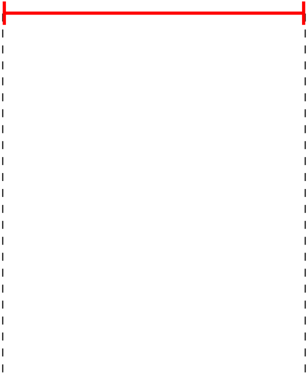
Temporal Representation and Reasoning in Interval Temporal Logics

Part III: expressiveness issues

- expressiveness classification
  - comparing expressive power of two (interval temporal) logics
  - bisimulation: a handy tool
- case study: ITL and the search for happiness

Dario Della Monica
ICE-TCS, School of Computer Science, Reykjavik University, Iceland

Halpern and Shoham's logic HS and Allens's relations



J. Halpern and Y. Shoham, *A propositional modal logic of time intervals*, Journal of the ACM, 1991

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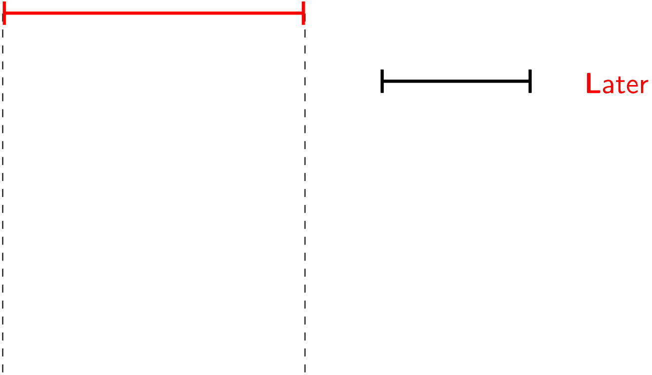

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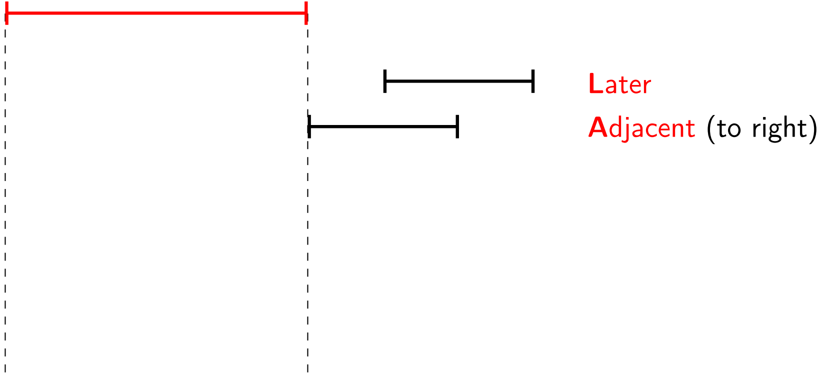

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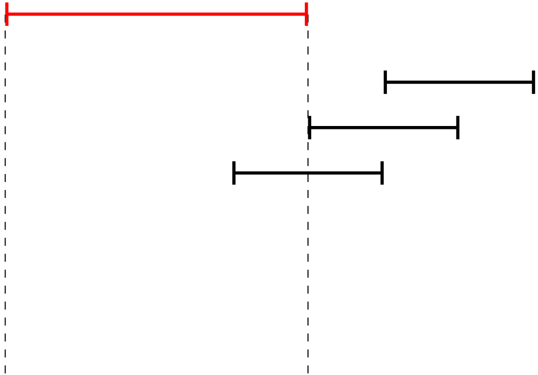

Later

Adjacent (to right)

Overlaps (to right)

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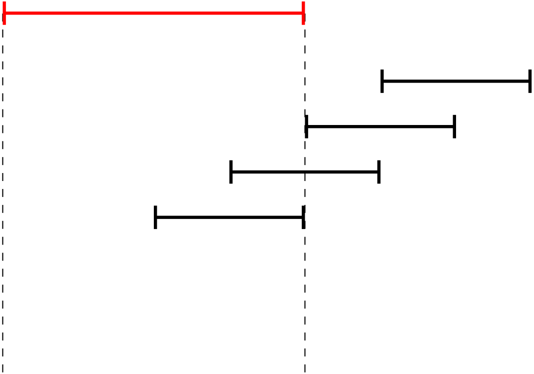

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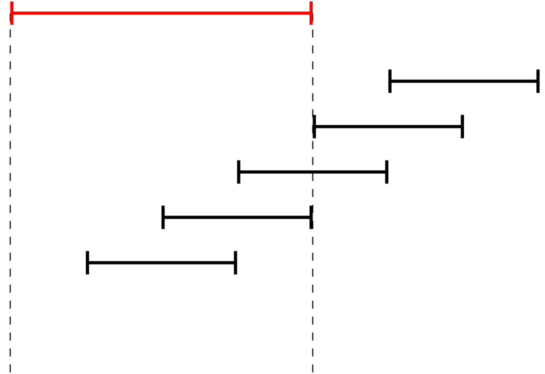

Later

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During (subinterval)

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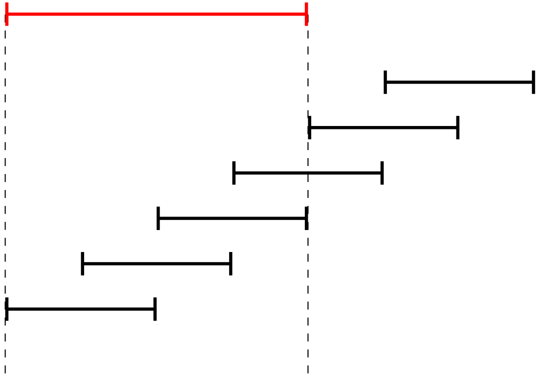

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During (subinterval)

Begins

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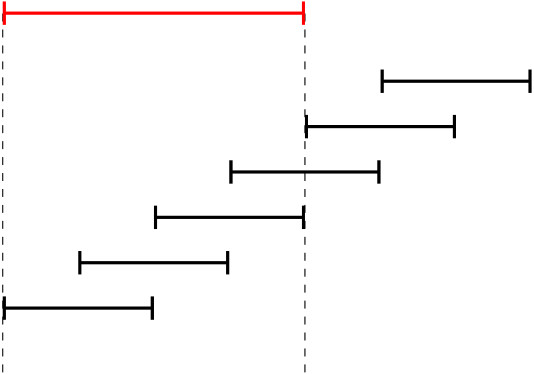

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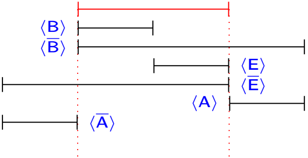
6 relations + their inverses = 12 Allen's relations

J. F. Allen, *Maintaining knowledge about temporal intervals*, Communications of the ACM, volume 26(11), pages 832-843, 1983

Definabilities among modalities

All modalities are definable in terms of $\langle B \rangle$, $\langle \overline{B} \rangle$, $\langle E \rangle$, $\langle \overline{E} \rangle$, $\langle A \rangle$, $\langle \overline{A} \rangle$

$HS \equiv \mathsf{B}\overline{\mathsf{B}}\mathsf{E}\overline{\mathsf{E}}\mathsf{A}\overline{\mathsf{A}}$



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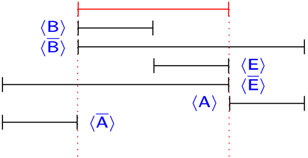

Defining the other interval modalities:

- Later: $\langle L\rangle\varphi \equiv \langle A\rangle\langle A\rangle\varphi$
- Before: $\langle \overline{L}\rangle\varphi \equiv \langle \overline{A}\rangle\langle \overline{A}\rangle\varphi$

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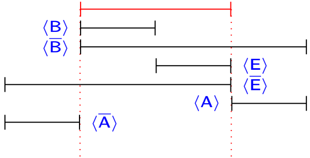

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- During (strict sub-interval): $\langle D \rangle \varphi \equiv \langle B \rangle \langle E \rangle \varphi$ $(\equiv \langle E \rangle \langle B \rangle \varphi)$
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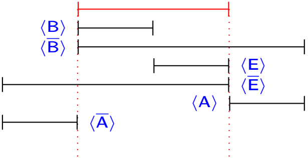

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- Overlaps on the right: $\langle O \rangle \varphi \equiv \langle E \rangle \langle \overline{B} \rangle \varphi$
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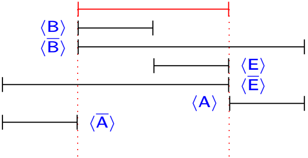

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In general, it is possible defining HS modalities in terms of others

The zoo of fragments of HS

- $2^{12} = 4096$ fragments of HS (syntactic)
- only ~ 1000 expressively different fragments
- expressiveness classification wrt. several classes of interval structures
  - all, dense, discrete, finite, ???

The expressiveness classification programme

Expressiveness classification programme: classify the fragments of HS with respect to their expressiveness, relative to important classes of interval models.

Comparing expressive power of HS fragments

L_1, L_2 HS-fragments

L_1 L_2

Comparing expressive power of HS fragments

L_1, L_2 HS-fragments

$$L_1 \ \{\prec, \equiv, \succ, \not\approx\} \ L_2$$

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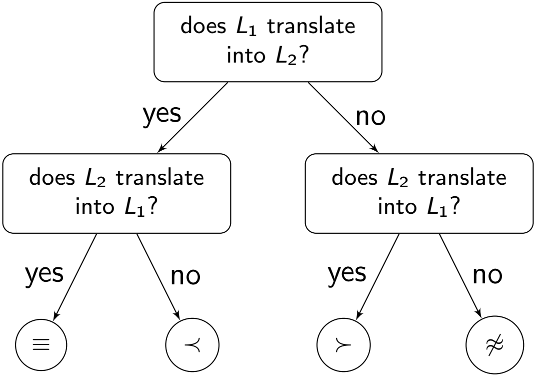

Truth-preserving translation

There exists a truth-preserving translation of L_1 into L_2

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Each modality $\langle X \rangle$ of L_1 is definable in L_2
(i.e., $\exists$ a L_2-formula φ s.t. $\langle X \rangle p \equiv \varphi$)

Example: $\langle \mathrm{L} \rangle p \equiv \langle \mathrm{A} \rangle \langle \mathrm{A} \rangle p$

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2^{12} fragments... $\frac{2^{12} \cdot (2^{12}-1)}{2}$ comparisons

Our approach

Solution:
To find a complete set of definabilities among modalities

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Notation:

$X_1X_2\ldots X_n$
$=$
HS-fragment with modalities $\langle X_1\rangle, \langle X_2\rangle, \ldots, \langle X_n\rangle$

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$$\overbrace{\mathsf{X_1 X_2 \ldots X_n}}^{\mathcal{X}} \qquad \overbrace{\mathsf{Y_1 Y_2 \ldots Y_m}}^{\mathcal{Y}}$$

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$\langle X_1\rangle \sqsubseteq \mathsf{Y_1\dots Y_m}$??

$\dots$??

$\langle X_n\rangle \sqsubseteq \mathsf{Y_1\dots Y_m}$??

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$\langle X_1\rangle \sqsubseteq Y_1\ldots Y_m$	??
	$\wedge$
$\ldots$	??
	$\wedge$
$\langle X_n\rangle \sqsubseteq Y_1\ldots Y_m$	??
———	—
$\mathcal{X} \preceq \mathcal{Y}$	??

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$\langle X_1 \rangle \sqsubseteq Y_1 \ldots Y_m$	??	`true`
	$\wedge$	$\wedge$
$\ldots$	??	`true`
	$\wedge$	$\wedge$
$\langle X_n \rangle \sqsubseteq Y_1 \ldots Y_m$	??	`true`
$\mathcal{X} \preceq \mathcal{Y}$	??	`true`

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$\langle X_1 \rangle \sqsubseteq Y_1 \ldots Y_m$	??	true	...
	$\wedge$	$\wedge$	$\wedge$
...	??	true	false
	$\wedge$	$\wedge$	$\wedge$
$\langle X_n \rangle \sqsubseteq Y_1 \ldots Y_m$	??	true	...
$\mathcal{X} \preceq \mathcal{Y}$	??	true	false

Our approach - cont'd

$\mathcal{Y} \preceq \mathcal{X}?$

$\mathcal{X} \preceq \mathcal{Y}?$

Our approach - cont'd

		$\mathcal{Y} \preceq \mathcal{X}$? yes	$\mathcal{Y} \preceq \mathcal{X}$? no
$\mathcal{X} \preceq \mathcal{Y}$?	yes	$\mathcal{X} \equiv \mathcal{Y}$	$\mathcal{X} \prec \mathcal{Y}$
	no	$\mathcal{X} \succ \mathcal{Y}$	$\mathcal{X} \not\equiv \mathcal{Y}$

Complete set of definabilities among modalities

$\langle L \rangle \sqsubseteq A$ $\quad \langle L \rangle p \equiv \langle A \rangle \langle A \rangle p$

$\langle D \rangle \sqsubseteq BE$ $\quad \langle D \rangle p \equiv \langle B \rangle \langle E \rangle p$

$\langle O \rangle \sqsubseteq \overline{B}E$ $\quad \langle O \rangle p \equiv \langle E \rangle \langle \overline{B} \rangle p$

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D. Della Monica, V. Goranko, A. Montanari, and G. Sciavicco, *Expressiveness of the Interval Logics of Allen's Relations on the Class of all Linear Orders: Complete Classification*, IJCAI 2011

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$\langle L \rangle \sqsubseteq DO$ $\quad \langle L \rangle p \equiv \langle O \rangle (\langle O \rangle \top \wedge [O](\langle O \rangle p \vee \langle D \rangle p \vee \langle D \rangle \langle O \rangle p))$

$\langle L \rangle \sqsubseteq \overline{B}D$ $\quad \langle L \rangle p \equiv \langle \overline{B} \rangle [D] \langle \overline{B} \rangle \langle D \rangle \langle \overline{B} \rangle p$

$\langle L \rangle \sqsubseteq EO$ $\quad \langle L \rangle p \equiv \langle O \rangle [E] \langle O \rangle \langle O \rangle p$

$\langle L \rangle \sqsubseteq BO$ $\quad \langle L \rangle p \equiv \langle O \rangle (\langle O \rangle \top \wedge [O] \langle B \rangle \langle O \rangle \langle O \rangle p)$

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L. Aceto, D. Della Monica, A. Ingolfsdottir, A. Montanari, and G. Sciavicco, *A complete classification of the expressiveness of interval logics of Allen's relations over dense linear orders*, TIME 2013

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L. Aceto, D. Della Monica, A. Ingolfsdottir, A. Montanari, and G. Sciavicco, *A complete classification of the expressiveness of interval logics of Allen's relations over dense linear orders*, TIME 2013

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(The first four definabilities hold over the class of all linear orders; all nine hold over the class of dense linear orders.)

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Proving non-existence

Existence is easy... **...non-existence is hard**

Proving non-existence

Existence is easy...

a new land

...non-existence is hard

Proving non-existence

Existence is easy... **...non-existence is hard**

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a bearded Icelander

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Proving non-existence

Existence is easy...

a new
land

a bearded
Icelander

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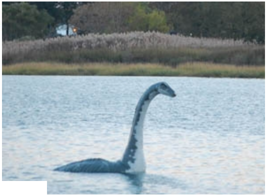

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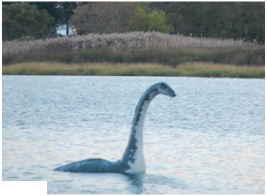

elfs

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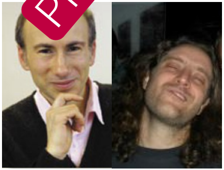

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Bisimulations

Bisimulation between interval structures

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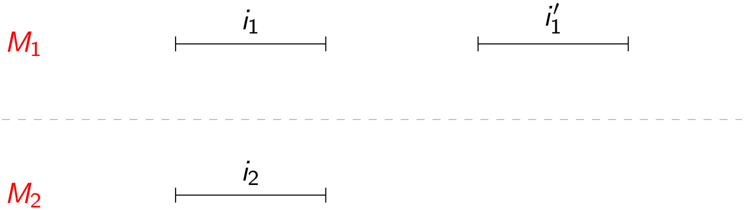

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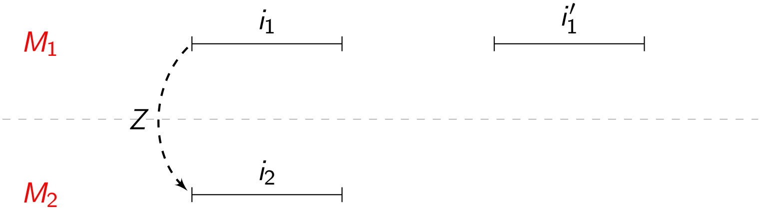

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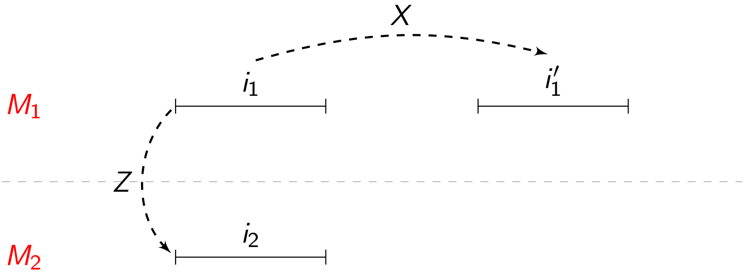

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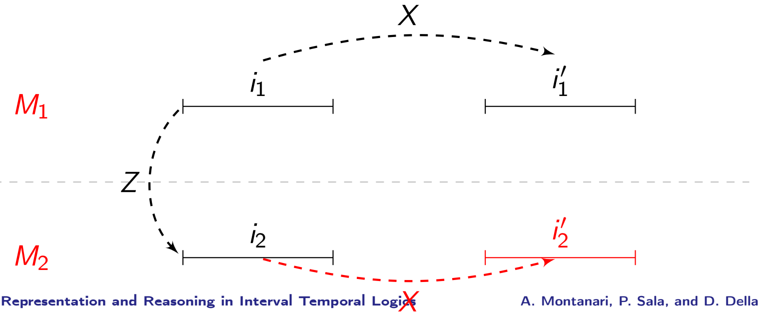

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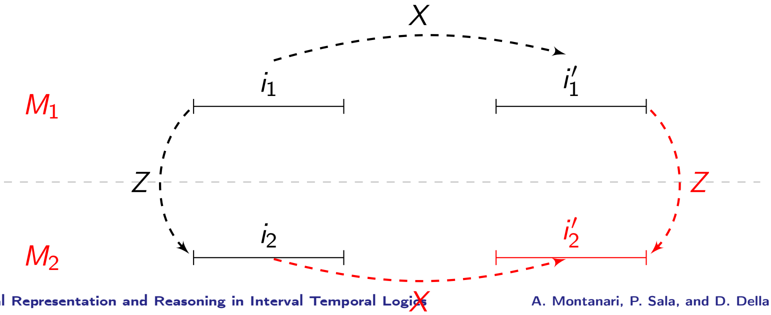

Bisimulation between interval structures - cont'd

Theorem A bisimulation for $\mathcal{L}$ preserves the truth of $\mathcal{L}$-formulae

$[a, b]$ and $[c, d]$ are bisimilar
φ is a $\mathcal{L}$-formula

φ is true in $[a, b]$ iff φ is true in $[c, d]$

How to use bisimulations to disprove definability

Suppose that we want to prove:

$\langle X \rangle$ is not definable in terms of $\mathcal{L}$

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An example: the operator $\langle D \rangle$

skip

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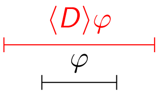
$$M, [a, b] \Vdash \langle D \rangle \varphi \overset{def}{\Leftrightarrow} \exists c, d \text{ such that } a < c < d < b \text{ and } M, [c, d] \Vdash \varphi$$



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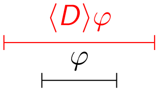

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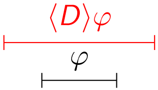

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To prove that $\langle D \rangle$ is not definable in terms of any other fragment, we must prove that:

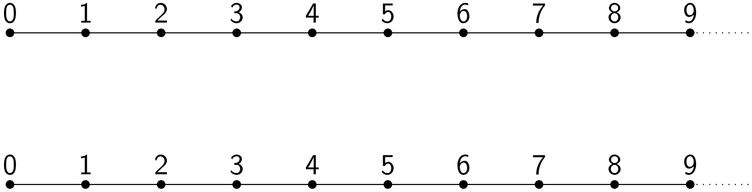
1) $\langle D \rangle$ is not definable in terms of ALBO$\overline{\text{ALBEDO}}$
2) $\langle D \rangle$ is not definable in terms of ALEO$\overline{\text{ALBEDO}}$

$\langle D \rangle$ is not definable in terms of A

A bisimulation wrt fragment A but not D

Bisimulation wrt A ($\mathcal{AP} = \{p\}$):

- models: $M_1 = \langle \mathbb{I}(\mathbb{N}), V_1 \rangle$,$M_2 = \langle \mathbb{I}(\mathbb{N}), V_2 \rangle$



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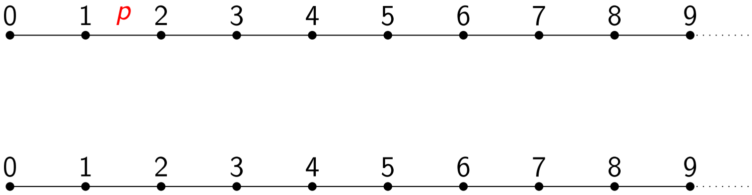

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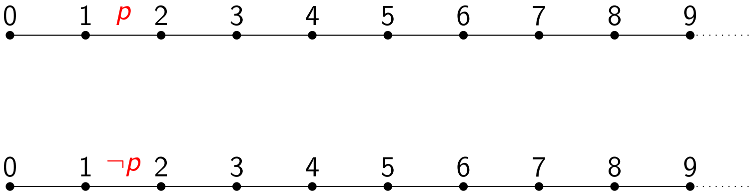

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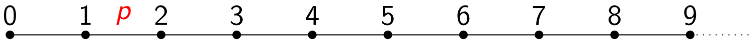

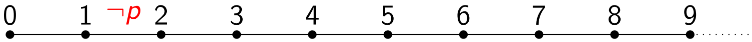

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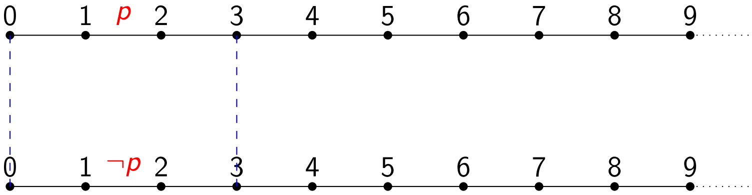

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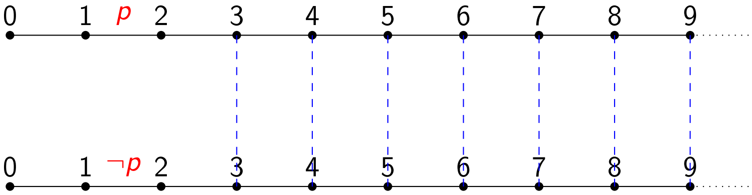

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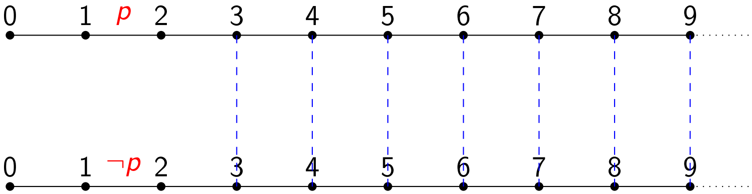

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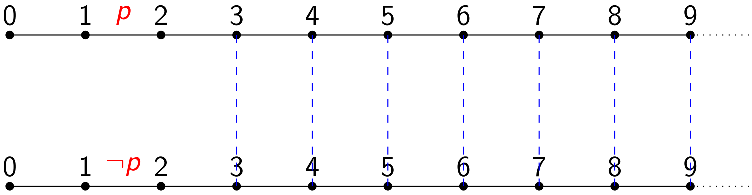

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$\Rightarrow$ the thesis

To sum up on the expressiveness classification

DONE:

- class of all linear orders (1347 fragments)
- classes of dense linear orders (966 fragments)

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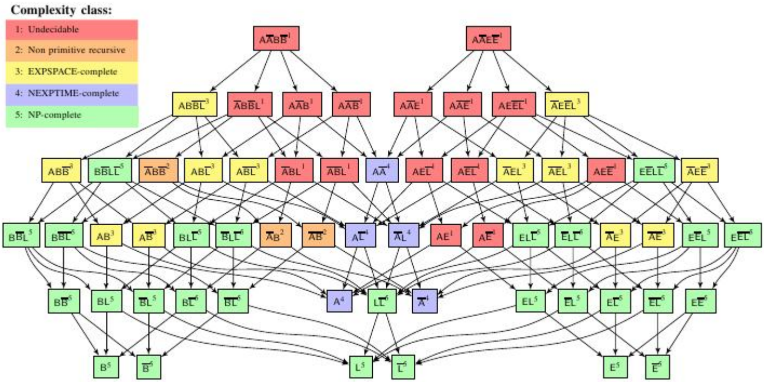
MISSING PIECES:

- $\langle O \rangle$ over finite/discrete linear orders — $\langle \overline{O} \rangle$ for free

Expressiveness/satisfiability classification over natural numbers



A case study

ITL and the search for happiness

Planning as satisfiability paradigm

The motion planning problem

- e.g., a robot moving into an environment trying to accomplish goals

Logical formalization

- environmental constraint (φ_E)
- goal specification (φ_G)

Models of $\varphi_E \wedge \varphi_G$ represent valid plans

- satisfiability: existence of a solution
- tableaux: solution

ITL and the search for happiness

A sample planning problem
Can a researcher be happy and still make good research?!?

Formalization within AB

- $\varphi_E =$ I do my job
- $\varphi_G =$ I am happy

does a plan to reach happiness exist?

$\equiv$

is $\varphi_E \wedge \varphi_G$ satisfiable

The formalism AB

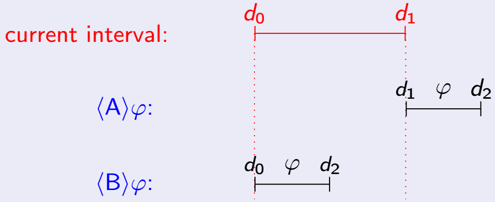
Syntax

$$\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \langle A \rangle\varphi \mid \langle B \rangle\varphi$$

Semantics

$\langle A \rangle$: $\mathbf{M}, [d_0, d_1] \Vdash \langle A \rangle\varphi$ iff $\exists d_2$ s.t. $d_1 < d_2$ and $\mathbf{M}, [d_1, d_2] \Vdash \varphi$

$\langle B \rangle$: $\mathbf{M}, [d_0, d_1] \Vdash \langle B \rangle\varphi$ iff $\exists d_2$ s.t. $d_0 \leq d_2 < d_1$ and $\mathbf{M}, [d_0, d_2] \Vdash \varphi$



The environmental constraint “I do my job”

φ_E = I do my job — environmental constraints

- time structure (years, months, weeks, days)
- physiological needs (sleeping, having fun)
- the rule of the research world

Metric capabilities

Atomic propositions $\mathrm{len}_{=\mathrm{k}}$, $\mathrm{len}_{<\mathrm{k}}$, $\mathrm{len}_{\leq\mathrm{k}}$, $\mathrm{len}_{>\mathrm{k}}$, $\mathrm{len}_{\geq\mathrm{k}}$ for all k
(over discrete orders, e.g., $\mathbb{N}$)

- $M, [a, b] \Vdash \mathrm{len}_{=\mathrm{k}}$ iff $b - a = k$ (analogously the others)

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$$\text{len}_{=k} \overset{def}{\Longleftrightarrow} \underbrace{\langle B \rangle \langle B \rangle \ldots \langle B \rangle}_{\substack{k-1 \text{ times} \\ \text{len}_{\geq k}}} \top \wedge \underbrace{[B][B] \ldots [B]}_{\substack{k \text{ times} \\ \text{len}_{\leq k}}} \bot$$

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$$\text{len}_{=k} \overset{def}{\iff} \underbrace{\langle B \rangle \langle B \rangle \ldots \langle B \rangle}_{\substack{k-1 \text{ times} \\ \text{len}_{\geq k}}} \top \wedge \underbrace{[B][B] \ldots [B]}_{\substack{k \text{ times} \\ \text{len}_{\leq k}}} \bot$$

- $\text{len}_{=k} + AB \Rightarrow$ (a metric version of) other modalities
  - $\text{len}_{=k} \wedge \langle D \rangle \varphi \overset{def}{\iff} \text{len}_{=k} \wedge \bigvee_{i=1}^{k-1} (\langle B \rangle \text{len}_{=i} \wedge \langle A \rangle (\text{len}_{<k-i} \wedge \varphi))$

Metric capabilities

Atomic propositions $\mathsf{len}_{=k}$, $\mathsf{len}_{<k}$, $\mathsf{len}_{\leq k}$, $\mathsf{len}_{>k}$, $\mathsf{len}_{\geq k}$ for all k
(over discrete orders, e.g., $\mathbb{N}$)

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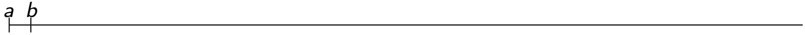
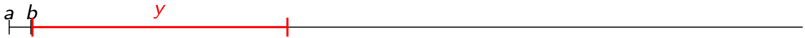
  - $\mathsf{len}_{=k} \wedge \langle E \rangle \varphi \overset{def}{\iff} \mathsf{len}_{=k} \wedge \bigvee_{i=1}^{k-1} (\langle B \rangle \mathsf{len}_{=i} \wedge \langle A \rangle (\mathsf{len}_{=k-i} \wedge \varphi))$
  - $\mathsf{len}_{=k} \wedge \langle O \rangle \varphi \overset{def}{\iff} \mathsf{len}_{=k} \wedge \bigvee_{i=1}^{k-1} (\langle B \rangle \mathsf{len}_{=i} \wedge \langle A \rangle (\mathsf{len}_{>k-i} \wedge \varphi))$

I do my job



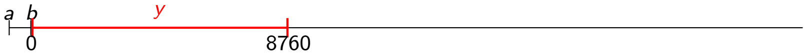
I do my job



there exists an interval starting in *b* where *y* (*year*) holds

$$\langle A \rangle y$$

I do my job



there exists an interval starting in b where y (*year*) holds

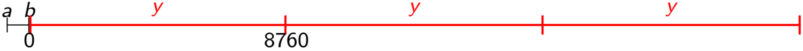
$$\langle \mathsf{A} \rangle y$$

all years are long 8760 hours (365*24)

$$[G](y \rightarrow \mathsf{len}_{=8760})$$

$[G]\varphi$ means that φ is true *globally* in the future ($[G]\varphi \stackrel{def}{\Longleftrightarrow} \varphi \wedge [\mathsf{A}]\varphi \wedge [\mathsf{A}][\mathsf{A}]\varphi$)

I do my job



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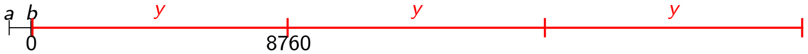
every year is followed by another year

$$[G](y \rightarrow \langle \mathrm{A} \rangle y)$$

I do my job



there exists an interval starting in b where y (*year*) holds

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all years are long 8760 hours (365*24)

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$[G]\varphi$ means that φ is true *globally* in the future ($[G]\varphi \overset{def}{\Longleftrightarrow} \varphi \wedge [A]\varphi \wedge [A][A]\varphi$)

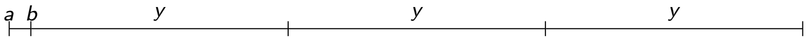
every year is followed by another year

$$[G](y \rightarrow \langle A \rangle y)$$

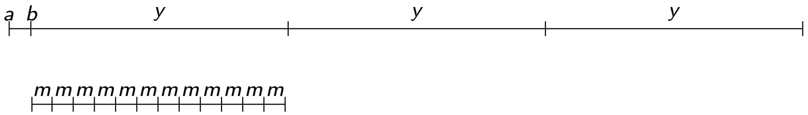
years do not overlap

$$[G](y \rightarrow [B]\neg\langle A \rangle y)$$

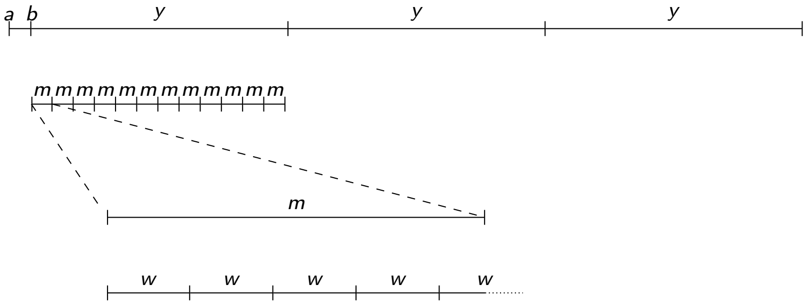
I do my job - cont'd



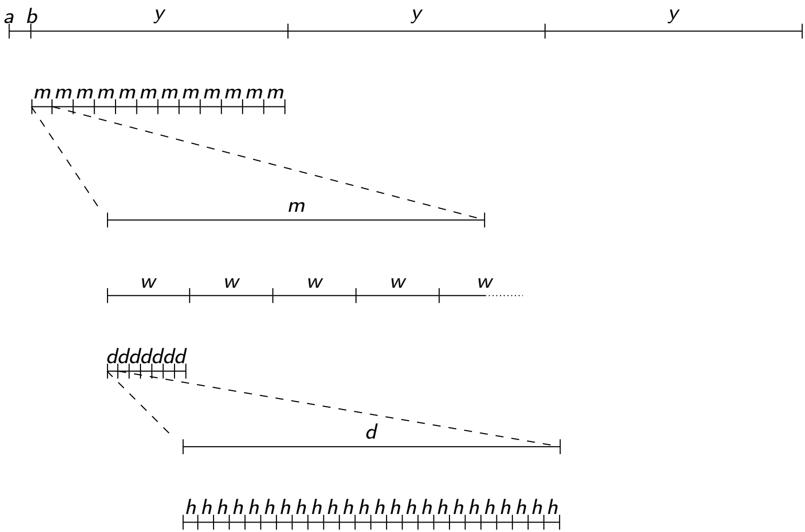
I do my job - cont'd



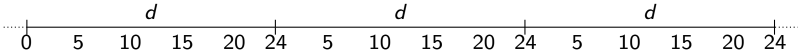
I do my job - cont'd



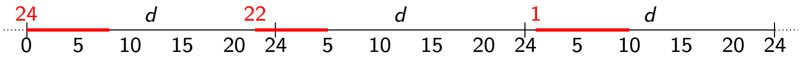
I do my job - cont'd



I do my job - cont'd



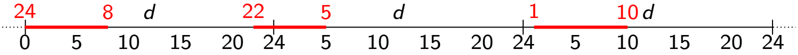
I do my job - cont'd



every day I get to sleep between 10 pm and 1 am

$$[G](\langle A \rangle d \rightarrow \langle A \rangle(\mathrm{len}_{\geq 22} \wedge \mathrm{len}_{\leq 25} \wedge \langle A \rangle sleep))$$

I do my job - cont'd



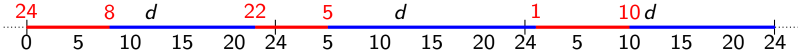
every day I get to sleep between 10 pm and 1 am

$$[G](\langle A \rangle d \to \langle A \rangle (\mathsf{len}_{\geq 22} \wedge \mathsf{len}_{\leq 25} \wedge \langle A \rangle sleep))$$

I sleep for 7-9 hours

$$[G](sleep \to \mathsf{len}_{\geq 7} \wedge \mathsf{len}_{\leq 9})$$

I do my job - cont'd



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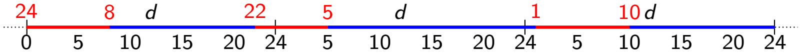
I sleep for 7-9 hours

$$[G](sleep \rightarrow \mathsf{len}_{\geq 7} \wedge \mathsf{len}_{\leq 9})$$

when I don't sleep I am awake

$$[G]((sleep \rightarrow \langle \mathsf{A} \rangle awake) \wedge (awake \rightarrow \langle \mathsf{A} \rangle sleep))$$

I do my job - cont'd



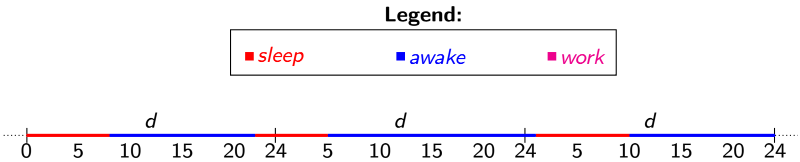
I sleep (and wake up) once a day

$$[G](day \rightarrow [\mathrm{B}](\langle \mathrm{A} \rangle awake \rightarrow \neg \langle \mathrm{B} \rangle \langle \mathrm{A} \rangle awake))$$

when I am awake I don't sleep and vice versa ($sleep \cap awake = \emptyset$)

$$\begin{aligned}[G](&\neg(\langle \mathrm{A} \rangle awake \wedge \langle \mathrm{A} \rangle sleep) \\ &\wedge (awake \rightarrow \neg \langle \mathrm{B} \rangle \langle \mathrm{A} \rangle sleep) \\ &\wedge (sleep \rightarrow \neg \langle \mathrm{B} \rangle \langle \mathrm{A} \rangle awake))\end{aligned}$$

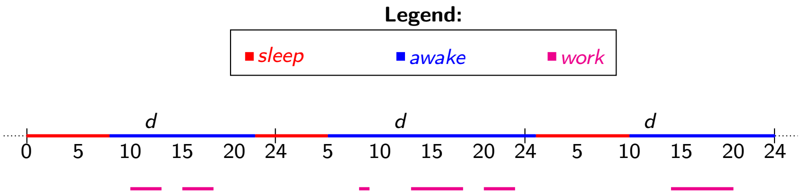
I do my job - cont'd



I work only when I am awake ($work \cap sleep = \emptyset$)

$$[G](\neg(\langle A\rangle work \wedge \langle A\rangle sleep) \wedge (work \rightarrow \neg\langle B\rangle\langle A\rangle sleep) \wedge (sleep \rightarrow \neg\langle B\rangle\langle A\rangle work))$$

I do my job - cont'd



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I do my job - cont'd

work enjoy homogeneity

I do my job - cont'd

work enjoy homogeneity

I do my job - cont'd

work enjoy homogeneity

RECALL !!!

we can express:	$\text{len}_{=k} \wedge \langle D \rangle \varphi$
	$\text{len}_{=k} \wedge \langle E \rangle \varphi$
and thus:	$\text{len}_{=k} \wedge [D] \varphi$
	$\text{len}_{=k} \wedge [E] \varphi$

I do my job - cont'd

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work enjoy homogeneity

$$[G](work \rightarrow \bigvee_{i=1}^{20} (\text{len}_{=i} \wedge [D](\text{len}_{=1} \rightarrow work)))$$

I do my job - cont'd

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work enjoy homogeneity

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$$[G](\mathit{work} \rightarrow \bigvee_{i=1}^{20} (\mathrm{len}_{=i} \wedge \langle E \rangle(\mathrm{len}_{=1} \wedge \mathit{work})))$$

I do my job - cont'd

RECALL !!!

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work enjoy homogeneity

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$$[G](work \rightarrow \bigvee_{i=1}^{20} (\text{len}_{=i} \wedge \langle E \rangle (\text{len}_{=1} \wedge work)))$$

$$[G](work \rightarrow \langle B \rangle (\text{len}_{=1} \wedge work))$$

I do my job - cont'd

counting work hours

$$\varphi_{wh}^{\geq 3} \overset{def}{\iff} \langle B \rangle (\langle A \rangle work \wedge \langle B \rangle (\langle A \rangle work \wedge \langle B \rangle \langle A \rangle work))$$

$$\varphi_{wh}^{\leq 3} \overset{def}{\iff} [B](\langle A \rangle work \rightarrow [B](\langle A \rangle work \rightarrow [B](\langle A \rangle work \rightarrow [B]\neg \langle A \rangle work)))$$

I do my job - cont'd

counting work hours

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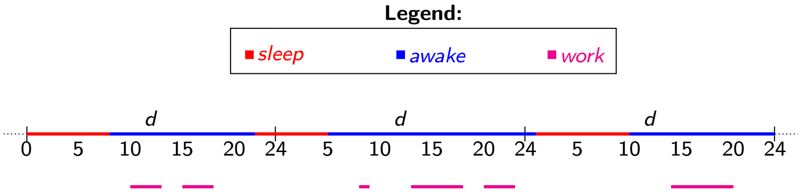
$$\varphi_{wh}^{\leq 3} \stackrel{def}{\iff} [B](\langle A \rangle \mathit{work} \rightarrow [B](\langle A \rangle \mathit{work} \rightarrow [B](\langle A \rangle \mathit{work} \rightarrow [B]\neg \langle A \rangle \mathit{work})))$$

$$\varphi_{wh}^{\geq k} \stackrel{def}{\iff} \langle B \rangle(\langle A \rangle \mathit{work} \wedge \langle B \rangle(\ldots \wedge \langle B \rangle(\langle A \rangle \mathit{work} \wedge \langle B \rangle \langle A \rangle \mathit{work})))$$

$$\varphi_{wh}^{\leq k} \stackrel{def}{\iff} [B](\langle A \rangle \mathit{work} \rightarrow [B](\ldots \rightarrow [B](\langle A \rangle \mathit{work} \rightarrow [B]\neg \langle A \rangle \mathit{work})))$$

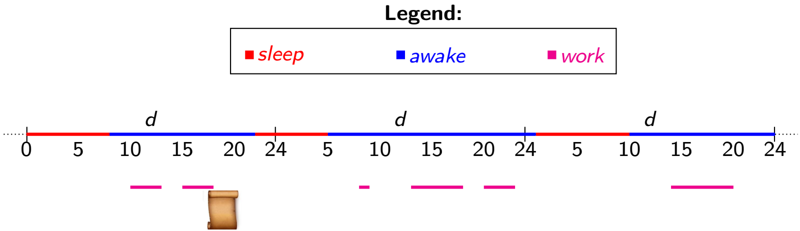
$$\varphi_{wh}^{=k} \stackrel{def}{\iff} \varphi_{wh}^{\geq k} \wedge \varphi_{wh}^{\leq k}$$

I do my job - cont'd



I get satisfied after χ hours of work

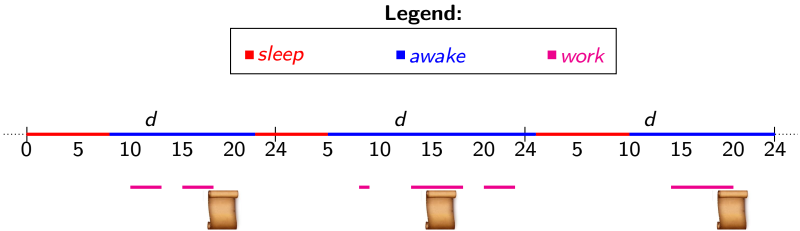
I do my job - cont'd



I get satisfied after χ hours of work

$$[A](\varphi_{wh}^{=\chi} \rightarrow \langle A \rangle(\text{len}_{=1} \wedge \mathit{paper\_accepted}))$$

I do my job - cont'd

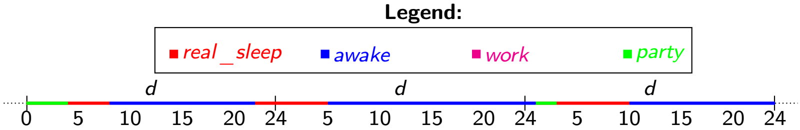


I get satisfied after χ hours of work

$$[A](\varphi_{wh}^{=\chi} \rightarrow \langle A \rangle(\text{len}_{=1} \wedge paper\_accepted))$$

$$[G](\langle A \rangle paper\_acc \rightarrow [A](\varphi_{wh}^{=\chi} \rightarrow \langle A \rangle(\text{len}_{=1} \wedge paper\_accepted)))$$

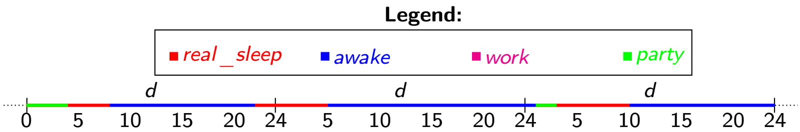
I do my job - cont'd



sometimes I party instead of sleeping

$$[G](\langle A\rangle party \rightarrow \langle A\rangle sleep)$$

I do my job - cont'd



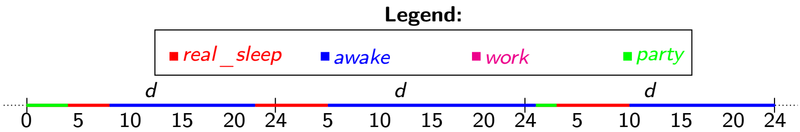
sometimes I party instead of sleeping

$$[G](\langle A \rangle party \rightarrow \langle A \rangle sleep)$$

I can't party all night long

$$[G](party \rightarrow \neg sleep \wedge \neg \langle B \rangle sleep)$$

I do my job - cont'd



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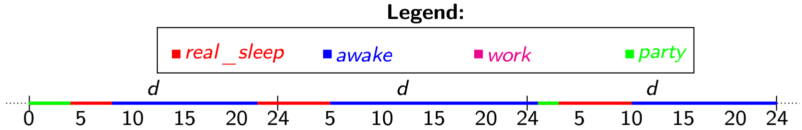
my actual sleep time is when I sleep and do not party

$$[G](party \rightarrow [A](\langle A\rangle awake \wedge \neg\langle B\rangle\langle A\rangle awake \rightarrow real\_sleep))$$

I do my job - cont'd



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$$[G](party \rightarrow [A](\langle A\rangle awake \wedge \neg\langle B\rangle\langle A\rangle awake \rightarrow real\_sleep))$$

when I have a party night, the next night I will sleep

$$[G](party \rightarrow \langle A\rangle(real\_sleep \wedge \langle A\rangle(awake \wedge \langle A\rangle(real\_sleep))))$$

The goal specification "I am happy"

φ_G = I am happy — goal specification

- subjective needs to live a happy life

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I want to publish at least κ papers per year

$$[G](year \rightarrow \langle B\rangle(\langle A\rangle paper\_accepted \wedge \langle B\rangle(\ldots \wedge \wedge \langle B\rangle(\langle A\rangle paper\_accepted \wedge \langle B\rangle\langle A\rangle paper\_accepted))))$$

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I want to work at most 40 hours in week days and 5 hours in week ends

$$[G](((\langle \mathrm{A} \rangle week \rightarrow [\mathrm{A}](\mathrm{len}_{=120} \rightarrow \varphi_{wh}^{\leq 40}))\wedge(\mathrm{len}_{=48} \wedge \langle \mathrm{A} \rangle week \rightarrow \varphi_{wh}^{\leq 5}))$$

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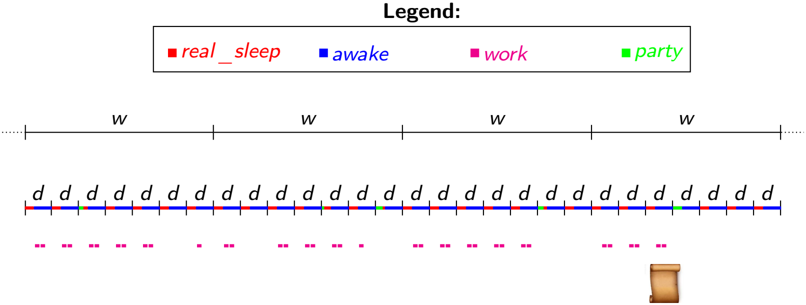
I want to work at most 40 hours in week days and 5 hours in week ends

$$[G](((\langle A\rangle week \rightarrow [A](\mathrm{len}_{=120} \rightarrow \varphi_{wh}^{\leq 40}))\wedge(\mathrm{len}_{=48}\wedge\langle A\rangle week \rightarrow \varphi_{wh}^{\leq 5}))$$

I want to party at least once a week

$$[G](week \rightarrow \langle B\rangle\langle A\rangle party)$$

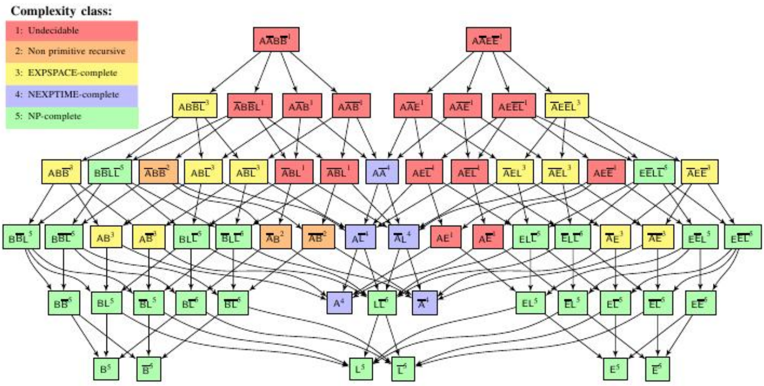
A plan to reach happiness



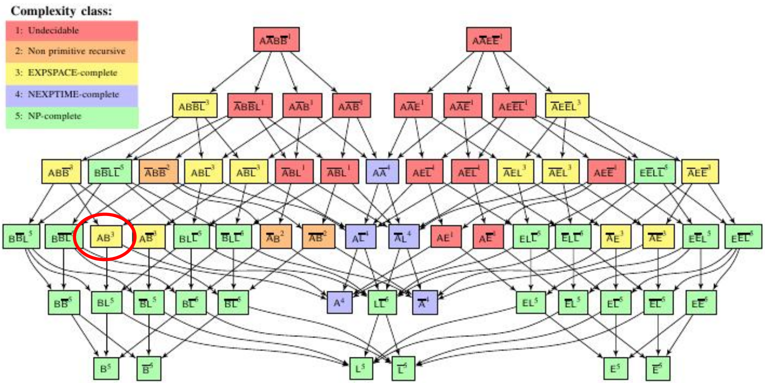
A model for $\varphi_E \wedge \varphi_G$

Satisfiability for AB decidable in EXPSPACE

Expressiveness/satisfiability classification over natural numbers



Expressiveness/satisfiability classification over natural numbers



Current research agenda

- To obtain a complete classification of the family of HS fragments with respect to decidability/undecidability of their satisfiability problem and with respect to their relative expressive power
- To extend the study of metric variants of interval logics (we already did it for $A\overline{A}$ over natural numbers, integers, and finite linear orders) to other HS fragments and over other metrizable linear orders, notably that of rational numbers

D. Bresolin, D. Della Monica, V. Goranko, A. Montanari, G. Sciavicco, Metric Propositional Neighborhood Logics on Natural Numbers, Software and Systems Modeling 2013

Current research agenda (cont'd)

D. Bresolin, A. Montanari, G. Sciavicco, P. Sala, Optimal decision procedures for MPNL over finite structures, the natural numbers, and the integers, Theoretical Computer Science 2013

- To explore possible connections between interval temporal logics and description logics

A. Artale, D. Bresolin, A. Montanari, V. Ryzhikov, G. Sciavicco, DL-Lite and Interval Temporal Logics: a Marriage Proposal, ECAI 2014

Mid-term research agenda

- Systematic application of game-theoretic techniques in interval-based synthesis
- Quest for automaton-based techniques for proving decidability of interval temporal logics
- Identification and development of major applications of interval temporal logics. Besides system specification, verification, and synthesis, planning and plan validation (to represent and to reason about actions/events with duration, accomplishments, and interval constraints), temporal databases (to deal with temporal aggregation), workflow systems (to cope with additional temporal constraints), and natural language processing (to model features like progressive tenses)

People

- **Aceto, Luca** — Reykjavik University, Iceland
- **Bresolin, Davide** — University of Bologna, Italy
- **Della Monica, Dario** — Reykjavik University, Iceland
- **Goranko, Valentin** — DTU, Lyngby, Copenhagen, Denmark
- **Hodkinson, Ian** — Imperial College, UK
- **Ingólfsdóttir, Anna** — Reykjavik University, Iceland
- **Kieroński, Emanuel** — University of Wroclaw, Poland
- **Lomuscio, Alessio** — Imperial College, UK
- **Marcinkowski, Jerzy** — University of Wroclaw, Poland
- **Michaliszyn, Jakub** — Imperial College, London
- **Montanari, Angelo** — University of Udine, Italy
- **Pratt-Hartmann, Ian** — University of Manchester, UK
- **Puppis, Gabriele** — CNRS researcher at LaBRI, France
- **Sala, Pietro** — University of Verona, Italy
- **Sciavicco, Guido** — University of Murcia, Spain
- and others (**Willem Conradie**, **Salih Durhan**, **Emilio Muñoz-Velasco**, **Aniello Murano**, **Giuseppe Perelli**, **Adriano Peron**, **Nicola Vitacolonna**, ..)

In summary: The land of interval logics is intriguing, exotic, rich with challenges, and worth exploring

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Recall that ITL make it possible to find the way to happiness

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And, it is fun!

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And, it is fun!

Everyone is invited to join the party!
Have fun at http://itl.dimi.uniud.it/

In summary: The land of interval logics is intriguing, exotic, rich with challenges, and worth exploring

Recall that ITL make it possible to find the way to happiness

And, it is fun!

Everyone is invited to join the party!
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The end.