Interval vs. Point Temporal Logic Model Checking: an Expressiveness Comparison

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HS (state-based) semantics and model checking

Truth of a formula ψ over a track ρ of a Kripke structure $\mathcal{K} = (\mathcal{AP}, W, \delta, \mu, w_0)$:

- $\mathcal{K}, \rho \models p$ iff $p \in \bigcap_{w \in \text{states}(\rho)} \mu(w)$, for any letter $p \in \mathcal{AP}$ (homogeneity assumption); (?)
- negation, disjunction, and conjunction are standard;
- $\mathcal{K}, \rho \models \langle A \rangle \psi$ iff there is a track ρ' s.t. $\text{lst}(\rho) = \text{fst}(\rho')$ and $\mathcal{K}, \rho' \models \psi$;
- $\mathcal{K}, \rho \models \langle B \rangle \psi$ iff there is a prefix ρ' of ρ s.t. $\mathcal{K}, \rho' \models \psi$;
- $\mathcal{K}, \rho \models \langle E \rangle \psi$ iff there is a suffix ρ' of ρ s.t. $\mathcal{K}, \rho' \models \psi$;
- the semantic clauses for $\langle \overline{A} \rangle$, $\langle \overline{B} \rangle$, and $\langle \overline{E} \rangle$ are similar

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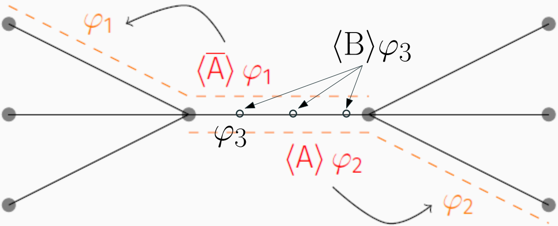
Model Checking

$\mathcal{K} \models \psi \iff$ for all *initial* tracks ρ of $\mathcal{K}$, it holds that $\mathcal{K}, \rho \models \psi$

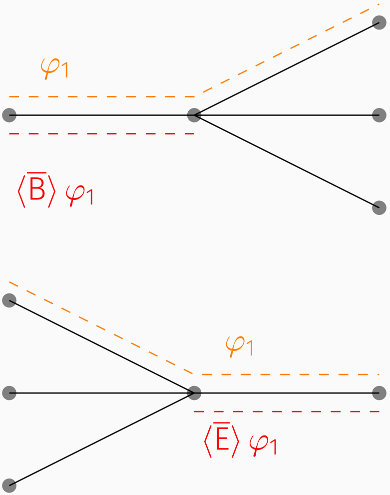
Possibly infinitely many tracks!

Branching (state-based) semantic var. of HS (HS_{st})



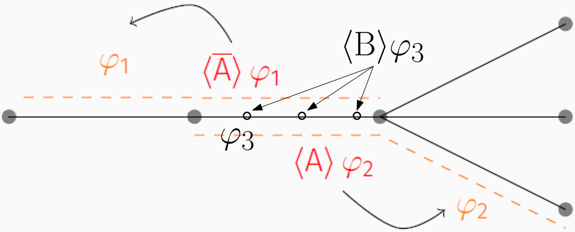
- Branching semantics of past/future operators [5]

Branching (state-based) semantic var. of HS (HS_{st})



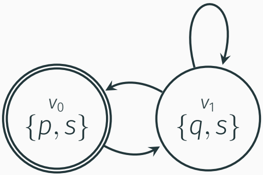
- Branching semantics of past/future operators [5]

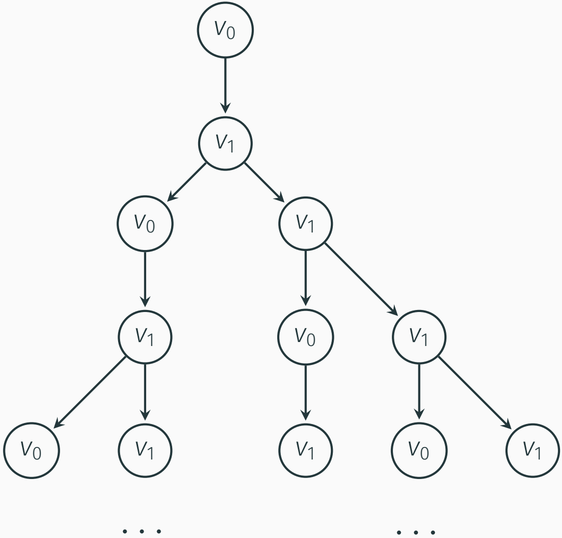
Linear-past (computation-tree-based) semantic var. of HS (HS_{lp})



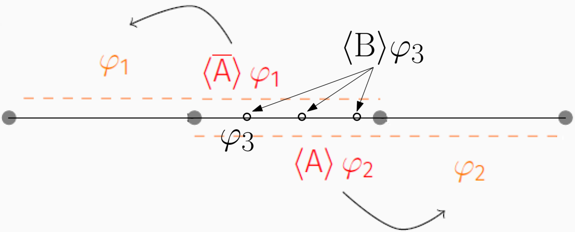
- Similar to CTL + linear past
- The past is linear, finite and cumulative [3, 4]

A computation tree



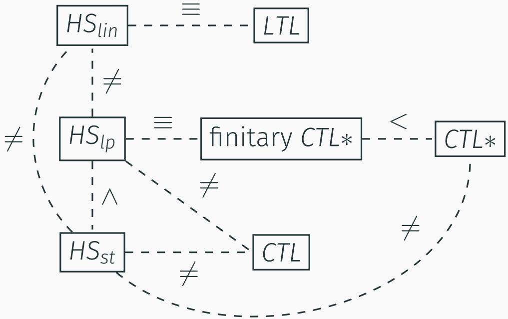


Linear (trace-based) semantic var. of HS (HS_{lin})

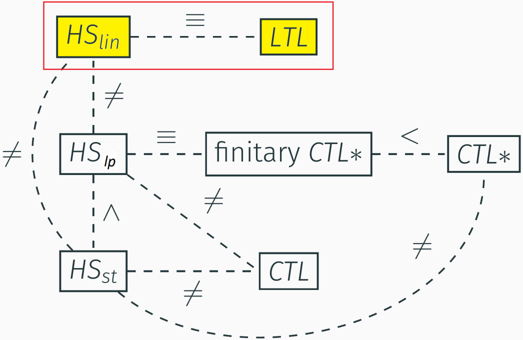


- Similar to LTL + PAST
- No branching in past/future
- $\mathcal{K} \models_{\text{lin}} \psi$, iff for each initial infinite path π and for each initial interval $[0, i]$, it holds that $\mathcal{IS}_{\mathcal{K},\pi}, [0, i] \models \psi$

Expressiveness comparison



Equivalence between LTL and HS_{lin}



Equivalence between LTL and HS_{lin}

FO formulas φ:

$$\varphi := p \in x \mid x = y \mid x < y \mid \neg \varphi \mid \varphi \wedge \varphi \mid \exists x.\varphi \,.$$

Given a HS_{lin} formula $\psi \rightsquigarrow \psi_{FO} = \exists x((\forall z.z \geq x) \wedge \forall y.h(\psi, x, y))$

$$\begin{aligned}
h(p, x, y) &= \forall z.((z \geq x \wedge z \leq y) \rightarrow p \in z), \\
h(\langle E \rangle \psi, x, y) &= \exists z.(z > x \wedge z \leq y \wedge h(\psi, z, y)), \\
h(\langle B \rangle \psi, x, y) &= \exists z.(z \geq x \wedge z < y \wedge h(\psi, x, z)), \\
h(\langle \overline{E} \rangle \psi, x, y) &= \exists z.(z < x \wedge h(\psi, z, y)), \\
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\end{aligned}$$

Theorem (Kamp's Theorem [2])

Given a FO sentence φ over $\mathcal{AP}$, one can construct an LTL formula ψ such that for all Kripke structures $\mathcal{K}$ over $\mathcal{AP}$ and infinite paths π,

$$\pi \models \varphi \iff \pi, 0 \models \psi.$$

Equivalence between LTL and $\mathsf{HS}_{\mathsf{lin}}$

Theorem (Kamp's Theorem [2])

Given a FO sentence φ over $\mathcal{AP}$, one can construct an LTL formula ψ such that for all Kripke structures $\mathcal{K}$ over $\mathcal{AP}$ and infinite paths π,

$$\pi \models \varphi \iff \pi, 0 \models \psi.$$

Theorem

LTL $\geq$ $\mathsf{HS}_{\mathsf{lin}}$.

Equivalence between LTL and HS_{lin}

Conversely:

Theorem

Given an LTL formula φ, we can construct an AB formula ψ such that φ in LTL is equivalent to ψ in AB_{lin}.

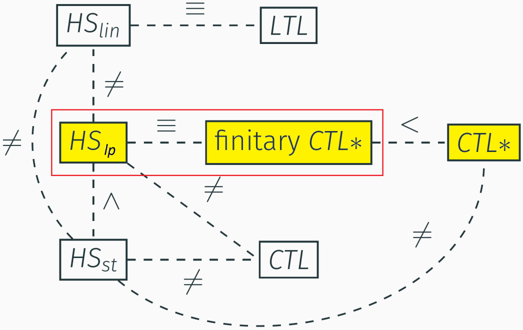
$$f(p) = p, \qquad f(X\psi) = \langle A\rangle(length_2 \wedge \langle A\rangle(length_1 \wedge f(\psi))),$$
$$f(\psi_1 U \psi_2) = \langle A\rangle\Big(\langle A\rangle(length_1 \wedge f(\psi_2)) \wedge [B](\langle A\rangle(length_1 \wedge f(\psi_1))\Big).$$

It holds that $\mathcal{K} \models \psi$ iff $\mathcal{K} \models_{\text{lin}} length_1 \rightarrow f(\psi)$

Corollary

HS_{lin} and LTL have the same expressiveness.

A characterization of HS_{lp}



A characterization of HS_{lp}

- finitary $CTL^* \leq HS_{lp}$:
  - BE and LTL define the same finitary languages, as

  (a) BE $\rightsquigarrow$ LTL: Kamp's th. over finite words

  (b) LTL $\rightsquigarrow$ BE: LTL-closure [6]
  - A to simulate $\forall / \exists$
  - finitary $CTL^* \leq HS_{st}$

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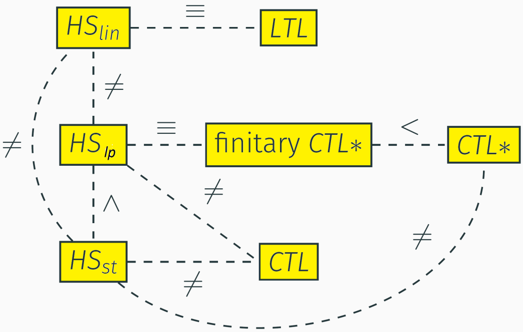
  - A to simulate $\forall/\exists$
  - finitary CTL* $\leq HS_{st}$
- $HS_{lp} \leq$ finitary CTL* (CTL*)
  - via hybrid logics: hybrid-CTL^*_{lp}

Expressiveness comparison of HS_{lin}, HS_{st}, HS_{lp}



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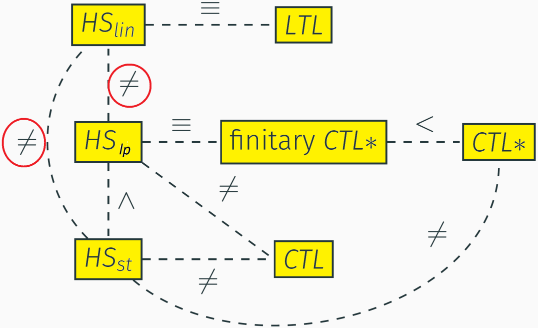

- The reachability property $\forall G \exists F p$ is not LTL-expressible, but it is expressible in HS_{lp} and HS_{st} (by $\langle \overline{B} \rangle \langle E \rangle p$)
- but the LTL formula Fp cannot be expressed in HS_{lp} or HS_{st}

Expressiveness comparison of HS_{lin}, HS_{st}, HS_{lp}

Proposition

The LTL formula $\mathsf{F}\, p$ (equivalent to the CTL formula $\forall \mathsf{F}\, p$) cannot be expressed in either HS_{lp} or HS_{st}.

Proof: We exhibit two families of Kripke structures $(\mathcal{K}_n)_{n \geq 1}$ and $(\mathcal{M}_n)_{n \geq 1}$ over $\{p\}$ such that

- for all $n \geq 1$ the LTL formula $\mathsf{F}\, p$ distinguishes $\mathcal{K}_n$ and $\mathcal{M}_n$,
- but for every HS_{st} formula ψ of size at most n, ψ does not distinguish $\mathcal{K}_n$ and $\mathcal{M}_n$.

Expressiveness comparison of $\mathsf{HS}_{\mathsf{lin}}$, $\mathsf{HS}_{\mathsf{st}}$, $\mathsf{HS}_{\mathsf{lp}}$

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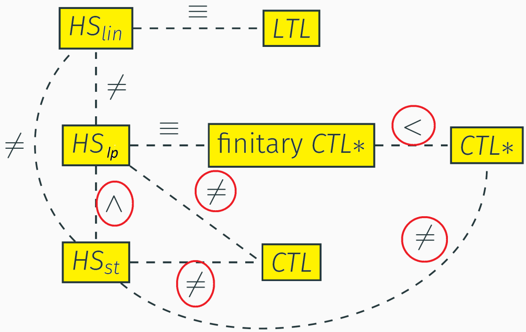
Expressiveness comparison of HS_{lin}, HS_{st}, HS_{lp}



- Already proved: $CTL^* \geq HS_{lp}$, $HS_{st} \geq HS_{lp}$(= finitary CTL^*)
- HS_{lp}, CTL, CTL^* are not sensitive to unravelling, HS_{st} is
- the CTL formula $\forall F p$ cannot be expressed in HS_{lp} or HS_{st}
- the (finitary) CTL^* formula $\exists(((p_1 U p_2) \vee (q_1 U q_2)) \, U \, r)$ cannot be expressed in CTL [1]

- HS_{lp}, CTL, CTL* are not sensitive to unravelling.

"*each state reachable from the initial one where p holds has a predecessor where p holds as well*" can be expressed in HS_{st} by:

$$\psi = \langle E\rangle(p \wedge length_1) \rightarrow \langle E\rangle(length_1 \wedge \langle \overline{A}\rangle(p \wedge \neg length_1)).$$

Corollary

$HS_{lp} \not\geq HS_{st}$, hence $HS_{st} > HS_{lp}$.

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