Bounded Model Checking

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Verification and Validation Techniques in AI and Cybersecurity

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Introduction

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  - it costed $\approx$ US $475 million;
  - big investment in formal verification.

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- distinctive features:
  - fully automatic;
  - exhaustive;
  - it generates a counterexample trace if the specification does not hold.

Linear Temporal Logic

We consider LTL model checking.

- LTL syntax:

$$p \mid \neg\phi \mid \phi \vee \phi \mid \mathsf{X}\,\phi \mid \phi\,\mathcal{U}\,\phi$$

  with $p \in \Sigma$

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- LTL syntax:

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  with $p \in \Sigma$
- shortcuts:
  - $\phi_1\,\mathcal{R}\,\phi_2 \equiv \neg(\neg\phi_1\,\mathcal{U}\,\neg\phi_2)$,
  - $\mathsf{F}\,\phi_1 \equiv \top\,\mathcal{U}\,\phi_1$
  - $\mathsf{G}\,\phi_1 \equiv \neg\,\mathsf{F}\,\neg\phi_1$

Linear Temporal Logic

- Semantics. LTL formulas are interpreted over infinite *state sequences* $\sigma = \langle \sigma_0, \sigma_1, \ldots \rangle \in (2^{\Sigma})^{\omega}$ of sets of propositions $\sigma_i \in 2^{\Sigma}$:

$$
\begin{array}{lll}
\sigma \models_i p & \text{iff} & p \in \sigma_i \\
\sigma \models_i \mathsf{X}\,\phi & \text{iff} & \sigma \models_{i+1} \phi \\
\sigma \models_i \phi_1 \,\mathcal{U}\, \phi_2 & \text{iff} & \text{there exists } j \geq i \text{ such that} \\
 & & \sigma \models_j \phi_2 \text{ and } \sigma \models_k \phi_1 \text{ for all} \\
 & & i \leq k < j \\
\ldots & & \\
\sigma \models_i \mathsf{F}\,\phi & \text{iff} & \exists j \geq i \,.\, \sigma \models_j \phi \\
\sigma \models_i \mathsf{G}\,\phi & \text{iff} & \forall j \geq i \,.\, \sigma \models_j \phi
\end{array}
$$

LTL model checking

- LTL model checking:
  - decide if $\mathcal{M}, s \models \phi$, where $\mathcal{M} = (S, I, T, L)$ is a Kripke structure, $s \in I$ is an initial state and ϕ is an LTL formula; in many contexts, you may find the notation: $\mathcal{M}, s \models A\phi$;
  - PSPACE-complete.

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$$\mathcal{M}, s \not\models \phi$$

State-space Explosion Problem

- the previous algorithm belongs to the class of explicit model checking algorithms:
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  is exponential in n;
- the size of system that could be verified by explicit model checkers was restricted to $\approx 10^6$ states.

From the Turing Award citation of Ed. Clarke

EDMUND MELSON CLARKE

United States – 2007

CITATION

Together with E. Allen Emerson and Joseph Sifakis, for their role in developing Model-Checking into a highly effective verification technology that is widely adopted in the hardware and software industries.

https://amturing.acm.org/award_winners/clarke_1167964.cfm

Averting the state space explosion

Although the 1981 paper [2] demonstrated that the model checking was possible in principle, its application to practical systems was severely limited. The most pressing limitation was the number of states to search. Early model checkers required explicitly computing every possible configuration of values the program might assume. For example, if a program counts the millimeters of rain at a weather station each day of the week, it will need 7 storage locations. Each location will have to be big enough to hold the largest rain level expected in a single day. If the highest rain level in a day is 1 meter, this simple program will have 10^{21} possible states, slightly less than the number of stars in the observable universe. Early model checkers would have to verify that the required property was true for every one of those states.

Tackling the explosion...

Three main techniques have been proposed:

- BDD-based symbolic model checking
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- SAT-based symbolic model checking, aka Bounded Model Checking.

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They allowed for the verification of systems with $> 10^{120}$ states.

- substantially larger than the number of atoms in the observable universe (around 10^{80})

Symbolic Transition Systems

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- why not in DNF?

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- there are several efficient algorithms for solving SAT (*e.g.*, DPLL, CDCL...) along with many heuristics (*e.g.*, 2 watching literals, glue clauses...)
- some numbers:
  - $> 100{\cdot}000$ variables;
  - $> 1{\cdot}000{\cdot}000$ clauses;

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The corresponding symbolic Kripke structure is the tuple $(s, f_I, f_T, \{f_{p_1}, \ldots, f_{p_k}\})$.

Symbolic Transition Systems

- we will write simply $\mathcal{M} = (S, I, T, L)$, meaning a symbolic transition system

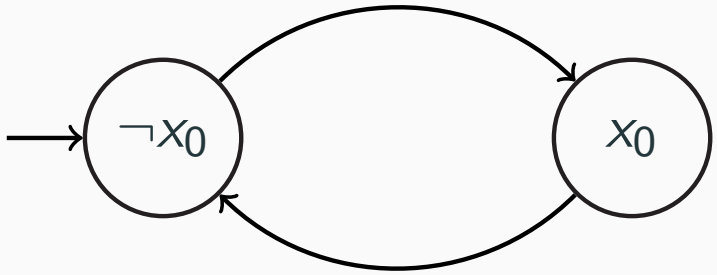
Real HW/SW systems are much easier to model with symbolic Kripke structures rather than with explicit-state ones:

- symbolic Kripke structures resemble the code of a program (or the logic of a circuit) in a much more natural fashion.

Example 1

simple-example.smv

Example 1 - SMV

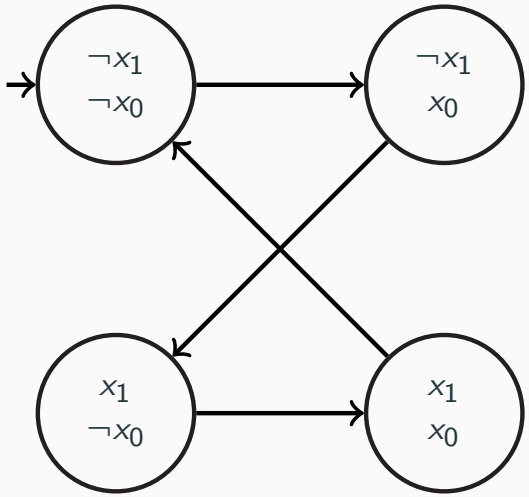


MODULE main
VAR
x_0 : boolean;
INIT
$\neg x_0$;
TRANS
$x_0 \leftrightarrow \neg x_0'$;

Example - 2

modulo-4-counter.smv

Example 2 - SMV



MODULE main

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x_0 : boolean ;

x_1 : boolean ;

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TRANS

$(x_0' \leftrightarrow \neg x_0)$

$\wedge$

$(x_1' \leftrightarrow ((x_0 \wedge \neg x_1) \vee (\neg x_0 \wedge x_1)))$;

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Bounded Model Checking

- recall that we can reduce $\mathcal{M}, s \models \psi$ to checking the emptiness of $\mathcal{M} \times \mathcal{A}_{\neg\psi}$;
  - the universal problem $\mathcal{M}, s \models A\psi$ is reduced to the existential problem $\mathcal{M}, s \models E\phi$, where $\phi := \neg\psi$;
- Bounded Model Checking (BMC) solves the problem $\mathcal{M}, s \models E\phi$ by proceeding incrementally:
  - we start with $k = 0$;
  - check if there exists and execution π of $\mathcal{M}$ of length k that satisfies ϕ; encode this problem into a SAT instance and call a SAT-solver;
  - if so, we have found a counterexample to ψ; if not, $k{+}{+}$.

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- ...but LTL formulas are defined over infinite state sequences;

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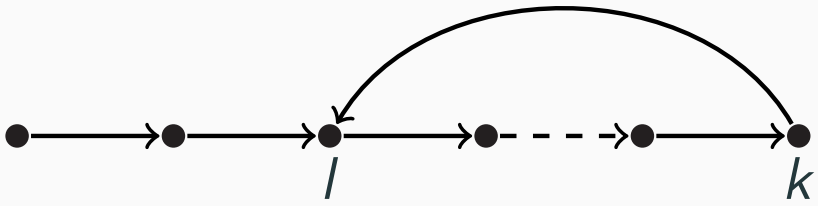
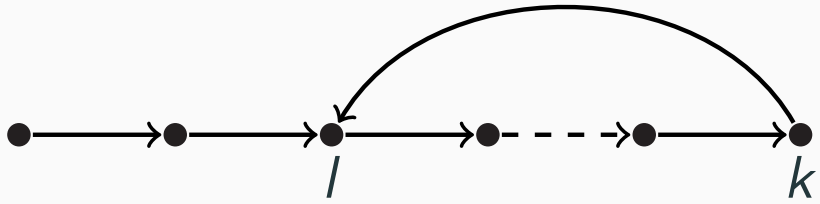
Crucial observation:

- a finite trace can still represent an infinite state sequence, if it contains a loop-back.



k-loop, aka Lasso-Shaped Models



Definition (k-loop)

A path π is a (k, l)-loop, with $l \leq k$, if $T(\pi(k), \pi(l))$ holds and $\pi = u \cdot v^{\omega}$, where:

- $u = \pi(1) \dots \pi(l-1)$;
- $v = \pi(l) \dots \pi(k)$.

We call π a k-loop if there exists $l \leq k$ for which π is a (k, l)-loop.

BMC

Given a finite trace π of the system $\mathcal{M}$, BMC distinguishes between two cases:

- either π contains a loop-back (π is lasso-shaped):
  $\Rightarrow$ apply standard LTL semantics to check if $\pi \models \phi$;
- or π is loop-free:
  $\Rightarrow$ apply bounded semantics
  $\Rightarrow$ if a path is a model of ϕ under bounded semantics then any extension of the path is a model of ϕ under standard semantics (conservative semantics)

Bounded Semantics for LTL

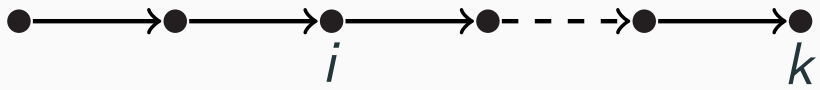
If π is not a k-loop, we introduce bounded semantics for LTL.

Definition (Bounded semantics for LTL)

Let $k \geq 0$ and π a path that is not a k-loop. An LTL formula ϕ is valid along π with bound k, written $\pi \models_k^0 \phi$, iff:

- $\pi \models_k^i p \quad$ *iff* $\quad p \in L(\pi(i))$
- $\pi \models_k^i \neg p \quad$ *iff* $\quad p \notin L(\pi(i))$

Bounded Semantics for LTL



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- $\pi \models_k^i \phi_1 \vee \phi_2$ *iff* $\pi \models_k^i \phi_1$ *or* $\pi \models_k^i \phi_2$
- $\pi \models_k^i \phi_1 \wedge \phi_2$ *iff* $\pi \models_k^i \phi_1$ *and* $\pi \models_k^i \phi_2$

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- $\pi \models_k^i \mathsf{X}\,\phi_1$ *iff* $i < k$ *and* $\pi \models_k^{i+1} \phi_1$
- $\pi \models_k^i \phi_1\,\mathcal{U}\,\phi_2$ *iff* $\exists i \leq j \leq k$ *such that* $\pi \models_k^j \phi_2$ *and* $\forall i \leq n < j$ *it holds that* $\pi \models_k^n \phi_1$

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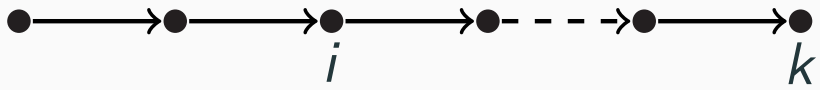

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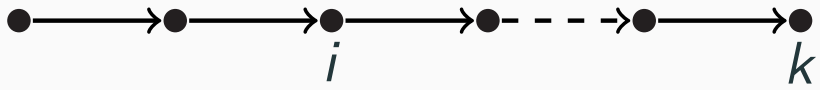
- $\pi \models_k^i \mathsf{G}\,\phi_1$ *iff* *???*
- $\pi \models_k^i \mathsf{F}\,\phi_1$ *iff* *???*

Bounded Semantics for LTL



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- $\pi \models_k^i \mathsf{G}\,\phi_1$ *is always false*
- $\pi \models_k^i \mathsf{F}\,\phi_1$ *iff* $\exists i \leq j \leq k$ *such that* $\pi \models_k^j \phi_1$

SAT-based encoding of BMC

Now we see how to reduce BMC to SAT.

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Definition (Unfolding of the Transition Relation)

For a Kripke structure $\mathcal{M}$ and $k \geq 0$, we define:

$$[\![\mathcal{M}]\!]_k := I(s_0) \wedge \bigwedge_{i=0}^{k-1} T(s_i, s_{i+1})$$

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What does a model of $[\![\mathcal{M}]\!]_k$ represent?

Encoding of the LTL formula

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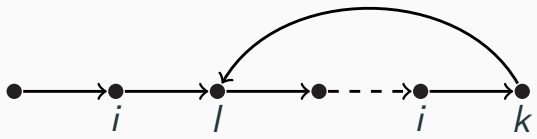
We have seen that BMC distinguishes between lasso-shaped (k-loop) and loop-free paths:

- we start with the encoding in case of k-loops.

Encoding of a loop



Definition (Loop Encoding)

Let $l \leq k$. We define:

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- $L_k := \bigvee_{l=0}^{k} {}_lL_k$

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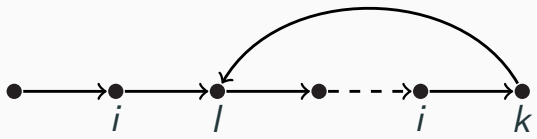

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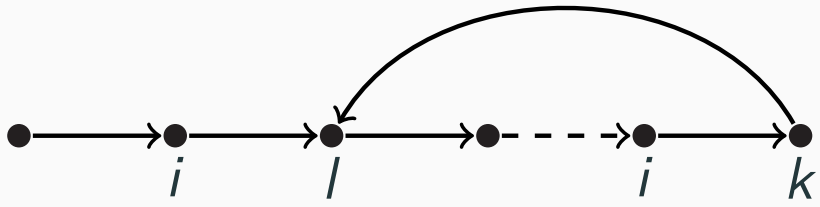
Definition (Successor in a Loop)

Let $l, i \leq k$ and π be a (k, l)-loop. We define the successor $succ(i)$ of i in π as:

- $succ(i) := i + 1$ *if* $i < k$;
- $succ(i) := l$ *if* $i = k$.

Encoding in case of Loop



Definition (Encoding of an LTL formula for a (k, l)-loop)

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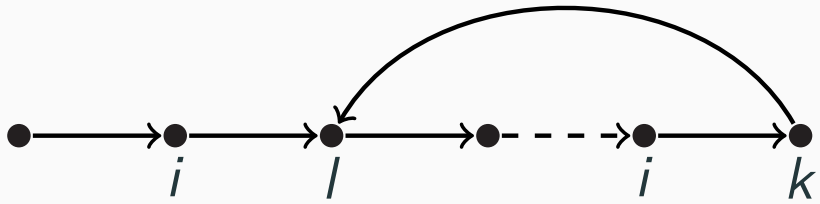

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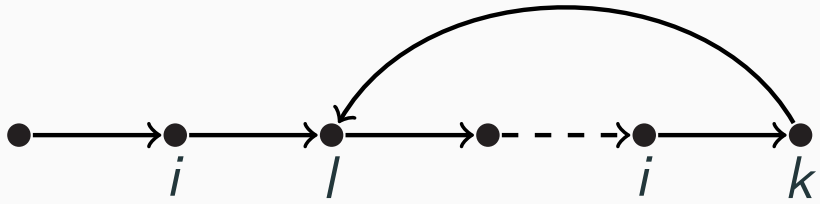

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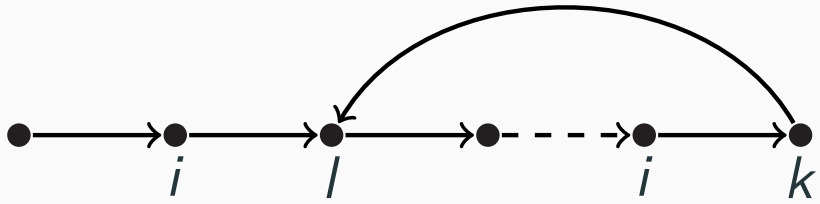

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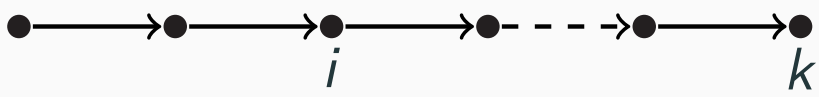
Encoding in case of NO Loops



Definition (Encoding of an LTL formula for a loop-free path)

Let ϕ be an LTL formula and $i, k \geq 0$. We define $[\![\phi]\!]_k^i$ recursively as follows:

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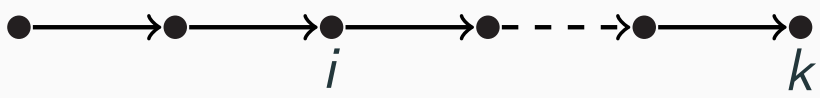
Encoding in case of NO Loops



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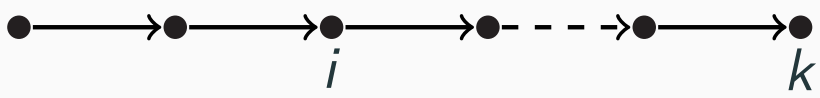

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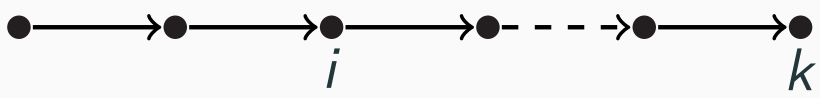

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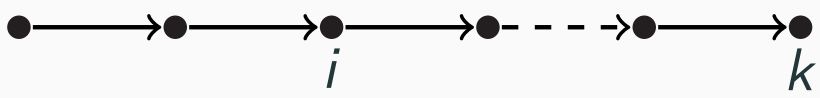

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Overall encoding

Definition (Overall encoding)

Let ϕ be an LTL formula, $\mathcal{M}$ be a Kripke structure and $k \geq 0$:

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Theorem (Soundness)

$[\![\mathcal{M}, \phi]\!]_k$ is satisfiable iff $\mathcal{M} \models_k E\phi$.

Completeness

Algorithm:

- start with $k = 0$
- call a SAT-solver on $[\![\mathcal{M}, \phi]\!]_k$
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What happens if $\mathcal{M} \not\models \phi$?

- the procedure does not terminate
- in order to be complete, BMC needs to compute the recurrence diameter: very costly
- BMC is mainly used as a bug finder, rather than as a prover.

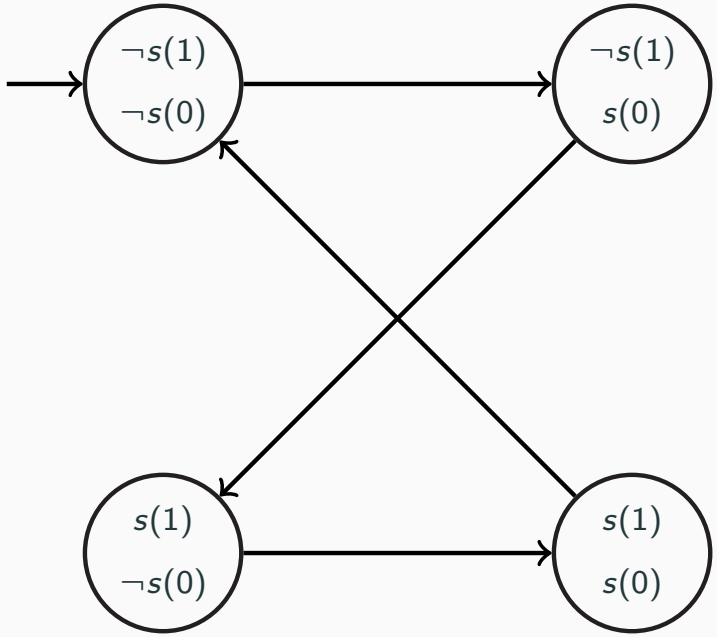
Conclusions

Questions?

Appendix

Example

modulo-4-counter.smv



- $\phi_1 := \mathsf{G}\,\mathsf{F}(s(0) \wedge s(1))$ ✓
- $\phi_2 := \mathsf{F}\,\mathsf{G}(\neg s(0) \wedge \neg s(1))$ ✗

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- satisfiability checking

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BLACK

- we developed this tool based on the idea of *bounded satisfiability checking*
- BLACK = Bounded Ltl sAtisfiability ChecKer [1]

[1] https://github.com/black-sat/black