Model Checking Complex Systems against ITL Specifications with Regular Expressions

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joint work with Laura Bozzelli, Angelo Montanari, and Adriano Peron

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HS (state-based) semantics and model checking

Truth of a formula ψ over a track ρ of a Kripke structure $\mathcal{K} = (\mathcal{AP}, W, \delta, \mu, w_0)$:

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Possibly infinitely many tracks!

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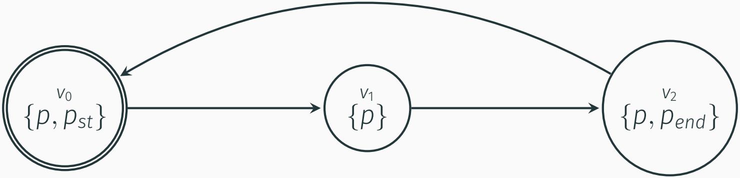
Example—Printer

Adapted from [3].



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- Under homogeneity,
  $v_0 v_1 v_2$ labeled by $p \Rightarrow v_0 v_1$ and $v_1 v_2$ labeled by p

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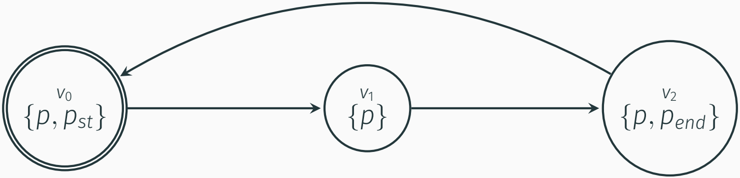

Imagine we want to label the process of printing a single sheet of paper with p.

- Under endpoint-based labeling, assuming $p \in \mu(v_0, v_2)$, then $(v_0 v_1 v_2)^n$ are all labeled by p

(Usual) regular expressions

$$r ::= \varepsilon \mid a \mid r \cup r \mid r \cdot r \mid r^*$$

for $a \in \mathcal{A}$.

Examples:

- $r_1 = a \cdot (b \cup c)^* \cdot b$
  - $abb, acb, abccbb, \ldots \in \mathcal{L}(r_1)$

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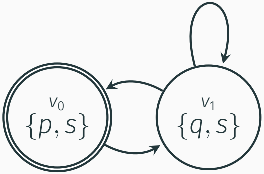
- $r_3 = \varepsilon \cdot (a \cup c)^*$
    - $\varepsilon, a, ca, aac, \ldots \in \mathcal{L}(r_3)$

Our regular expressions

$$r ::= \varepsilon \mid \phi \mid r \cup r \mid r \cdot r \mid r^*$$

where ϕ is a Boolean (propositional) formula over $\mathcal{AP}$.



- $\rho = v_0 v_1 v_0 v_1 v_1$
- $\mu(\rho) = \{p, s\}\{q, s\}\{p, s\}\{q, s\}\{q, s\}$
- $\rho' = v_0 v_1 v_1 v_1 v_0$
- $\mu(\rho') = \{p, s\}\{q, s\}\{q, s\}\{q, s\}\{p, s\}$

Examples:

- $r_1 = (\mathsf{p} \wedge \mathsf{s}) \cdot \mathsf{s}^* \cdot (\mathsf{p} \wedge \mathsf{s})$
  - $\mu(\rho) \notin \mathcal{L}(r_1)$, but $\mu(\rho') \in \mathcal{L}(r_1)$
- $r_2 = (\neg \mathsf{p})^*$
  - $\mu(\rho) \notin \mathcal{L}(r_2)$, and $\mu(\rho) \notin \mathcal{L}(r_2)$

Nondeterministic finite automata (NFA)

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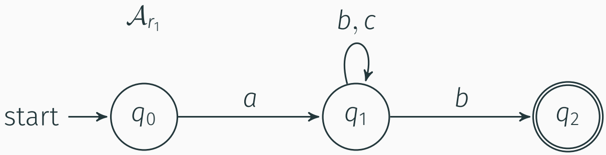

- $abb, acb, abccbb, \ldots \in \mathcal{L}(r_1) = \mathcal{L}(\mathcal{A}_{r_1})$

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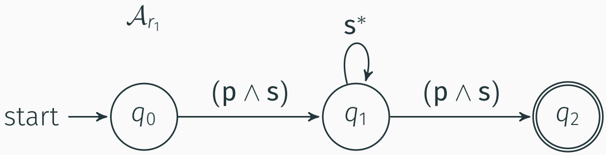

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- To force homogeneity, all regular expressions in the formula:

$$p \cdot (p)^*$$

- for endpoint-based labeling, regular expressions in the formula:

$$\bigcup_{(i,j)\in I} (q_i \cdot \top^* \cdot q_j)$$

for some $I \subseteq \{1, \ldots, |W|\}^2$, where $q_i \in \mathcal{AP}$ labels only $w_i \in W$.

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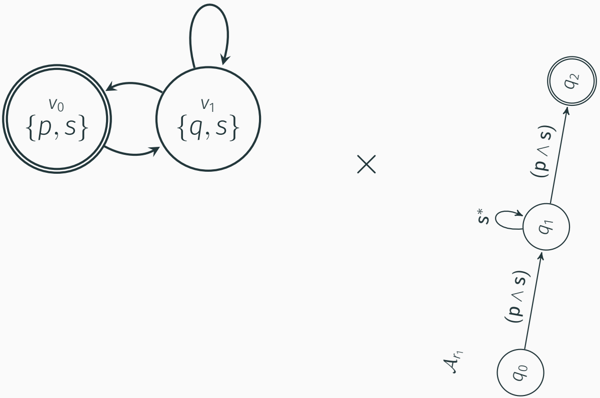
Idea: for a regular expression r



Decidability of MC for HS + regular expressions

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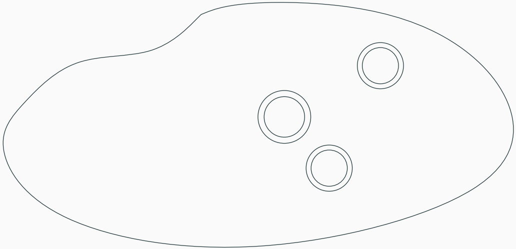

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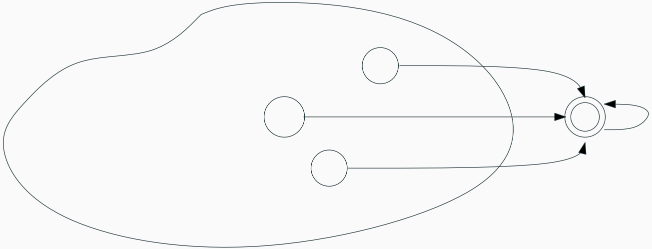

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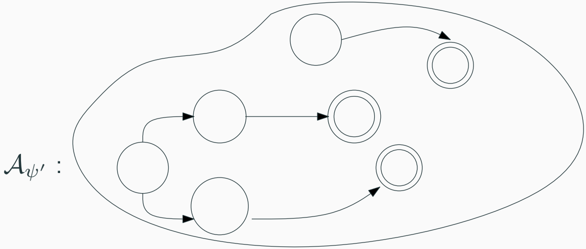

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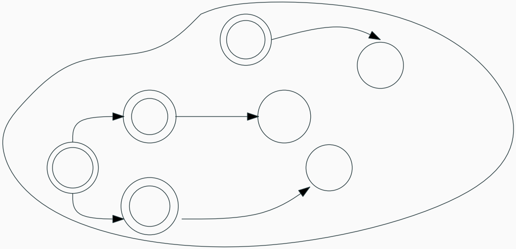


The $A\overline{A}B\overline{B}$ fragment + regular expressions

We want to show that formulas of $A\overline{A}B\overline{B}$ + regular expressions can be checked by using polynomial working space.

To start with, we prove the following theorem, which is a building block of the **PSPACE**-model checking algorithm for $A\overline{A}B\overline{B}$.

Theorem

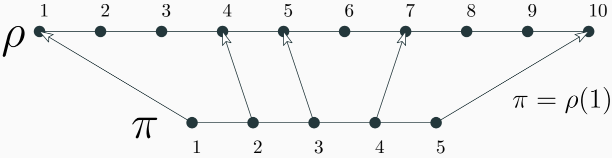
Let ρ be a track of $\mathcal{K}$ and φ be an $A\overline{A}B\overline{B}$ formula with RE's $r_1, \ldots, r_u$ such that

$$\mathcal{K}, \rho \models \varphi.$$

Then, there exists a track π of $\mathcal{K}$ such that

$$\mathcal{K}, \pi \models \varphi \quad \textit{and} \quad |\pi| \leq |W| \cdot (|\varphi| + 1) \cdot 2^{2 \sum_{\ell=1}^{u} |r_\ell|}.$$

Small model for $A\overline{A}B\overline{B}$



$\pi = \rho(1)\rho(4)\rho(5)\rho(7)\rho(10)$

We want to guarantee that: for all π-positions j, with corresponding ρ-positions i_j, and for all $s = 1, \ldots, u$, $\mathcal{A}^s(\mu(\pi^j)) = \mathcal{A}^s(\mu(\rho^{i_j}))$

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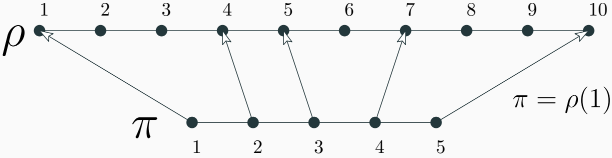

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Theorem (Exponential small-model for A$\overline{\text{A}}$B$\overline{\text{B}}$)

Let ρ be a track of $\mathcal{K}$ and φ be an A$\overline{\text{A}}$B$\overline{\text{B}}$ formula with RE's $r_1, \ldots, r_u$ such that $\mathcal{K}, \rho \models \varphi$.
Then, there exists a track π of $\mathcal{K}$, induced by ρ, such that

$$\mathcal{K}, \pi \models \varphi \quad \text{and} \quad |\pi| \leq |W| \cdot (|\varphi| + 1) \cdot 2^{2\sum_{\ell=1}^{u} |r_\ell|}.$$

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The small model is “strict”:

- Let pr_i be the i-th smallest prime.
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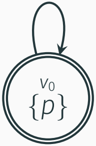
- Let $\mathcal{K} =$ 

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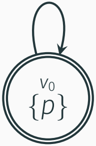
- Let $\mathcal{K} =$ 

- Let us fix some $n \in \mathbb{N}$. The shortest track satisfying

$$\psi = \bigwedge_{i=1}^{n} (p^{pr_i})^*$$

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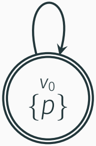
- Let $\mathcal{K} =$ 

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- The length of ψ is $O(n \cdot pr_n) = O(n^2 \log n)$,
  but the length of ρ is $pr_1 \cdots pr_n \geq 2^n$.

A PSPACE MC algorithm for $A\overline{A}B\overline{B}$—Trials

- The algorithm can consider only tracks having length bounded by the exponential small model
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A PSPACE MC algorithm for $\mathsf{A\overline{A}B\overline{B}}$—Trials

- The algorithm can consider only tracks having length bounded by the exponential small model
- However, they are still too long! $\Rightarrow$ triples $(G, D(\psi), w)$ summarizing tracks, where
  - $G \subseteq \mathrm{Subf}_{\langle B \rangle}(\psi)$ contains the subformulas that hold on some prefix
  - $D(\psi)$ is the configuration of the DFAs after reading the track,
  - and w is the last state of the track

Lemma

For all formulas ψ of $\mathsf{B\overline{B}}$, and for all tracks ρ, ρ' of $\mathcal{K}$, if ρ and ρ' are summarized by the same triple $(G, D(\psi), w)$, then

$$\mathcal{K}, \rho \models \psi \iff \mathcal{K}, \rho' \models \psi.$$

A PSPACE MC algorithm for $A\overline{A}B\overline{B}$—Trials

- The algorithm cannot explicitly store the DFAs for the regular expressions occurring in $\psi \Rightarrow$ just store the current states of the computations of the DFAs and calculates on-the-fly the successor states

A PSPACE MC algorithm for $\mathsf{A}\overline{\mathsf{A}}\mathsf{B}\overline{\mathsf{B}}$

Algorithm 1 Check($\mathcal{K}, \psi, w, G, D(\psi)$)

```
if ψ = r then                                          ◁ r is a regular expression
    if the current state of the DFA for r in advance(D(ψ), μ(w)) is final then
        return ⊤
    else
        return ⊥
else if ψ = ¬ψ′ or ψ = ψ1 ∧ ψ2 then
    Recursively
else if ψ = ⟨B⟩ ψ′ then
    return ψ′ ∈ G
else if ψ = ⟨B̄⟩ ψ′ then
    for all b ∈ {1, ..., |W| · (2|ψ′| + 1) · 2^(2 Σ_{ℓ=1}^{u} |r_ℓ|) − 1}, all (G′, D(ψ)′, w′) do
        if Reach(K, ψ′, (G, D(ψ), w), (G′, D(ψ)′, w′), b) and Check(K, ψ′, w′, G′, D(ψ)′) then
            return ⊤
    return ⊥
```

A PSPACE MC algorithm for $\mathsf{A\overline{A}B\overline{B}}$

Algorithm 2 Check($\mathcal{K}, \psi, w, G, D(\psi)$)

if $\psi = r$ **then** ◁ *r is a regular expression*
  if the current state of the DFA for r in advance($D(\psi), \mu(w)$) is final **then**
    return $\top$
  else
    return $\bot$
else if $\psi = \neg\psi'$ or $\psi = \psi_1 \wedge \psi_2$ **then**
  Recursively
else if $\psi = \langle \mathsf{B} \rangle\, \psi'$ **then**
  return $\psi' \in G$
else if $\psi = \langle \overline{\mathsf{B}} \rangle\, \psi'$ **then**
  for all $b \in \{1, \ldots, |W| \cdot (2|\psi'| + 1) \cdot 2^{2^{\sum_{\ell=1}^{u} |r_\ell|}} - 1\}$, all $(G', D(\psi)', w')$ **do**
    if Reach($\mathcal{K}, \psi', (G, D(\psi), w), (G', D(\psi)', w'), b$) and Check($\mathcal{K}, \psi', w', G', D(\psi)'$) **then**
      return $\top$
  return $\bot$

$\mathsf{A\overline{A}B\overline{B}}$ is PSPACE-complete

- We replace the sub-formulas $\langle A \rangle\, \psi$ and $\langle \overline{A} \rangle\, \psi$ with the regular expressions $r_{\langle A \rangle\, \psi}$ and $r_{\langle \overline{A} \rangle\, \psi}$:

$$r_{\langle A \rangle\, \psi} := \top^* \cdot \Big(\bigcup_{w \in W_{\langle A \rangle\, \psi}} q_w \Big); \qquad r_{\langle \overline{A} \rangle\, \psi} := \Big(\bigcup_{w \in W_{\langle \overline{A} \rangle\, \psi}} q_w \Big) \cdot \top^*.$$

- To determine $W_{\langle A \rangle\, \psi}$ and $W_{\langle \overline{A} \rangle\, \psi}$, we iterate the previous algorithm

Theorem

The MC problem for formulas of $\mathsf{A\overline{A}B\overline{B}}$ *over finite Kripke structures is* PSPACE*-complete.*

$\mathsf{A\bar{A}B\bar{B}}$ is PSPACE-complete

- We replace the sub-formulas $\langle \mathsf{A} \rangle\, \psi$ and $\langle \bar{\mathsf{A}} \rangle\, \psi$ with the regular expressions $r_{\langle \mathsf{A} \rangle\, \psi}$ and $r_{\langle \bar{\mathsf{A}} \rangle\, \psi}$:

$$r_{\langle \mathsf{A} \rangle\, \psi} := \top^* \cdot \Big(\bigcup_{w \in W_{\langle \mathsf{A} \rangle\, \psi}} q_w \Big); \qquad r_{\langle \bar{\mathsf{A}} \rangle\, \psi} := \Big(\bigcup_{w \in W_{\langle \bar{\mathsf{A}} \rangle\, \psi}} q_w \Big) \cdot \top^*.$$

- To determine $W_{\langle \mathsf{A} \rangle\, \psi}$ and $W_{\langle \bar{\mathsf{A}} \rangle\, \psi}$, we iterate the previous algorithm

Theorem

The MC problem for formulas of $\mathsf{A\bar{A}B\bar{B}}$ *over finite Kripke structures is* PSPACE*-complete.*

Proof.

The purely propositional fragment of HS is hard for **PSPACE**: we prove this fact by a reduction from the **PSPACE**-complete *universality problem for regular expressions.* □

Complexity results

	Homogeneity	Regular expressions
Full HS, BE	non-elementary **EXPSPACE**-hard	non-elementary **EXPSPACE**-hard
$A\bar{A}B\bar{B}\bar{E}$, $A\bar{A}E\bar{B}\bar{E}$	**EXPSPACE** **PSPACE**-hard	non-elementary **PSPACE**-hard
$A\bar{A}\bar{B}\bar{E}$	**PSPACE**-complete	non-elementary **PSPACE**-hard
$A\bar{A}B\bar{B}$, $B\bar{B}$, $\bar{B}$, $A\bar{A}E\bar{E}$, $E\bar{E}$, $\bar{E}$	**PSPACE**-complete	**PSPACE**-complete
$A\bar{A}B$, $A\bar{A}E$, AB, $\bar{A}E$	$\mathbf{P}^{\mathbf{NP}}$-complete	**PSPACE**-complete
$A\bar{A}$, $\bar{A}B$, AE, A, $\bar{A}$	$\mathbf{P}^{\mathbf{NP}[O(\log^2 n)]}$ $\mathbf{P}^{\mathbf{NP}[O(\log n)]}$-hard	**PSPACE**-complete
Prop, B, E	**co-NP**-complete	**PSPACE**-complete

Complexity results

	Homogeneity	Regular expressions
Full HS, BE	non-elementary **EXPSPACE**-hard	non-elementary **EXPSPACE**-hard
$A\overline{A}B\overline{B}\overline{E}$, $A\overline{A}E\overline{B}\overline{E}$	**EXPSPACE** **PSPACE**-hard	non-elementary **PSPACE**-hard
$A\overline{A}\overline{B}\overline{E}$	**PSPACE**-complete	non-elementary **PSPACE**-hard
$A\overline{A}B\overline{B}$, $B\overline{B}$, $\overline{B}$, $A\overline{A}E\overline{E}$, $E\overline{E}$, $\overline{E}$	**PSPACE**-complete	**PSPACE**-complete
$A\overline{A}B$, $A\overline{A}E$, AB, $\overline{A}E$	$\mathbf{P}^{\mathbf{NP}}$-complete	**PSPACE**-complete
$A\overline{A}$, $\overline{A}B$, AE, A, $\overline{A}$	$\mathbf{P}^{\mathbf{NP}[O(\log^2 n)]}$ $\mathbf{P}^{\mathbf{NP}[O(\log n)]}$-hard	**PSPACE**-complete
Prop, B, E	**co-NP**-complete	**PSPACE**-complete

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