# Translations from LTL to Büchi automata

# Learning scenarios

Some tools:

- **LTL2BA** www.lsv.fr/~gastin/ltl2ba
  *by Paul Gastin & Denis Oddoux*

  (simple,
  does not always give a pruned automaton)

- **Rabinizer 4** https://www7.in.tum.de/~kretinsk/rabinizer4.html
  *by Jan Kretinsky, Tobias Meggendorfer, Salomon Sickert (et al.)*

  (more efficient,
  supports several types of automata: Büchi, Generalised Büchi, Rabin, parity,
  deterministic, non-deterministic, ...
  uses acceptance conditions on transitions rather than on states)

# Simple (inefficient) translation from LTL

| | | |
|---|---|---|
| LTL vocabulary | Propositional letters: | $\Sigma = \{p, q, r, \ldots\}$ |
| | Boolean connectives: | $\vee, \wedge, \neg, \rightarrow, \leftrightarrow$ |
| | Modal operators: | F, G, U, X |

Syntax $\quad \phi :$

$$\phi : \quad p \quad | \qquad \text{(for all } p \in \Sigma)$$
$$\phi \vee \phi \;|\; \phi \wedge \phi \;|\; \neg\phi \;|\; \phi \rightarrow \phi \;|\; \phi \leftrightarrow \phi$$
$$F\,\phi \;|\; G\,\phi \;|\; \phi\, U\, \phi \;|\; X\,\phi$$

Goal: translate $\phi$ to an automaton $A_\phi$ s.t. $L(A_\phi) = \{\ w \in \wp(\Sigma)^\omega \mid w \vDash \phi\ \}$

# Simple (inefficient) translation from LTL

Goal: translate $\phi$ to an automaton $A_\phi$ s.t. $L(A_\phi) = \{ \; w \in \wp(\Sigma)^\omega \mid w \models \phi \; \}$

<u>Expanded closure</u> of $\phi$

$cl(\phi)$ = smallest set of formulas such that

- $\phi \in cl(\phi)$
- if $\alpha \in cl(\phi)$ and $\beta$ subformula of $\alpha$, then $\beta \in cl(\phi)$
- if $\alpha \in cl(\phi)$, then $\neg\alpha \in cl(\phi)$ ($\neg\neg\,\beta$ becomes $\beta$)
- if $\alpha \in cl(\phi) \cap \{ F\ldots, G\ldots, \ldots U\ldots\}$, then $X\alpha \in cl(\phi)$

<u>State</u> of $A_\phi$

(= tableau atom)

(P initial if $\phi \in P$)

any set $P \subseteq cl(\phi)$ such that

- $\alpha \in P$ iff $\neg\alpha \notin P$
- $\alpha \wedge \beta \in P$ iff $\alpha, \beta \in P$, etc…
- $F\alpha \in P$ iff $\alpha \in P$ or $XF\alpha \in P$
- $G\alpha \in P$ iff $\alpha \in P$ and $XG\alpha \in P$
- $\alpha U \beta \in P$ iff $\beta \in P$ or $\{\alpha, X(\alpha U \beta)\} \subseteq P$

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State of $A_\phi$ (= tableau atom): any set $P \subseteq cl(\phi)$ such that

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(P initial if $\phi \in P$)

Transition of $A_\phi$ (= tableau edge): P —S—> P' when

- $p \in P \leftrightarrow p \in S$ for all $p \in \Sigma \cap cl(\phi)$
- $X\alpha \in P \leftrightarrow \alpha \in P'$ for all $X\alpha \in cl(\phi)$

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<u>Acceptance</u> of $A_\phi$: (= tableau acceptance)

For every $\alpha \in \{ F\beta, \gamma U \beta \} \cap cl(\phi)$
there is a state $P_\alpha$ that is <u>$\alpha$-fulfilling</u> (i.e. $\alpha \in P_\alpha \rightarrow \beta \in P_\alpha$)
and that is visited infinitely often

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Muller condition

enforcing $In(\rho) = F$ for some $F \in \mathcal{F}$

$\mathcal{F} = \{ F \subseteq States \mid \forall \alpha \in \ldots \; \exists P_\alpha \in F$ s.t. $P_\alpha$ $\alpha$-fulfilling $\}$

Generalised Büchi

enforcing $In(\rho) \cap F \neq \varnothing$ for all $F \in \mathcal{F}$

$\mathcal{F} = \{ F_\alpha \mid \alpha \in \ldots \}$ where $F_\alpha = \{ \alpha$-fulfilling states $\}$

Büchi

enforcing $In(\rho) \cap F \neq \varnothing$

similar to closure under intersection (on the tablet...)