

# Interval Temporal Logic Model Checking

(Invited talk)

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# Model checking

**Model checking:** the desired properties of a system are checked against a model of it

- ▶ the **model** is usually a (finite) state-transition system
- ▶ system properties are specified by a **temporal logic** (LTL, CTL, and the like)

Distinctive features of model checking:

- ▶ **exhaustive** check of all the possible behaviours
- ▶ **fully automatic** process
- ▶ a **counterexample** is produced for a violated property

# Point-based vs. interval-based model checking

Model checking is usually **point-based**:

- ▶ properties express requirements over points (snapshots) of a computation (states of the state-transition system)
- ▶ they are specified by means of point-based temporal logics such as LTL and CTL

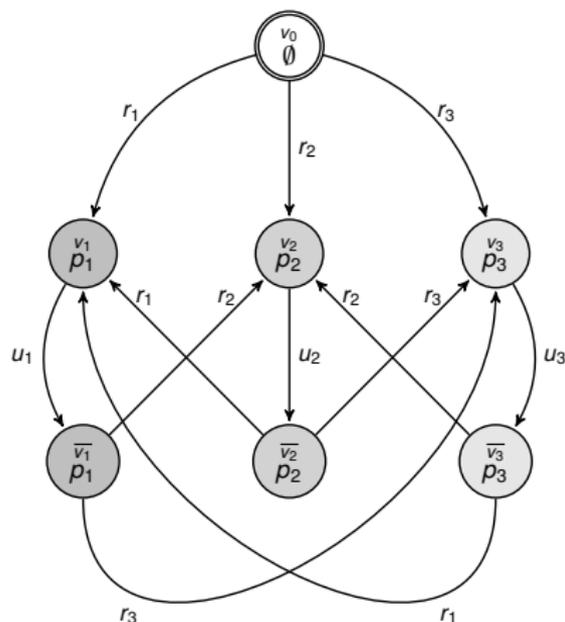
**Interval-based** properties express conditions on computation stretches, e.g., actions with duration, accomplishments, and temporal aggregations, instead of on computation states

Little work has been done on **interval temporal logic (ITL) model checking** (Bozzelli, Lomuscio, Michaliszyn, Molinari, Montanari, Murano, Perelli, Peron, Sala)

# Outline of the talk

- ▶ the model checking problem for interval temporal logics
- ▶ complexity results: the general picture
- ▶ the case of the interval temporal logic  $\overline{A\overline{A}B\overline{B}E}$
- ▶ model checking epistemic interval temporal logics
- ▶ a comparison with LTL, CTL, and CTL\* model checking

# The modeling of the system: Kripke structures



- ▶ HS formulas are interpreted over (finite) state-transition systems, whose states are labeled with sets of proposition letters (**Kripke structures**)
- ▶ An interval is a **trace** (finite path) in a Kripke structure

A finite Kripke structure

# HS: the modal logic of Allen's interval relations

The thirteen **binary ordering relations** between two intervals on a linear order form the set of *Allen's interval relations*

They give rise to corresponding unary modalities over frames where intervals are primitive entities:

- ▶ HS features **a modality for any Allen ordering relation** between pairs of intervals (except for equality)

Allen rel.	HS	Definition	Example
<i>meets</i>	$\langle A \rangle$	$[x, y] \mathcal{R}_A [v, z] \iff y = v$	
<i>before</i>	$\langle L \rangle$	$[x, y] \mathcal{R}_L [v, z] \iff y < v$	
<i>started-by</i>	$\langle B \rangle$	$[x, y] \mathcal{R}_B [v, z] \iff x = v \wedge z < y$	
<i>finished-by</i>	$\langle E \rangle$	$[x, y] \mathcal{R}_E [v, z] \iff y = z \wedge x < v$	
<i>contains</i>	$\langle D \rangle$	$[x, y] \mathcal{R}_D [v, z] \iff x < v \wedge z < y$	
<i>overlaps</i>	$\langle O \rangle$	$[x, y] \mathcal{R}_O [v, z] \iff x < v < y < z$	

All modalities can be expressed by means of  $\langle A \rangle$ ,  $\langle B \rangle$ ,  $\langle E \rangle$ , and their transposed modalities only

# HS semantics and model checking

Truth of a formula  $\psi$  over a trace  $\rho$  of a Kripke structure  $\mathcal{K} = (\mathcal{AP}, W, \delta, \mu, w_0)$  defined by induction on the complexity of  $\psi$ :

- ▶  $\mathcal{K}, \rho \models p$  iff  $p \in \bigcap_{w \in \text{states}(\rho)} \mu(w)$ , for any letter  $p \in \mathcal{AP}$  (**homogeneity assumption**);
- ▶ negation, disjunction, and conjunction are standard;
- ▶  $\mathcal{K}, \rho \models \langle \mathbf{A} \rangle \psi$  iff there is a trace  $\rho'$  s.t.  $\text{fst}(\rho) = \text{fst}(\rho')$  and  $\mathcal{K}, \rho' \models \psi$ ;
- ▶  $\mathcal{K}, \rho \models \langle \mathbf{B} \rangle \psi$  iff there is a prefix  $\rho'$  of  $\rho$  s.t.  $\mathcal{K}, \rho' \models \psi$ ;
- ▶  $\mathcal{K}, \rho \models \langle \mathbf{E} \rangle \psi$  iff there is a suffix  $\rho'$  of  $\rho$  s.t.  $\mathcal{K}, \rho' \models \psi$ ;
- ▶ the semantic clauses for  $\langle \bar{\mathbf{A}} \rangle$ ,  $\langle \bar{\mathbf{B}} \rangle$ , and  $\langle \bar{\mathbf{E}} \rangle$  are similar

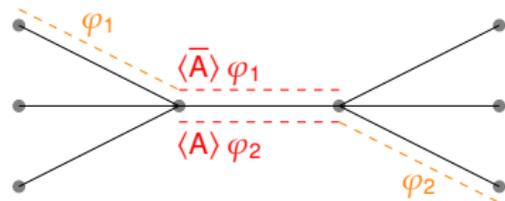
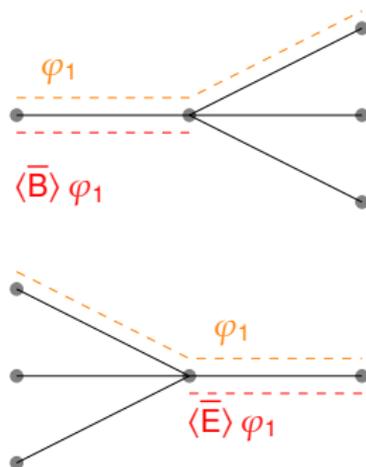
## Model Checking

$\mathcal{K} \models \psi \iff$  for all *initial* traces  $\rho$  of  $\mathcal{K}$ , it holds that  $\mathcal{K}, \rho \models \psi$

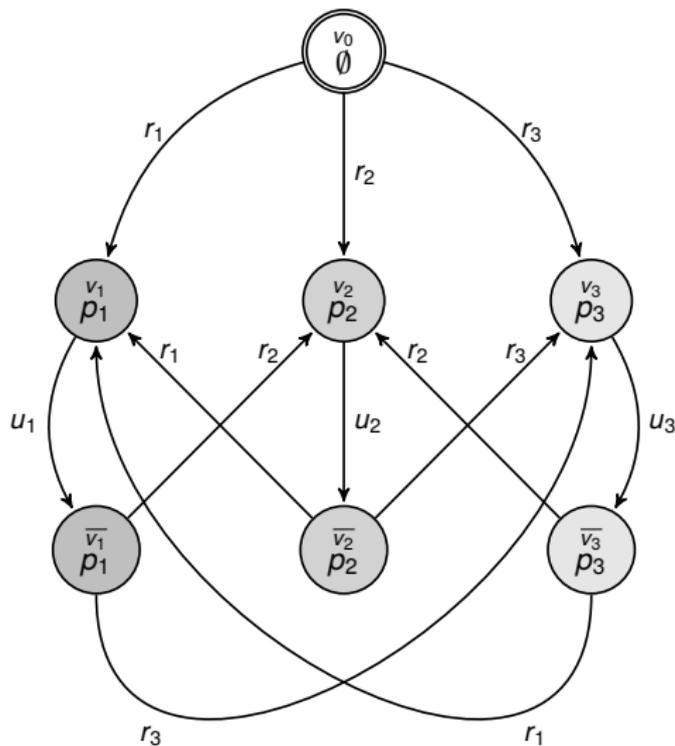
**Possibly infinitely many traces!**

## Remark: HS state semantics ( $HS_{st}$ )

- ▶ According to the given semantics, HS modalities allow one to branch both in the past and in the future



# An example: the Kripke structure $\mathcal{K}_{Sched}$



## A short account of $\mathcal{K}_{Sched}$

$\mathcal{K}_{Sched}$  models the behaviour of a **scheduler** serving 3 processes which are continuously requesting the use of a common resource

**Initial state:**  $v_0$  (no process is served in that state)

In  $v_i$  and  $\bar{v}_i$  the  **$i$ -th process** is served ( $p_i$  holds in those states)

The scheduler **cannot serve the same process twice** in two successive rounds:

- ▶ process  $i$  is served in state  $v_i$ , then, after “some time”, a transition  $u_i$  from  $v_i$  to  $\bar{v}_i$  is taken; subsequently, process  $i$  cannot be served again immediately, as  $v_i$  is not directly reachable from  $\bar{v}_i$
- ▶ a transition  $r_j$ , with  $j \neq i$ , from  $\bar{v}_i$  to  $v_j$  is then taken and process  $j$  is served

It can be **easily generalised** to an arbitrary number of processes

## Some meaningful properties to be checked over $\mathcal{K}_{Sched}$

Validity of properties over all legal computation intervals can be forced by modality  $[E]$  (they are suffixes of at least one initial trace)

**Property 1:** in any computation interval of length at least 4, at least 2 processes are witnessed (**YES**/no process can be executed twice in a row)

$$\mathcal{K}_{Sched} \models [E](\langle E \rangle^3 \top \rightarrow (\chi(p_1, p_2) \vee \chi(p_1, p_3) \vee \chi(p_2, p_3))),$$

where  $\chi(p, q) = \langle E \rangle \langle \bar{A} \rangle p \wedge \langle E \rangle \langle \bar{A} \rangle q$

**Property 2:** in any computation interval of length at least 11, process 3 is executed at least once (**NO**/the scheduler can postpone the execution of a process ad libitum)

$$\mathcal{K}_{Sched} \not\models [E](\langle E \rangle^{10} \top \rightarrow \langle E \rangle \langle \bar{A} \rangle p_3)$$

**Property 3:** in any computation interval of length at least 6, all processes are witnessed (**NO**/the scheduler should be forced to execute them in a strictly periodic manner, which is not the case)

$$\mathcal{K}_{Sched} \not\models [E](\langle E \rangle^5 \rightarrow (\langle E \rangle \langle \bar{A} \rangle p_1 \wedge \langle E \rangle \langle \bar{A} \rangle p_2 \wedge \langle E \rangle \langle \bar{A} \rangle p_3))$$

# Model checking: the key notion of $BE_k$ -descriptor

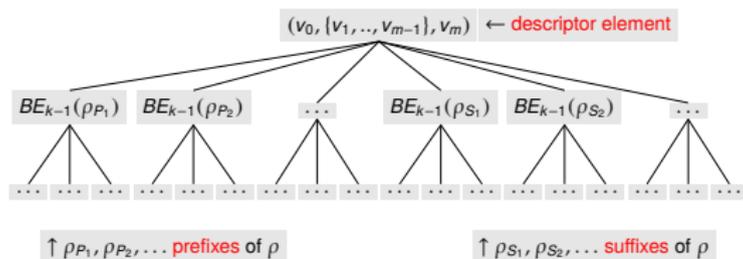
- ▶ The **BE-nesting depth** of an HS formula  $\psi$  ( $\text{Nest}_{BE}(\psi)$ ) is the maximum degree of nesting of modalities  $B$  and  $E$  in  $\psi$
- ▶ Two traces  $\rho$  and  $\rho'$  of a Kripke structure  $\mathcal{K}$  are  **$k$ -equivalent** if and only if  $\mathcal{K}, \rho \models \psi$  iff  $\mathcal{K}, \rho' \models \psi$  for all HS-formulas  $\psi$  with  $\text{Nest}_{BE}(\psi) \leq k$

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We provide a suitable tree representation for a trace, called a  $BE_k$ -descriptor

The  **$BE_k$ -descriptor** for a trace  $\rho = v_0 v_1 \dots v_{m-1} v_m$ , denoted  $BE_k(\rho)$ , is defined as follows:

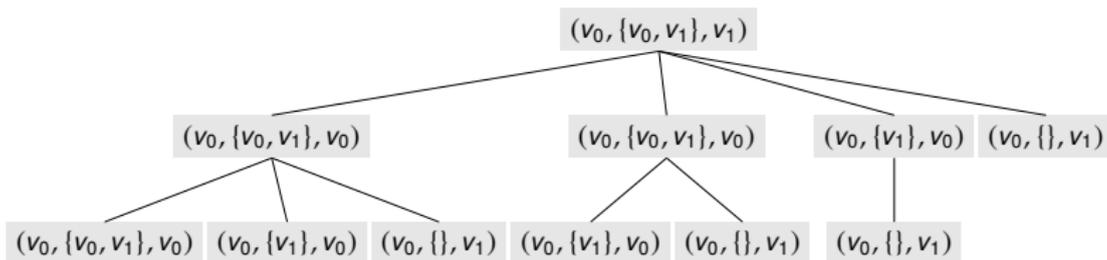


Remark: the descriptor does not feature sibling isomorphic subtrees

# An example of a $BE_2$ -descriptor



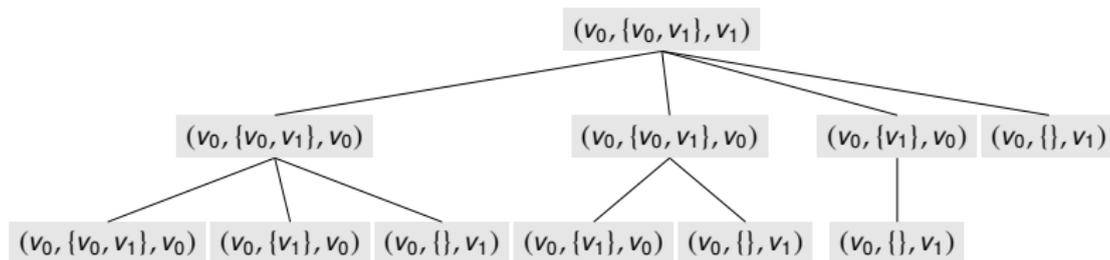
The  $BE_2$ -descriptor for the trace  $\rho = v_0 v_1 v_0^4 v_1$  (for the sake of readability, only the subtrees for prefixes are displayed)



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**Remark:** the subtree to the left is associated with both prefixes  $v_0 v_1 v_0^3$  and  $v_0 v_1 v_0^4$  (there are no sibling isomorphic subtrees in the descriptor)

# Decidability of model checking for full HS

**FACT 1:** For any Kripke structure  $\mathcal{K}$  and any BE-nesting depth  $k \geq 0$ , the number of different  $BE_k$ -descriptors is **finite** (and thus at least one descriptor has to be associated with infinitely many traces)

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## Theorem

*The model checking problem for full HS on finite Kripke structures is decidable (with a non-elementary algorithm)*



A. Molinari, A. Montanari, A. Murano, G. Perelli, and A. Peron, Checking Interval Properties of Computations, Acta Informatica, Special Issue: Temporal Representation and Reasoning (TIME'14), Vol. 56, n. 6-8, October 2016, pp. 587-619

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What about lower bounds?

# The logic BE

## Theorem

*The model checking problem for BE, over finite Kripke structures, is EXPSPACE-hard*



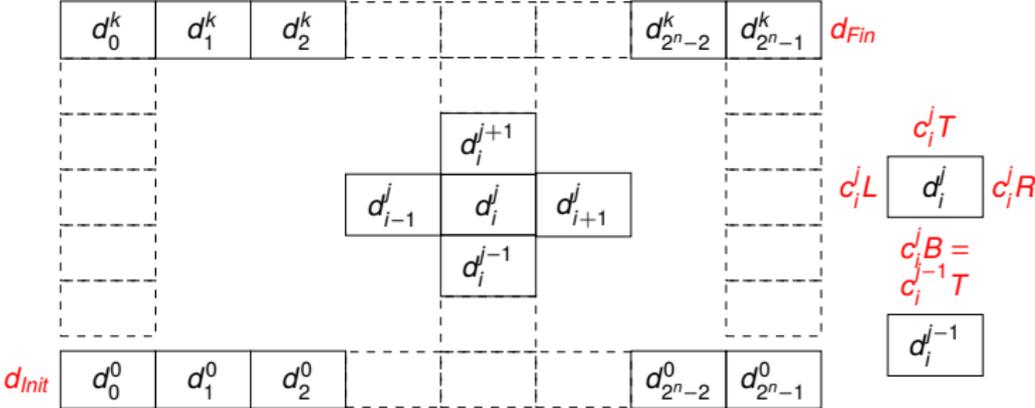
L. Bozzelli, A. Molinari, A. Montanari, A. Peron, and P. Sala, Interval Temporal Logic Model Checking: The Border Between Good and Bad HS Fragments, IJCAR 2016

Proof (sketch): a polynomial-time **reduction from a domino-tiling problem** for grids with rows of single exponential length

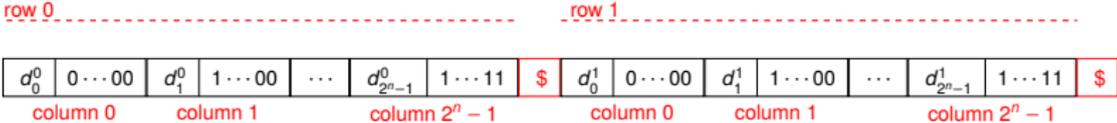
- ▶ for an instance  $\mathcal{I}$  of the problem, we build a Kripke structure  $\mathcal{K}_{\mathcal{I}}$  and a BE formula  $\varphi_{\mathcal{I}}$  in polynomial time
- ▶ there is an initial trace of  $\mathcal{K}_{\mathcal{I}}$  satisfying  $\varphi_{\mathcal{I}}$  iff there is a tiling of  $\mathcal{I}$
- ▶  $\mathcal{K}_{\mathcal{I}} \models \neg\varphi_{\mathcal{I}}$  iff there exists no tiling of  $\mathcal{I}$

# BE hardness: encoding of the domino-tiling problem

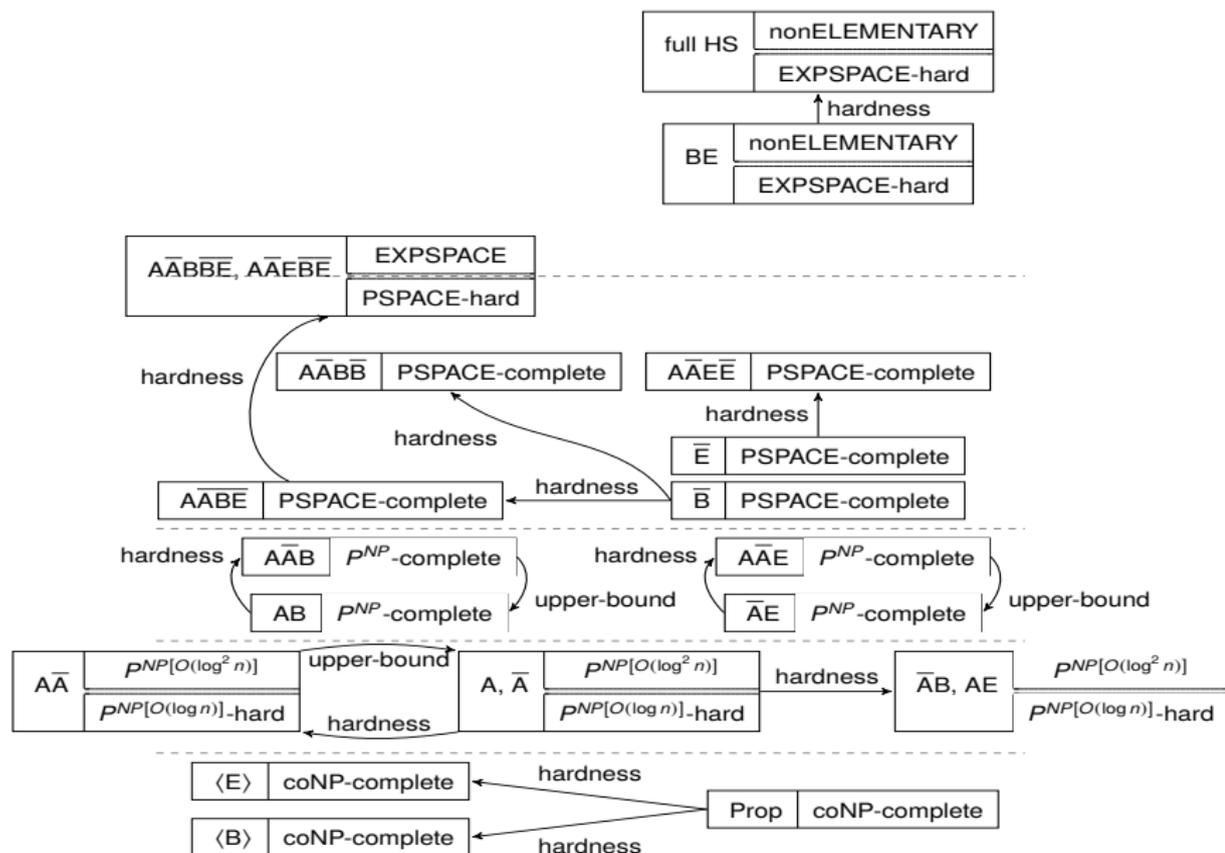
Instance of the tiling problem:  $(C, \Delta, n, d_{init}, d_{final})$ , with  $C$  a finite set of colors and  $\Delta \subseteq C \times C \times C \times C$  a set of tuples  $(c_B, c_L, c_T, c_R)$



## String (interval) encoding of the problem



# The complexity picture



# Three main gaps to fill

The picture shows that there three main gaps to fill:

- ▶ full HS and BE are in between **nonELEMENTARY** and **EXPSPACE**
- ▶  $A\bar{A}B\bar{B}E$ ,  $A\bar{A}E\bar{B}E$ ,  $AB\bar{B}E$ ,  $AE\bar{B}E$ ,  $\bar{A}B\bar{B}E$ , and  $\bar{A}E\bar{B}E$  are in between **EXPSPACE** and **PSPACE**
- ▶  $A$ ,  $\bar{A}$ ,  $A\bar{A}$ ,  $\bar{A}B$ , and  $AE$  are in between  $P^{NP[O(\log^2 n)]}$  and  $P^{NP[O(\log n)]}$

# The logic $A\bar{A}B\bar{B}E$

Let us consider the case of the logic  $A\bar{A}B\bar{B}E$ , which is obtained from full HS ( $A\bar{A}B\bar{B}E\bar{E}$ ) by removing modality  $\langle E \rangle$

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- ▶ the size of the tree representation of  $B_k$ -descriptors is larger than necessary (**redundancy**) and it prevents their efficient exploitation in model checking algorithms
- ▶ a **trace representative** can be chosen to represent a (possibly infinite) set of traces with the same  $B_k$ -descriptor
- ▶ a **bound**, which depends on both the number  $|W|$  of states of the Kripke structure and the B-nesting depth  $k$ , can be given to the length of trace representatives

# Prefix-bisimilarity

## Definition (Prefix-bisimilarity)

Two traces  $\rho$  and  $\rho'$  are  **$h$ -prefix bisimilar** if the following conditions inductively hold:

- ▶ for  $h = 0$ :  
 $\text{fst}(\rho) = \text{fst}(\rho')$ ,  $\text{lst}(\rho) = \text{lst}(\rho')$ , and  $\text{states}(\rho) = \text{states}(\rho')$
- ▶ for  $h > 0$ :  
 $\rho$  and  $\rho'$  are 0-prefix bisimilar and for each proper prefix  $v$  of  $\rho$  (resp., proper prefix  $v'$  of  $\rho'$ ), there exists a proper prefix  $v'$  of  $\rho'$  (resp., proper prefix  $v$  of  $\rho$ ) such that  $v$  and  $v'$  are  $(h - 1)$ -prefix bisimilar
- ▶  $h$ -prefix bisimilarity is an **equivalence relation** over the set of traces
- ▶  $h$ -prefix bisimilarity **propagates downwards**

# $h$ -prefix bisimilarity $\Rightarrow h$ -equivalence

## Proposition

Let  $h \geq 0$ , and  $\rho$  and  $\rho'$  be two  $h$ -prefix bisimilar traces of a Kripke structure  $\mathcal{K}$ . For each  $\overline{A\overline{A}B\overline{B}E}$  formula  $\psi$ , with  $B$ -nesting of  $\psi$  less than or equal to  $h$ , it holds that

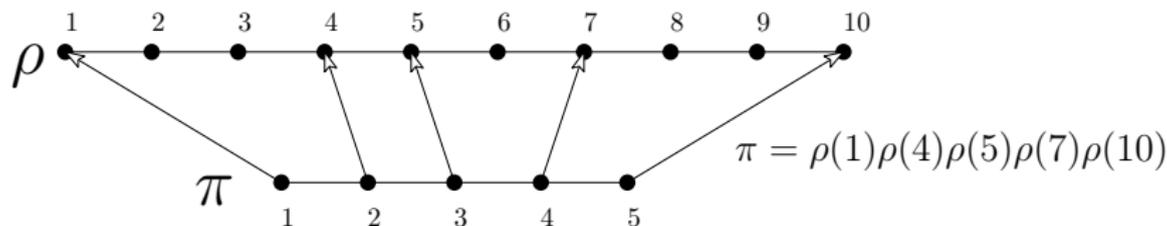
$$\mathcal{K}, \rho \models \psi \iff \mathcal{K}, \rho' \models \psi$$

# Induced trace

## Definition (Induced trace)

Let  $\rho$  be a trace of length  $n$  of a Kripke structure  $\mathcal{K}$ . A **trace induced by  $\rho$**  is a trace  $\pi$  of  $\mathcal{K}$  such that there exists an increasing sequence of  $\rho$ -positions  $i_1 < \dots < i_k$ , where  $i_1 = 1$ ,  $i_k = n$ , and

$$\pi = \rho(i_1) \cdots \rho(i_k)$$



If  $\pi$  is induced by  $\rho \Rightarrow \text{fst}(\pi) = \text{fst}(\rho)$ ,  $\text{lst}(\pi) = \text{lst}(\rho)$ , and  $|\pi| \leq |\rho|$

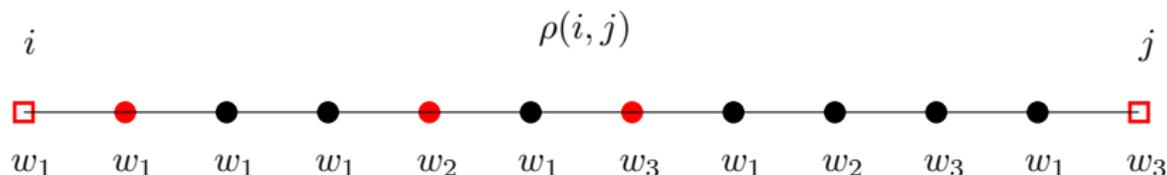
# Prefix-skeleton sampling

## Definition (Prefix-skeleton sampling)

Let  $\rho$  be a trace of a Kripke structure  $\mathcal{K} = (\mathcal{AP}, W, \delta, \mu, w_0)$ .

Given two  $\rho$ -positions  $i$  and  $j$ , with  $i \leq j$ , the **prefix-skeleton sampling** of  $\rho(i, j)$  is the **minimal set  $P$**  of  $\rho$ -positions in the interval  $[i, j]$  satisfying:

- ▶  $i, j \in P$ ;
- ▶ for each state  $w \in W$  occurring along  $\rho(i + 1, j - 1)$ , the minimal position  $k \in [i + 1, j - 1]$  such that  $\rho(k) = w$  is in  $P$



$$P = \{i, i + 1, i + 4, i + 6, j\}$$

# $h$ -prefix sampling

## Definition ( $h$ -prefix sampling)

For each  $h \geq 1$ , the  $h$ -prefix sampling of  $\rho$  is the minimal set  $P_h$  of  $\rho$ -positions inductively satisfying the following conditions:

- ▶ for  $h = 1$ :  $P_1$  is the prefix-skeleton sampling of  $\rho$ ;
- ▶ for  $h > 1$ :
  - ▶  $P_h \supseteq P_{h-1}$  and
  - ▶ for all pairs of consecutive positions  $i, j$  in  $P_{h-1}$ , the prefix-skeleton sampling of  $\rho(i, j)$  is in  $P_h$

## Proposition

The  $h$ -prefix sampling  $P_h$  of (any)  $\rho$  is such that  $|P_h| \leq (|W| + 2)^h$

## A small model (trace) result

Given a trace  $\rho$ , we can derive another trace  $\rho'$ , induced by  $\rho$  and  $h$ -prefix bisimilar to  $\rho$ , such that  $|\rho'| \leq (|W| + 2)^{h+2}$  as follows:

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$\rho$  and  $\rho'$  can be proved to be  $h$ -prefix bisimilar, and thus  $\rho'$  is indistinguishable from  $\rho$  with respect to the fulfilment of any formula  $\psi$ , with B-nesting of  $\psi$  (abbreviated  $\text{Nest}_B(\psi) \leq h$ )

By the previous bound on  $|P_h|$ , it holds that  $|\rho'| \leq (|W| + 2)^{h+2}$

# An EXPSPACE model checking algorithm for $\overline{A\overline{A}B\overline{B}E}$

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## Algorithm 1 ModCheck( $\mathcal{X}, \psi$ )

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- 1:  $h \leftarrow \text{Nest}_{\mathbb{B}}(\psi)$
  - 2:  $u \leftarrow \text{New}(\text{Unravelling}(\mathcal{X}, w_0, h))$   $\triangleleft w_0$  initial state of  $\mathcal{X}$
  - 3: **while**  $u.\text{hasMoreTracks}()$  **do**
  - 4:      $\rho' \leftarrow u.\text{getNextTrack}()$
  - 5:     **if**  $\text{Check}(\mathcal{X}, h, \psi, \rho') = 0$  **then return** 0: " $\mathcal{X}, \rho' \not\models \psi$ "  $\triangleleft$  Counterexample found ✗
  - return** 1: " $\mathcal{X} \models \psi$ "  $\triangleleft$  Model checking OK ✓
- 



L. Bozzelli, A. Molinari, A. Montanari, A. Peron, and P. Sala, Interval Temporal Logic Model Checking Based on Track Bisimilarity and Prefix Sampling, ICTCS 2016

# PSPACE-hardness of $\overline{A\overline{A}B\overline{B}E}$ model checking

**PSPACE-hardness** of the model checking problem for the fragment  $\overline{B}$  (and thus for  $\overline{A\overline{A}B\overline{B}E}$ ) can be proved by a reduction from the QBF problem

## Theorem

*The model checking problem for  $\overline{B}$ , and thus for  $\overline{A\overline{A}B\overline{B}E}$ , over finite Kripke structures is PSPACE-hard*

$\overline{A\overline{A}B\overline{B}E}$  model checking is thus in between PSPACE and EXPSPACE (remind: BE model checking is EXPSPACE-hard)



A. Molinari, A. Montanari, A. Peron, and P. Sala, Model Checking Well-Behaved Fragments of HS: The (Almost) Final Picture, KR 2016

# Epistemic HS (Lomuscio and Michaliszyn)

**Distinctive feature** of Epistemic HS (EHS for short): the labelling function is defined on the **endpoints** of the (finite) traces/intervals

Lomuscio and Michaliszyn proved that the local model checking problem (verification of a given specification against a single initial trace) for the fragment EHS[BE] is **PSPACE-complete**

If epistemic modalities are removed, it is **in PTIME** (notice that modalities  $\langle B \rangle$  and  $\langle E \rangle$  allow one to access only sub-intervals of the given initial one, whose number is quadratic in the length of it)



A. Lomuscio and J. Michaliszyn, An Epistemic Halpern-Shoham Logic, IJCAI 2013

## Epistemic HS (Lomuscio and Michaliszyn) - cont'd

Later on, they showed that **the picture drastically changes** with other fragments of HS that allow one to access infinitely many traces

They proved that the model checking problem for the HS fragment  $\overline{AB}$ , extended with epistemic modalities, is decidable, with a **non-elementary** upper bound

Notice that formulas of this logic can possibly refer to infinitely many (future) traces



A. Lomuscio and J. Michaliszyn, Decidability of model checking multi-agent systems against a class of EHS specifications, ECAI 2014

# Epistemic HS (Lomuscio and Michaliszyn) - cont'd

In their most recent contribution, Lomuscio and Michaliszyn generalized the labeling function by allowing it to be given by any **regular expression on the states of intervals**

Such a generalization results in a considerable increase in the expressiveness of the specifications at no computational cost in terms of the corresponding model checking problem



A. Lomuscio and J. Michaliszyn, Model Checking Multi-Agent Systems against Epistemic HS Specifications with Regular Expressions, KR 2016

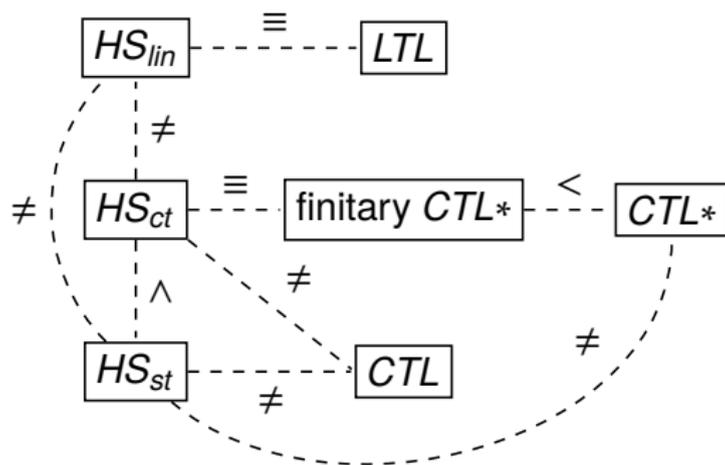
# A comparison with LTL, CTL, and CTL\* model checking

In order to compare the expressiveness of HS model checking with that of LTL, CTL, and CTL\* model checking, we define alternative semantics for HS:

- ▶ **trace semantics** ( $HS_{lin}$ ) - the infinite paths (computations) of the Kripke structure are the main semantic entities
- ▶ **computation tree semantics** ( $HS_{ct}$ ) - the future is branching, but the past is linear (as well as finite and cumulative)

Trace (resp., computation tree) semantics allowed us to establish a **bridge** between HS model checking and LTL (resp., CTL/CTL\*) model checking

# The expressiveness picture



L. Bozzelli, A. Molinari, A. Montanari, A. Peron, and P. Sala, Interval vs. Point Temporal Logic Model Checking: an Expressiveness Comparison, FSTTCS 2016

# Current and future research agenda

- ▶ To **complete the picture** of interval temporal logic model checking under the homogeneity assumption (and the  $HS_{st}$  state semantics)
- ▶ **Planning as Model Checking** in Interval Temporal Logic
- ▶ To remove the **homogeneity assumption**
- ▶ To replace **finite Kripke structures** with more complex ones (pushdown systems, other infinite state transition systems, systems based on timelines)

# People

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