Interval Temporal Logic Model Checking Based on Track Bisimilarity and Prefix Sampling

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Model checking

- Model checking: the desired properties of a system are checked against a model of the system
  - the model is a (finite) state-transition graph
  - system properties are specified by a temporal logic (e.g., LTL, CTL, CTL*, . . .)

- Distinctive features of model checking:
  - exhaustive verification of all the possible behaviours
  - fully automatic process
  - a counterexample is produced for a violated property

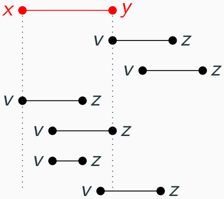
Point-based vs. interval-based model checking

- Model checking is usually point-based:
  - properties express requirements over points (snapshots) of a computation (states of the state-transition system)
  - they are specified by means of point-based temporal logics such as LTL and CTL and the like
- Interval-based model checking:
  - Interval-based properties express conditions on computation stretches: accomplishments, actions with duration, and temporal aggregations
  - they are specified by means of interval temporal logics such as HS and its fragments

The logic HS

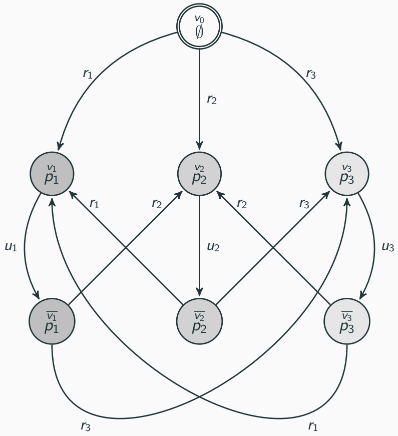
HS features a modality for any Allen ordering relation between pairs of intervals (except for equality)

Allen rel.	HS	Definition	Example
meets	$\langle A\rangle$	$[x,y]\mathcal{R}_A[v,z] \iff y = v$	
before	$\langle L\rangle$	$[x,y]\mathcal{R}_L[v,z] \iff y < v$	
started-by	$\langle B\rangle$	$[x,y]\mathcal{R}_B[v,z] \iff x = v \wedge z < y$	
finished-by	$\langle E\rangle$	$[x,y]\mathcal{R}_E[v,z] \iff y = z \wedge x < v$	
contains	$\langle D\rangle$	$[x,y]\mathcal{R}_D[v,z] \iff x < v \wedge z < y$	
overlaps	$\langle O\rangle$	$[x,y]\mathcal{R}_O[v,z] \iff x < v < y < z$	



All modalities can be expressed by means of $\langle A\rangle$, $\langle B\rangle$, $\langle E\rangle$ and their transposed modalities only

Kripke structures



An example of Kripke structure

- HS formulas are interpreted over (finite) state-transition systems whose states are labelled with sets of proposition letters (Kripke structures)
- An interval is a track (finite path/trace) in a Kripke structure

HS semantics and model checking

Truth of a formula ψ over a track ρ of a Kripke structure $\mathcal{K} = (\mathcal{AP}, W, \delta, \mu, w_0)$:

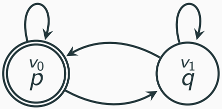
- $\mathcal{K}, \rho \models p$ iff $p \in \bigcap_{w \in \mathrm{states}(\rho)} \mu(w)$, for any letter $p \in \mathcal{AP}$ (homogeneity assumption);
- negation, disjunction, and conjunction are standard;
- $\mathcal{K}, \rho \models \langle \mathrm{A} \rangle \psi$ iff there is a track ρ' s.t. $\mathrm{lst}(\rho) = \mathrm{fst}(\rho')$ and $\mathcal{K}, \rho' \models \psi$;
- $\mathcal{K}, \rho \models \langle \mathrm{B} \rangle \psi$ iff there is a prefix ρ' of ρ s.t. $\mathcal{K}, \rho' \models \psi$;
- $\mathcal{K}, \rho \models \langle \mathrm{E} \rangle \psi$ iff there is a suffix ρ' of ρ s.t. $\mathcal{K}, \rho' \models \psi$;
- the semantic clauses for $\langle \overline{\mathrm{A}} \rangle$, $\langle \overline{\mathrm{B}} \rangle$, and $\langle \overline{\mathrm{E}} \rangle$ are similar

Model Checking

$\mathcal{K} \models \psi \iff$ for all *initial* tracks ρ of $\mathcal{K}$, it holds that $\mathcal{K}, \rho \models \psi$

Possibly infinitely many tracks!

BE-descriptors



BE_2-descriptor for the track $\rho = v_0 v_1 v_0^4 v_1$

(only the part for prefixes is shown)

- $(v_0, \{v_0, v_1\}, v_1)$
  - $(v_0, \{v_0, v_1\}, v_0)$
    - $(v_0, \{v_0, v_1\}, v_0)$
    - $(v_0, \{v_1\}, v_0)$
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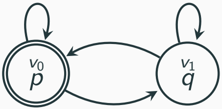

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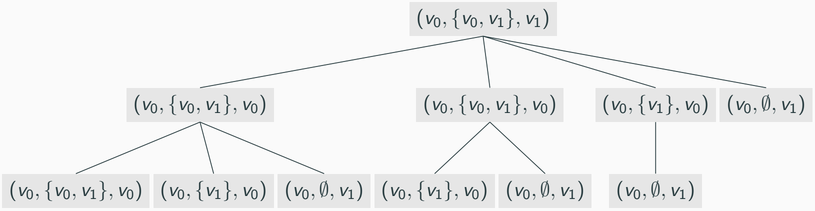

- FACT 1: For any Kripke structure $\mathcal{K}$ the number of different descriptors of bounded depth k is finite
- FACT 2: Two tracks ρ and ρ' of a Kripke structure $\mathcal{K}$ described by the same BE_k-descriptor are k-equivalent

Decidability of HS model checking

Theorem

The model checking problem for full HS on Kripke structures is decidable (non-elementary algorithm)

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A. Molinari, A. Montanari, A. Murano, G. Perelli, and A. Peron. Checking interval properties of computations.

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Theorem

The model checking problem for BE *on Kripke structures is EXPSPACE-hard*

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The logic $A\overline{A}B\overline{BE}$

In this paper, we focus our attention on the HS fragment $A\overline{A}B\overline{BE}$, which is obtained from full HS ($A\overline{A}BE\overline{BE}$) by removing modality $\langle \mathrm{E} \rangle$

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- the size of the tree representation of B_k-descriptors is larger than necessary (redundancy) and it prevents their efficient exploitation in model checking algorithms
- a track representative can be chosen to represent a (possibly infinite) set of tracks with the same B_k-descriptor
- a bound, which depends on both the number $|W|$ of states of the Kripke structure and the B-nesting depth k, can be given to the length of track representatives

Prefix-bisimilarity

Definition (Prefix-bisimilarity)

The tracks ρ and ρ' are *h*-prefix bisimilar if the following conditions inductively hold:

- for $h = 0$:
  $\mathsf{fst}(\rho) = \mathsf{fst}(\rho')$, $\mathsf{lst}(\rho) = \mathsf{lst}(\rho')$, and $\mathsf{states}(\rho) = \mathsf{states}(\rho')$.
- for $h > 0$:
  ρ and ρ' are 0-prefix bisimilar and for each proper prefix ν of ρ (resp., proper prefix ν' of ρ'), there exists a proper prefix ν' of ρ' (resp., proper prefix ν of ρ) such that ν and ν' are $(h-1)$-prefix bisimilar.

- *h*-prefix bisimilarity is an equivalence relation over $\mathsf{Trk}_{\mathcal{K}}$.
- *h*-prefix bisimilarity propagates downwards.

Proposition

Let $h \geq 0$, and ρ and ρ' be two h-prefix bisimilar tracks of a Kripke structure $\mathcal{K}$. For each $\mathsf{A\overline{A}B\overline{B}\overline{E}}$ *formula ψ, with B-nesting of ψ less than or equal to h, it holds that*

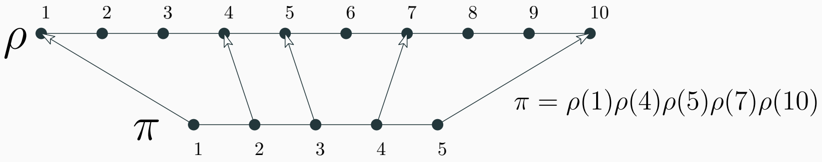
$$\mathcal{K}, \rho \models \psi \iff \mathcal{K}, \rho' \models \psi.$$

Induced track

Definition (Induced track)

Let ρ be a track of length n of a Kripke structure $\mathcal{K}$. A track induced by ρ is a track π of $\mathcal{K}$ such that there exists an increasing sequence of ρ-positions $i_1 < \ldots < i_k$, where $i_1 = 1$, $i_k = n$, and

$$\pi = \rho(i_1) \cdots \rho(i_k).$$



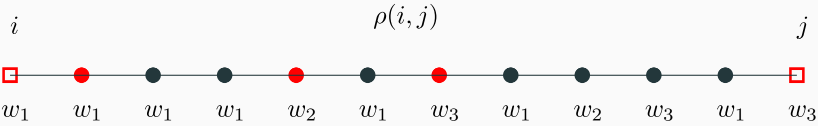
If π is induced by $\rho \Rightarrow \mathsf{fst}(\pi) = \mathsf{fst}(\rho)$, $\mathsf{lst}(\pi) = \mathsf{lst}(\rho)$, and $|\pi| \leq |\rho|$.

Prefix-skeleton sampling

Definition (Prefix-skeleton sampling)

Let ρ be a track of a Kripke structure $\mathcal{K} = (\mathcal{AP}, W, \delta, \mu, w_0)$.
Given two ρ-positions i and j, with $i \leq j$, the prefix-skeleton sampling of $\rho(i, j)$ is the minimal set P of ρ-positions in the interval $[i, j]$ satisfying:

- $i, j \in P$;
- for each state $w \in W$ occurring along $\rho(i+1, j-1)$, the minimal position $k \in [i+1, j-1]$ such that $\rho(k) = w$ is in P.



$$P = \{i, i+1, i+4, i+6, j\}$$

h-prefix sampling

Definition (h-prefix sampling)

For each $h \geq 1$, the h-prefix sampling of ρ is the minimal set P_h of ρ-positions inductively satisfying the following conditions:

- for $h = 1$: P_1 is the prefix-skeleton sampling of ρ;
- for $h > 1$:
  - $P_h \supseteq P_{h-1}$ and
  - for all pairs of consecutive positions i, j in P_{h-1}, the prefix-skeleton sampling of $\rho(i, j)$ is in P_h.

Property

The h-prefix sampling P_h of (any) ρ is such that $|P_h| \leq (|W| + 2)^h$.

Now what?

From a track ρ, we can derive another track ρ', induced by ρ and h-prefix bisimilar to ρ, such that $|\rho'| \leq (|W| + 2)^{h+2}$ in this way:

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1. we first compute the $(h+1)$-prefix sampling P_{h+1} of ρ;
2. then for all the pairs of consecutive ρ-positions $i, j \in P_{h+1}$, we consider a track induced by $\rho(i, j)$,
   with no repeated occurrences of any state,
   except at most the first and last ones (hence no longer than $(|W| + 2)$);

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ρ and ρ' can be proved to be h-prefix bisimilar,
$\Rightarrow \rho'$ is indistinguishable from ρ w.r.t. the fulfilment of any $\mathsf{A\overline{A}B\overline{B}\overline{E}}$ formula ψ, with B-nesting of ψ (abbreviated $\mathsf{d_B}(\psi)$) less than or equal to h;

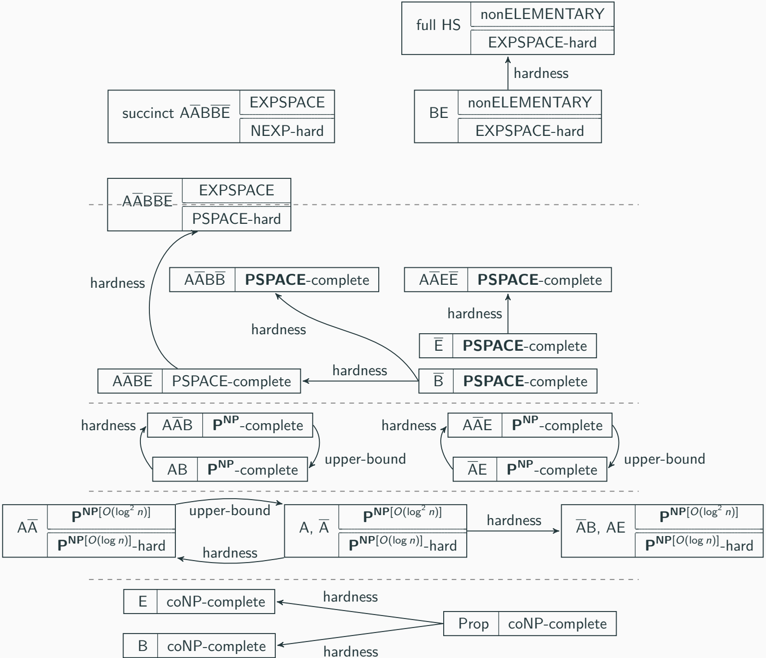
by the previous bound on $|P_h|$, we have $|\rho'| \leq (|W| + 2)^{h+2}$.

An EXPSPACE model checking algorithm for $A\overline{A}B\overline{B}\overline{E}$

Algorithm 1 $\texttt{ModCheck}(\mathcal{K}, \psi)$

```
h ← d_B(ψ)
u ← New (Unravelling(K, w_0, h))                    ◁ w_0 initial state of K
while u.hasMoreTracks() do
    ρ̃ ← u.getNextTrack()
    if Check(K, h, ψ, ρ̃) = 0 then
        return 0: "K, ρ̃ ⊭ ψ"                          ◁ Counterexample found ✗
return 1: "K ⊨ ψ"                                    ◁ Model checking OK ✓
```

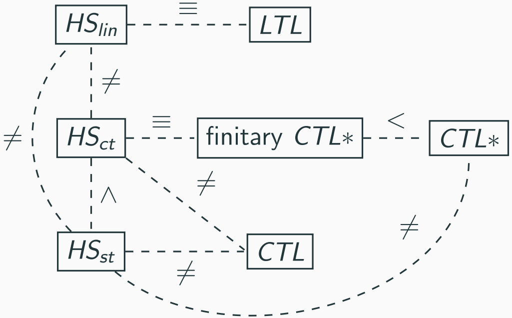
Complexity picture



Current and future work

- Comparison of HS model checking with LTL, CTL, and CTL* one (to this end, we introduced two semantic variants of the problem respectively based on the linear-past semantics and the linear semantics) - DONE
- Application: Planning as Model Checking in Interval Temporal Logic - IN PROGRESS
- Determining the precise complexity of full HS (and of a little subset of its fragments)
- Relaxing the homogeneity assumption

Expressiveness comparison



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