LLS) leg placement strategy reviewed in Section 4.1. Irrespective of the particular stability mechanism, independent quadrupedal running models (Herr and McMahon, 2001) and experience with RHex (Saranli et al., 2001) underscores the importance of coordinated leg recirculation as a dominant factor in determining the quality of a steady state running gait. Indeed, experience shows that the quality of RHex's locomotion (primarily on flat terrain, much less so on rugged terrain—at least for the task open loop version discussed in this section) is extraordinarily sensitive to the clock deformation and tracking gains (Weingarten et al., 2004). A great deal of time and care must be spent empirically tuning up the relative onset and duration of the slow and fast clock phases, and then matching the proportional and derivative motor shaft error gains before even stable, much less fast and efficient RHex gaits emerge.

For example, seeking to increase fore-aft speed, we quickly find that the obvious step of shortening the clock period merely leads to an often debilitating mismatch of

intended stride frequency with the natural frequency of the virtual, sagittal mass-spring system. Intuitively, the clock parameters must instantiate in an automated and, crucially, task open loop manner, the two aspects of control discovered by Raibert as described above. Roughly speaking, the relationships between the slow and fast phase intervals of the RHex clock seem responsible for the Hamiltonian or timing aspects of that control, whereas the proportional derivative controller gains seem responsible for its energy injecting properties. But there are many intuitively countervailing relationships encountered in translating the inertial frame touchdown angle conditions of the simple sagittal spring mass template into the body frame touchdown angle relationships of a sagittal spring mass bearing a rigid body like RHex. Moreover, when intuitively transforming from the mechanism's phase into an internal clock phase coordinate, the interrelationship between the Hamiltonian fore-aft speed regulation effects and the work performing 'spring energy replenishing' effects is complicated.

For example, consider the clock 'offset' parameter that dictates the angle (relative to the body) at which the slow (putative stance) phase of the hip motor shaft reference should occur. One way to imagine speeding up the gait is to decrease this offset, hoping to mimic the Raibert controller's assignment of a more vertical touchdown angle as a means of accelerating the fore-aft velocity component. However, the clock frequency and duty factor (ratio of slow to fast phase intervals) must be changed as well in exactly the right way to maintain the correspondence between clock phase interval and physical leg flight and stance interval or there will be unintentional (and generally undesirable) coupling effects. To see how this undesirable coupling occurs when the clock's slow phase interval is not exactly aligned with physical leg stance imagine a leg that is still being 'clocked' quickly when it physically touches down. Its load increases dramatically and errors build up between the commanded and true motor shaft angle that call out increased current into motor windings. This, in turn, generally increases its torque output, thereby increasing the ground reaction forces and, hence, possibly contributing to accelerating the robot's mass center as desired or, possibly, diluting or countering that effect, depending upon the angle of the true ground reaction force vector and the timing relative to the leg spring's compression–decompression cycle. On the other hand, if the duration of the slow phase is not appropriately shortened to match, then the leg's liftoff must occur 'prematurely' relative to the clock's cycle. The sudden loss of its leg's load will result in a sudden increase in motor shaft velocity—appropriate to the onset of protraction, but contrary to the clock's still slow command. The resulting large error in the derivative term of the tracking controller will instigate large opposing torques from the actuator whose resulting negative work will delay the needed protraction, defeating the initial center of mass acceleration, and, of course, wasting battery energy as well. Similar lines

of reasoning support the empirical evidence that speed increases generally result only from a complicated simultaneous adjustment of all clock parameters. Moreover, just as the clock parameters must be tuned in a delicate balance, so must the tracking gains themselves be adjusted in accord.

Because of the highly coupled nature of the clock and tracking parameters and their influence on locomotion performance, hand tuning of gaits is very challenging. Notably, unlike the improvements in leg design which were brought about by careful human tuned materials engineering as we have described in Section 3.3, RHex gait performance improvements beyond those initially reported (Saranli et al., 2001) have not been forthcoming from human intuition. Instead, automated gait tuning for specific behavioral traits (e.g. speed, or efficiency) within fixed environments (e.g. flat asphalt, rolling grass, sheer linoleum, and so on) has led to dramatic performance improvements including, for example a fivefold increase in top speed (to five body lengths per second, at ∼2.5 m/s) and a threefold efficiency gain (to 0.6 specific resistance; Weingarten et al., 2004). Automated though they are—adjustments being made by a simple learning algorithm on measurements taken automatically upon largely autonomously managed experiments—such a purely empirical means of gait improvement requires an unfortunately large number of repeated experiments (a typical battery entails 200–300 repetitive runs through an 8 m course; Weingarten et al., 2004). To replace our empirical search by actually prescribing a clock signal that will insure an effective run—that is, a gait with high efficiency, strong stability properties, and useful maneuverability—we must better understand the coupling of the clock, plant and environment.

4.3.3. Understanding the clock: toward the analysis of self-stability

The surprising biological hypothesis of self stabilization and its biologically inspired mathematical analysis, reviewed in Section 4.1 with respect to the lateral leg spring template for horizontal plane runners, has inspired a similar inquiry regarding locomotion in the sagittal plane. The resulting numerical (Seyfarth et al., 2002) and theoretical (Ghigliazza et al., 2003) confirmation of a sagittal spring mass self-stability regime has, in turn, stimulated the development of a new tool for the stability analysis of hybrid Hamiltonian systems with symmetry (Altendorfer et al., 2004) that advances prospects for a more rational approach to tuning useful machines such as RHex.

Developing a formal understanding of the relation of the RHex clock to locomotion performance remains a daunting prospect owing to the non-integrability of even simple spring mass mechanics (Holmes, 1990). Closed form approximations (Schwind and Koditschek, 2000) that underlie prior analytical results of (Ghigliazza et al., 2003) arise only for very special cases of the models we have been discussing. For RHex, and, indeed, any animal

posited to recruit no active sensory measurements one requires a template that can explain the stabilizing influence of a leg recirculation scheme implemented only in body coordinates, as opposed to the inertial frame measurements that would be required to implement the 'self-stabilizing' leg placement algorithms considered in the prior literature (Ghigliazza et al., 2003; Schmitt and Holmes, 2000b; Schwind and Koditschek, 2000; Seyfarth et al., 2003). This will necessarily take the form of a 3 degree of freedom sagittal springy mass carrying a rigid body in the form of a third link whose pitch is subjected in stance to the influence of gravity—a model for which no natural integrable approximation appears to be forthcoming.

The first analytical account of how the open loop RHex clock might stabilize a legged runner (Altendorfer et al., 2004) applies the novel framework for hybrid Hamiltonian systems with symmetry to a monopedal version of this heretofore unapproachable rigid body augmented sagittal spring mass. The new framework complements prior analysis by integrable approximation. The latter offers exact (both necessary and sufficient) conditions for stability over a very small region of the parameter space—typically not in regions of physical interest. The former provides very weak conditions (either necessary for stability or sufficient for instability) over the entirety of the parameter space.

For example, in the desirable steady state regime of 'small' pitching (i.e. when steady state body pitching velocity is dominated by the steady state leg touchdown velocity) we now know that the clock offset (the deformation parameter that determines the relative time of the slow phase interval) must be retarded (Altendorfer et al., 2003) or there can be no stable gait, strengthening the intuition gained during the empirical tuning studies discussed above. Similar partial qualitative conclusions can now be derived for all possible combinations of steady state pitch and clock phase.

Much more analytical effort will be required to develop a low dimensional model that is actually prescriptive—for example, whose stability conclusions concerning clock parameters can be shown to predict the steady state behavior of RHex. However, the new model is a source of novel hypotheses for robotic and animal runners than can further the precision with which we assess the nature of their locomotion competence.

4.3.4. Correcting the clock: experience with control points on the coordination plane

The RHex controller we have just explored clearly occupies one extreme corner in the space of coordination schemes—centralized, feedforward (lower right corner of Fig. 10; Klavins et al., 2002). This task open loop controller consistently manifests surprisingly competent locomotion even in environments that are far from what it was designed and tuned for: e.g. in the presence of significant ground perturbations during runs with long aerial phases (albeit the speed or specific resistance may no longer be as favorable).

In general, the centralized feedforward RHex clock is surprisingly hard to beat: performance typically degrades gradually over a large variety of perturbative environments prior to outright failure. For example, the original RHex presentation (Saranli et al., 2001) describes how specific resistance increases by an order of magnitude as the open loop feedforward excited machine is driven over 'fractal style' terrain. Notwithstanding its diminishing efficiency, we find that the centralized feedforward clock can endure such environmental uncertainty even up to roughly two bodylengths per second before outright failure due to destabilization (e.g. pitching onto its back or yawing off the course). In general, eventually, as the terrain gets sufficiently irregular, or steep, or slippery, or broken, the higher speed gaits begin to fail and the robot either destabilizes or gets stuck.

Intuition and simulation evidence (Klavins et al., 2002) has long supported the view articulated in H_3 that more reliance on feedback (moving the operating point upwards in Fig. 10) is necessitated by greater uncertainty in the environment while the efficacy of that feedback, particularly in centralized lockstep, can be compromised by task time constants that exceed its transmission rate. As we slowly add new sensors and exploit existing feedback channels in RHex, empirical evidence increasingly begins to confirm these same broad hypotheses.

Initial experiments with inclined planes (Komsuoglu et al., 2001) show clearly the key importance of feedback alterations to the RHex clock in the presence of even very simple environmental uncertainty. For example, let us return to the clock offset parameter discussed above in the context of open loop tuning. A slightly negative value, typically associated with default locomotion on level ground, results in successful ascents for slopes of less than 10° beyond which inclinations the robot may stall out, slipping back down the slope, or, more typically as the inclination increases, pitching backwards into an uncontrolled fall. Online sensory feedback-based adaptation of this offset parameter has been shown empirically to confer much greater fitness on slopes.

Using an accelerometer fixed in the body to detect average pitch, the robot's inclination was fed back to retard the clock offset in proportion to the perceived slope angle. Imagining the virtual leg of the SLIP template as aligned with the mean angle of a tripod's hips during the slow phase of the clock reference signal conveys the appropriate intuition underlying this scheme. In this view, retarding the offset places the angle of maximum virtual leg spring compression farther behind the body so that it is once again aligned with the gravitational acceleration vector, as if on level ground. Kinematically, such a SLIP configuration 'leans' the mass center forwards over the toe and closer to the slope's surface, thereby reducing the overturning torque. Dynamically, this delayed transition from 'fast' to 'slow' commanded hip velocity prolongs the period of greater power expenditure during stance, in proportion to the

additional work against gravity that will be required to advance the body mass.

Empirical outcomes suggest that such conceptualizations of the external perturbing influences and an effective means of countering them are endorsed by physical reality. The physical effect of the commanded shift in virtual leg kinematics indeed results in successful (Komsuoglu et al., 2001) ascents on slopes exceeding 25° (and still more with the advent of the advanced clock tuning reported in Weingarten et al. (2004). The physical effect of the prolonged energy injection at the beginning of tripod stance yields measurable increases of roughly 1/3 in the climbing work done against gravity by the motors with a roughly corresponding decrease in specific resistance (Komsuoglu et al., 2001).

These early experiments with inclination compensation offer a compelling example of the origins and constraining consequences of sensory bandwidth limitations in feedback controller performance. The accelerometers are very cheap, robust MEMs devices with appropriate survivability for the rough-and-tumble RHex application, but which exhibit a higher than desirable noise floor. Moreover, our very crude initial signal processing strategy used to extract effective pitch simply equated the magnitude of 'steady state' fore-aft acceleration as arising from the projection of the true gravitational acceleration vector consequent upon the body's orientation. Ascertaining 'steady state' introduces long processing lags—the result of simple low pass filters— significantly decreasing the overall sensory system's working bandwidth, adding significant delay to the feedback loop. In consequence, the benefits of the offset adjustment are only manifest on inclined planes smooth enough to exhibit an average (or 'DC') slope value that persists sufficiently long to inform the sluggish filters. Additional severe loop delays were introduced in the forward path arranged for these early experiments by the implementation of clock parameter changes only on a stride-to-stride stride basis. While it seems intuitively clear that the simple offset parameter compensation scheme should not be expected to stabilize locomotion on slopes of high spatial frequency (e.g. in the range below the robot's body length), we have in fact not come close to running quickly (above roughly one body length per second) over even modestly undulating terrain (e.g. at spatial frequency above the two body length range) because of the sensory bandwidth limitations.

More recent experimentation begins to explore the value of decentralized control authority in both feedforward and feedback schemes. Ongoing efforts (Weingarten and Koditschek, 2004) aim to corroborate the purpose and value of these heretofore unfamiliar (to RHex) corners in the architectural plane introduced by hypothesis H_3. The research depends upon a growing sensory suite incorporating more advanced signal processing methods (Lin et al., 2003). Even with the rudimentary hip motor shaft sensory capability of the initial RHex platform, growing experience with decentralized modifications to the rigid original clock scheme (leftward adjustments in Fig. 10) begins to confirm its importance in settings such as highly irregular or broken terrain where these corresponding broad hypotheses H_3 makes with regard to independence of limbs and other constituent parts emerge.

Generally, as the terrain becomes increasingly uneven, whether flat, undulating, or sloped on average, the centralized schemes begin to have trouble. For example, the centralized feedback slope climbing scheme outlined above, when presented with a very rocky and irregular slope, will often fail at even modest slopes and at modest speeds. The only gaits we presently have developed that are capable of ascending rocky slopes at average inclinations greater than $\sim 15°$ are very slow, centralized, open loop, quasi-static 'creepers' that attempt blindly to secure footholds and handholds, advance the body slowly enough to leave them intact, and then reposition trusting the body's 'grip' on the terrain to hold the ground already gained. When centralized gaits tuned for dynamical operation over homogeneous terrain are driven over the rough at comparable speeds they inevitably deteriorate. At the highest speeds, the centralized schemes fail in such environments— typically catastrophically, by uncontrollable yawing off course or pitching into a flip.

In a recent advance we have applied the decentralized clock coordination schemes mentioned above (Klavins and Koditschek, 2002) to a modified version of the original RHex centralized feedback scheme. The new feedback mechanism takes into account at least implicitly the intrinsic power limitations of RHex's actuators by saturating command voltages that would imply operation beyond the motor speed-torque limit. In the decentralized version, all the legs participating in a tripod are required to attempt a synchronization of their individual 'touchdown' angles. Here, the intuition is that legs arriving 'too early' at a touchdown arising from a local high point will contribute destabilizing yawing or pitching torques whereas those arriving 'too late' at a touchdown arising from a local low point will not counterbalance those already in contact. In contrast, all legs in the other tripod are required to attempt to remain mechanically out of phase with the first set—in other words, they must 'wait' to touchdown until the present set of stance legs are nearing the completion of stance. Obviously, this means that each individual leg must get its own individual clock whose 'time' must be repeatedly advanced and retarded to keep pace with the 'experiences' of those around it. The new coordination schemes are guaranteed to yield asymptotically stable alternating tripod reference trajectories in the feedforward sense because of the recent theory (Klavins and Koditschek, 2002). According to hypothesis H_3, they will confer mechanical stability in the feedback configuration as well for the intuitive reasons outlined above. Moreover, they will begin to degrade as the locomotion task is required to react increasingly quickly beyond the bandwidth limitations of the sensory capacity.

Initial evidence suggests that we are encountering just such empirical outcomes (Weingarten and Koditschek, 2004).

5. Conclusion

This paper reviews the locomotion control hierarchy as a series of biologically refutable hypotheses motivated by observations of animal behavior. Translated into the language of dynamical systems theory, these biological hints, combined with prior engineering practice and theory are then applied to the design and implementation of a novel robot of unprecedented mobility, RHex.

In hypothesis H_1 we summarize the last decade's evidence for dynamical gaits in multi-legged animal by proposing the centrality of stability as a requirement for and determinant of reliable running. Against the backdrop of this central feature of biological locomotion, we trace the success of the RHex platform back to its reliance on limit cycles with large basins rather than deliberatively planned reference trajectories.

In hypothesis H_2 we address the problem of understanding and then exploiting the emergence of a simple dynamical locomotion pattern for running. The same simple pattern emerges from highly varied and complex morphologies whose kinematic design seems strongly favorable to quasi-static operation, if not outright antithetical to agility. Rendering this observation in terms of dynamical systems theory yields the notion of a *template* around which is stabilized (Full et al., 2002) the body's high degree of freedom *anchor* (Full and Koditschek, 1999) and whose phase offers a tractable global surrogate variable for purposes of coordination with the other distant degrees of freedom. RHex's ability to anchor a similar template via a mechanically preferred posture suggests the further value of such designs.

In hypothesis H_3 we situate the surprising observation of mechanically self-stabilizing animal gaits within a whole plane of coordination architectures for running. This two-dimensional family of designs parametrizes the varied schemes of interconnection between and among mechanical degrees of freedom and their biologically concomitant internal pattern generators. We trace the development of RHex coordination algorithms from their 'simplest' origins in the centralized feedback corner of this plane through more costly operating points arising from the addition of more sophisticated sensory suites and individuated multiple clocks.

Arthropods offer breathtaking examples of locomotor behavior lying well beyond the most advanced capabilities of any present or near term future robotic platform. More specifically, there is no aspect of locomotion capability presently to be found on RHex or any other extant robot that can begin to compare to any legged animal. No doubt, some of the animals' wonderful capabilities result from materials properties, energy conversion methods, and sensory mechanisms still beyond the reach of human technology. However, the RHex experience demonstrates that even conventionally engineered materials, actuators and sensors can be re-engineered to elicit significant improvements in appropriately focused capabilities when fundamental principles underlying the algorithmic source of a correspondingly narrowed animal behavior are elucidated. In turn, biological inspiration, when applied to the design of robots required to work at some prescribed task in the real world, can stimulate mathematical insights and engineered artifacts that reach beyond description to generate quantitative hypotheses of the integrated system, not simply its parts in isolation. Thus, the synthesis of functioning systems promises to generate novel, testable biological hypotheses.

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