Università degli studi di Udine
Corso per il dottorato di ricerca: Temporal Logics: Satisfiability Checking, Model Checking, and Synthesis
January 2017
Lecture 01, Part 02: Temporal Logics

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Description

This short (about 60 minutes) part of the lectures introduces temporal logic as a means of formal verification of systems.

Lecture 01, Part 02

- Formal Verification
- Representing systems
- Representing Properties
- LTL
- CTL
- CTL*
- Reasoning tasks

Formal Verification:

Verification: making sure that the system meets its specification (compare to Validation, making sure the specification is what the customer wants).

Approaches to verification include testing, rigorous mathematical proof and formal verification of a model of the system against a formal specification.

Tasks:

The main task is model checking. This is to verify that a model of the system satisfies each formally expressed property capturing each part of the specification.

This is seen as an algorithm which takes as input a formal description of a system and a formal description of a property and outputs “yes” or “no”.

Other tasks we will mention include deciding satisfiability of a property, deciding validity of a property, deciding whether one property is a logical consequence of another, synthesis of a system to satisfy a property, and imperative programming of a system.

Associated Theoretical Questions:

We may mention the computability of a problem and its computational complexity and the computational complexity of an algorithmic solution to a problem.

Approaches:

Fully automatic model checking. Can be by exhaustive search of a model with finite states or using symmetry, symbolic evaluation, abstraction can be extended to some models where the system has an infinite number of states.

The search, especially with large numbers of states, or infinite states, can be "on the fly" where the system states are only explicitly constructed as needed, or only to a certain depth.

Interactive deductive proof is an alternative style of approach.

Modelling the System:

Typically the system is represented as a Finite State Machine.

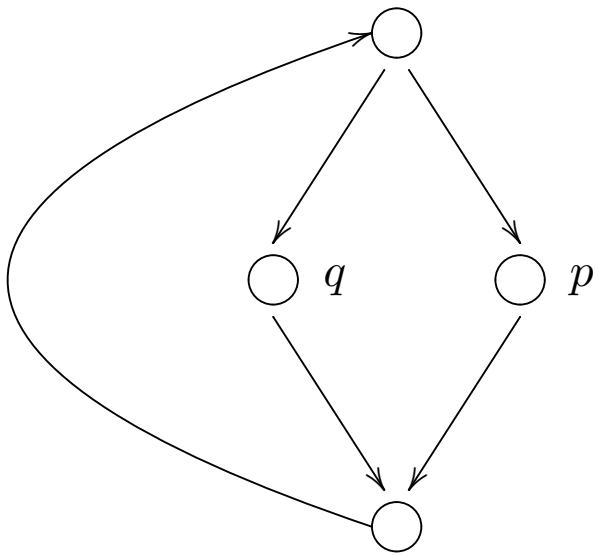
Software tools (e.g. Uppaal, NuSMV, SPIN, Spot, ...) for model checking often have their own formal system description languages.

Extensions may include timed automata, fair transition systems, ...

Model of a System:

Thus we suppose that we can model the system as a finite state machine: a set of states and a set of allowed transitions from some states to other states.

In order to allow abstraction of observable basic, or atomic properties, we also suppose that there are a set of atomic properties, and that some properties are true at some states.



Communicating Components:

Note that this model easily extends to the case of separate but communicating components.

If each component can be modelled as a FSM then the whole system can be taken to be the (Cartesian) product of the separate FSMs: also an FSM.

Each state of the combined system is a tuple of states from each component.

Properties:

Examples.
If a process stays in a waiting state then it will eventually be given a resource.
The program will eventually terminate.
The program will never return a null value.
The two processes will never both be in their critical states at the same time.
The program will never be in a logged-in state unless a correct PIN has been entered beforehand.
The system will always dispense a product after the correct money has been inserted.

Safety and Liveness:

Ideas from Lamport.

A safety property is one saying something of the form "something bad never happens". Eg, the program never returns null values. Eg, the program never allows two processes to be in the mutual exclusion states.

A liveness property is one saying something of the form "something good eventually happens". Eg, the program eventually terminates successfully. Eg, if a component is waiting for a resource then it will eventually be granted.

Deadlock:

Deadlock is a common property to want to check for (want to check that it is not possible).
It is simply a state which has no successor states.
It may seem easy to determine if such a state exists.
However, not so easy to tell if such a state is reachable from the initial state. Also not so clear when system FSM is generated as a product of component FSMs involving communication.
We assume in these lectures that all states do have successors. If you want to include deadlocks, then just introduce a deadlock sink state with an appropriate propositional label.

Fairness:

Typical examples of types of fairness constraints...

Every process should be executed infinitely often.

Every process that is enabled infinitely often should be executed infinitely often.

Every process that is eventually always enabled should be executed infinitely often.

Reachability:

Is a certain state (or set of states) reachable from the initial state?

Use a proposition to identify the interesting state(s) and see if that proposition is possibly sometime true, or never true.

Structures:

We assume a countable set $\mathcal{L}$ of propositional atoms, or atomic propositions.

A transition structure is a triple (S, R, g) with S a finite set of states, $R \subseteq S \times S$ a binary relation and for each $s \in S$, $g(s) \subseteq \mathcal{L}$.

R is the *transition relation* and *labelling* g tells us which atoms are true at each time.

R is assumed to be total: every state has at least one successor $\forall x \in S. \exists y \in S$ s.t.$(x, y) \in R$

Fullpaths:

Given a structure (S, R, g).
An ω-sequence of states $\langle s_0, s_1, s_2, ... \rangle$ from S is a fullpath (through (S, R, g)) iff for each i, $(s_i, s_{i+1}) \in R$.

If $\sigma = \langle s_0, s_1, s_2, ... \rangle$ is a fullpath then we write $\sigma_i = s_i$, $\sigma_{\geq j} = \langle s_j, s_{j+1}, s_{j+2}, ... \rangle$ (also a fullpath).

LTL Syntax:

Define some strings of symbols as (well formed) formulas, or wffs of LTL.

If $p \in \mathcal{L}$ then p is a wff.

If α and β are wff then so are $\neg\alpha$, $\alpha \wedge \beta$, $X\alpha$, and $\alpha U \beta$.

Read: Not, and(conjunction), tomorrow (or next), and until.

LTL Semantics:

Write $M, \sigma \models \alpha$ iff the formula α is true of the fullpath σ in the structure $M = (S, R, g)$ defined recursively by:

$$
\begin{array}{lll}
M, \sigma \models p & \text{iff} & p \in g(\sigma_0), \text{ for } p \in \mathcal{L} \\
M, \sigma \models \neg\alpha & \text{iff} & M, \sigma \not\models \alpha \\
M, \sigma \models \alpha \wedge \beta & \text{iff} & M, \sigma \models \alpha \text{ and } M, \sigma \models \beta \\
M, \sigma \models X\alpha & \text{iff} & M, \sigma_{\geq 1} \models \alpha \\
M, \sigma \models \alpha U \beta & \text{iff} & \text{there is some } i \geq 0 \text{ such that } M, \sigma_{\geq i} \models \beta \\
 & & \text{and for each } j, \text{ if } 0 \leq j < i \text{ then } M, \sigma_{\geq j} \models \alpha
\end{array}
$$

Abbreviations:

Classical: $\top \equiv p \vee \neg p$, $\bot \equiv \neg\top$, $\alpha \vee \beta \equiv \neg(\neg\alpha \wedge \neg\beta)$,
$\alpha \to \beta \equiv \neg\alpha \vee \beta$, $\alpha \leftrightarrow \beta \equiv (\alpha \to \beta) \wedge (\beta \to \alpha)$.
Read: truth, falsity, or(disjunction), implication, iff(equivalence).

Temporal: $F\alpha \equiv (\top U \alpha)$, $G\alpha \equiv \neg F(\neg\alpha)$.
Read: eventually, always.

Example Properties:

$G\neg(p \wedge q)$
$G(p \rightarrow (Xq \vee XXq \vee XXXq))$
$G(p \rightarrow Fq)$
$G(Gp \rightarrow Fq)$
$GFp \wedge GFq$
$G(t \rightarrow Gt)$
$Fp \rightarrow ((\neg p)U(q \wedge \neg p))$
$FGp \vee GFq$

CTL* Syntax:

If $p \in \mathcal{L}$ then p is a wff.

If α and β are wff then so are $\neg\alpha$, $\alpha \wedge \beta$, $X\alpha$, $\alpha U \beta$ and $A\alpha$.

Read: Not, and(conjunction), tomorrow (or next), until and all paths.

CTL* Semantics:

Write $M, \sigma \models \alpha$ iff the formula α is true of the fullpath σ in the structure $M = (S, R, g)$ defined recursively by:

$M, \sigma \models p$	iff	$p \in g(\sigma_0)$, for $p \in \mathcal{L}$
$M, \sigma \models \neg\alpha$	iff	$M, \sigma \not\models \alpha$
$M, \sigma \models \alpha \wedge \beta$	iff	$M, \sigma \models \alpha$ and $M, \sigma \models \beta$
$M, \sigma \models X\alpha$	iff	$M, \sigma_{\geq 1} \models \alpha$
$M, \sigma \models \alpha U \beta$	iff	there is some $i \geq 0$ such that $M, \sigma_{\geq i} \models \beta$ and for each j, if $0 \leq j < i$ then $M, \sigma_{\geq j} \models \alpha$
$M, \sigma \models A\alpha$	iff	for all fullpaths τ with $\tau_0 = \sigma_0$, $M, \tau \models \alpha$

CTL* Abbreviations:

Classical and linear temporal abbreviations as for LTL.

Also,
$E\alpha \equiv \neg A \neg \alpha$ meaning that there is a path on which α holds.

CTL* examples:

$XEXp \rightarrow EXXp$
$AG(p \rightarrow Ap)$
$AXFp \rightarrow XAFp$
$E(pU(E(pUq))) \rightarrow E(pUq)$
$AG(p \rightarrow q) \rightarrow (EFp \rightarrow EFq)$
$AG(p \rightarrow EXFp) \rightarrow (p \rightarrow EGFp)$

CTL Syntax:

CTL is a restriction of CTL* (but it can be presented directly, not via CTL*).
It includes, as wffs, all atoms p, and if α and β are wffs then so are $\neg\alpha$, $\alpha \wedge \beta$, $AX\alpha$, $EX\alpha$, $AF\alpha$, $EF\alpha$, $A(\alpha U \beta)$, and $E(\alpha U \beta)$.
So $AG\alpha$ and $EG\alpha$ are also included.

It uses the same semantics as CTL*.
However, ...

CTL Semantics:

It can be shown that the truth of CTL formulas in a structure only depend on the initial state of the fullpath. They are sometimes called state formulas.
That is, if $\pi_0 = \sigma_0$ then $M, \pi \models \alpha$ iff $M, \sigma \models \alpha$.

Not so for LTL or CTL* formulas in general.

This makes some of our reasoning tasks easier.

Examples:

$AG(p \rightarrow AFq)$
$EFq \rightarrow EFEGq$
$AG(p \rightarrow EXp) \rightarrow AG(p \rightarrow EGp)$
$EG(p \rightarrow A(qUr))$

Expressiveness:

CTL* extends both LTL and CTL.

CTL can say things that LTL can not: eg $AG(p \rightarrow EFq)$

LTL can say things that CTL can not. eg $G(p \rightarrow FGp)$

Example of Satisfiability:

Is the following part of a specification ever possibly true?
$p \wedge G(p \rightarrow EXFp) \wedge AFG\neg p$

Satisfiability:

A formula α is satisfiable iff there is some structure (S, R, g) with some fullpath σ through it such that $(S, R, g), \sigma \models \alpha$.

Eg, $\top$, p, Fp, $p \wedge Xp \wedge F\neg p$, Gp are each satisfiable.

Eg, $\bot$, $p \wedge \neg p$, $Fp \wedge G\neg p$, $p \wedge G(p \rightarrow Xp) \wedge F\neg p$ are each not satisfiable.

The Sat. Problem for LTL

Can we invent an algorithm for deciding whether an input formula (of LTL) is satisfiable or not?

We want to invent an algorithm which solves the LTL-SAT problem. Input should be a formula from LTL. Output is "yes" or "no" depending on whether the formula is satisfiable or not.

In a later lecture, we have a detailed look at an algorithm to decide the satisfiability of LTL formulas.

Example of Validity:

Does every system satisfy this?
$AG(p \rightarrow EX(qUp)) \rightarrow (p \rightarrow EG(p \vee q))$

Example of Consequence:

If we have a system which ensures the following
$AG(p \rightarrow GXAFq)$
then does it also make the following true?
$GFp \rightarrow GFq$

Example of Synthesis:

Make a structure with a fullpath making the following true.
$AG(p \rightarrow EXFp) \wedge p \wedge EF(p \wedge XEG\neg p)$

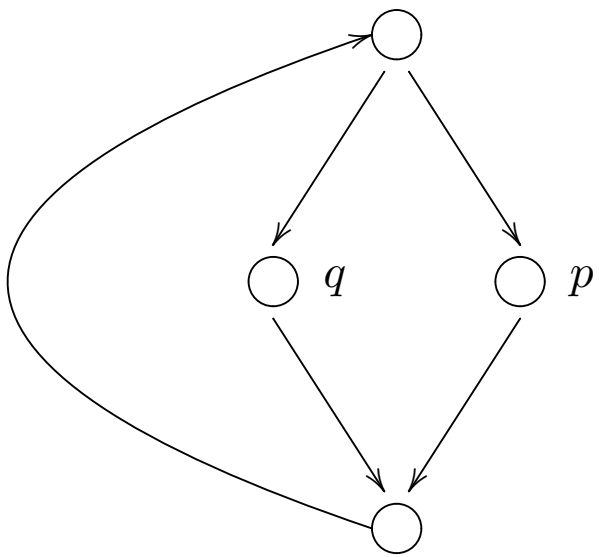
Example of Imperative Programming:

p
$G(p \rightarrow Xp)$
Fr
$G((p \wedge r) \rightarrow Xr)$

Example of Model Checking:

Does the following formula hold (at the top state) in the structure below?

$AG(GFEXp \rightarrow GFp)$



And that's all for this part of the lecture series.

Coming up we try to make algorithms to solve these problems.