Università degli Studi di Udine

Master degree in Artificial Intelligence and Cybersecurity

Fair Transition Systems - Part 3

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An example of reactive program

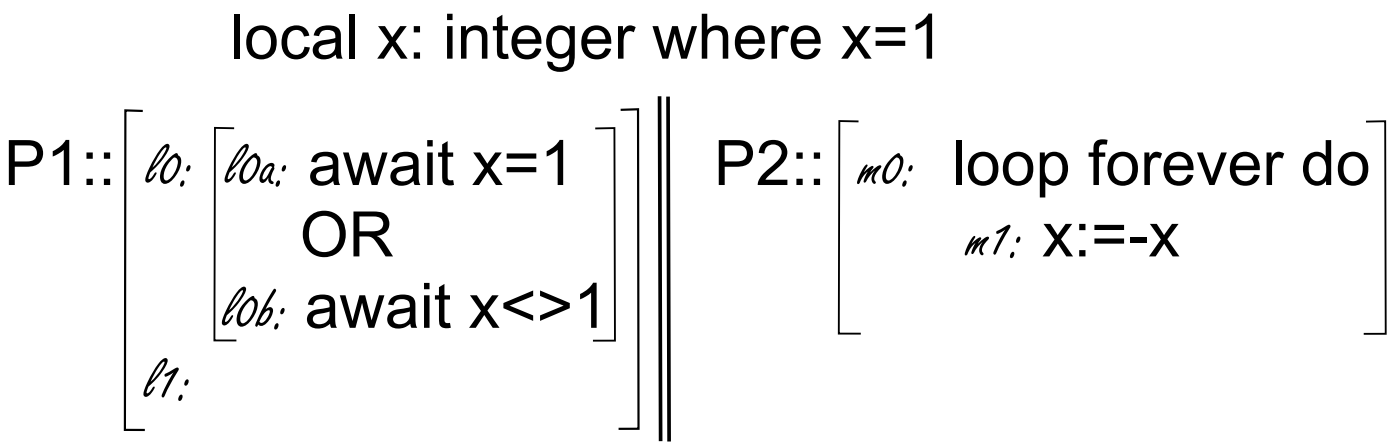
local x: integer where x=1

P1:: [ℓ_0: [ℓ_{0a}: await x=1 OR ℓ_{0b}: skip] ℓ_1:] $\|$ P2:: [m_0: loop forever do m_1: x:=-x]

Properties of the program

- $P2$ never ends.
- Are there computations where $P1$ does not terminate its execution?
  - The **only candidate computation** is:
    $\sigma = < \pi = \{l_o, m_0\}, x = 1 > \rightarrow^{m_0}$
    $< \pi = \{l_o, m_1\}, x = 1 > \rightarrow^{m_1}$
    $< \pi = \{l_o, m_0\}, x = -1 > \rightarrow^{m_0}$
    $< \pi = \{l_o, m_1\}, x = -1 > \rightarrow^{m_1}$
    $< \pi = \{l_o, m_0\}, x = 1 > \rightarrow^{m_0} \ldots$
  - **It is not an admissible computation**: the requirement of justice is satisfied by l_{0a}, since it is not constantly enabled, but not by l_{0b}, which is constantly enabled and never executed.

An example of reactive program - variant 1



Properties of the program - variant 1

- $P2$ never ends.
- Are there computations where $P1$ does not terminate its execution?
  - The **only candidate computation** is the previously identified one.
  - In such a case, it turns out to be an **admissible computation**: the requirement of justice is satisfied both by l_{0a} and by l_{0b}, as no one of them is constantly enabled.

An example of reactive program - variant 2

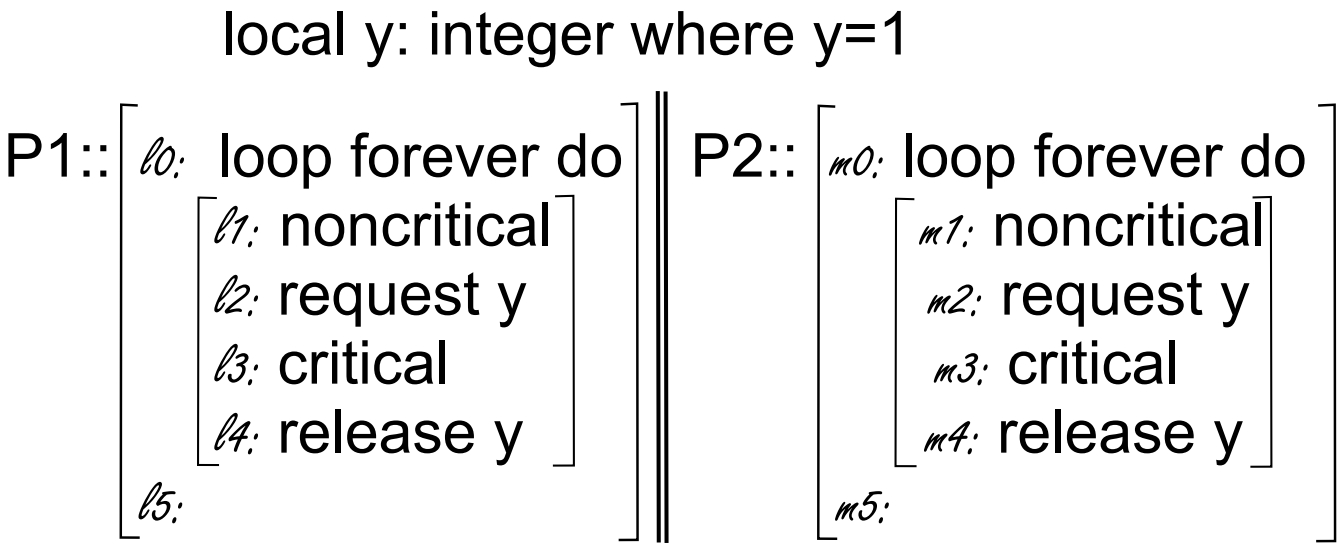
local x: integer where x=1

P1:: [ℓ_0: if x=1 then
ℓ_1: skip
else
ℓ_2: skip
ℓ_3:] || P2:: [m_0: loop forever do
m_1: x:=-x]

Properties of the program - variant 2

- $P2$ never ends.
- Are there computations where $P1$ does not terminate its execution?
  - The **candidate computations** are those where, from a certain point onwards, l_0 (respectively, l_1, l_2) constantly belongs to the value of π.
  - **They are not admissible computations**:
    (the transition corresponding to) the instruction `if` is always enabled and thus sooner or later it must be executed;
    if it is never executed, then the condition of justice is violated,
    if it is executed, then (the transition corresponding to) one of the two instructions `skip` is reached, and, since it is constantly enabled, sooner or later it must be executed;
    if it is never executed, then the condition of justice is violated.

The mutual exclusion problem: the SPL program MUX_SEM



Properties: mutual exclusion

- **Mutual exclusion**: no computation of the program may include a state where both processes are within their critical region ($P1$ at $l3$ and $P2$ at $m3$).
- If $P1$ is at $l3$, the instruction $l2$ has assigned the value 0 to y (by decrementing it by 1) and, until instruction $l4$ is executed, any attempt of $P2$ to execute $m2$ is bound to fail (as the enabling condition $y > 0$ is not satisfied). A similar argument holds in case $P2$ is at $m3$.

  It is worth pointing out that process $P1$ cannot stay forever in the critical region, that is, it cannot remain indefinitely at position $l3$, thanks to the justice requirement for instruction `critical` (the same remark applies to $P2$).

Properties: accessibility - 1

- **Accessibility**: every state of a computation where a process is at the end of its non-critical region, that is, $P1$ is at $l2$ (resp., $P2$ is at $m2$), must be followed by a state where it is within its critical region, that is, it must be followed by a state where $P1$ is at $l3$ (resp., $P2$ is at $m3$).
- Let us show now that every computation where this is not the case is not admissible.

  Let us assume that $P1$ is blocked at $l2$ (the case in which $P2$ is blocked at $m2$ is analogous). If, from a certain point onwards, y remains constantly equal to 1, that is, $P2$ remains indefinitely within its non-critical region, $l2$ is constantly enabled and never executed, and thus the requirement of justice, which is implied by the requirement of compassion, is violated.

Properties: accessibility - 2

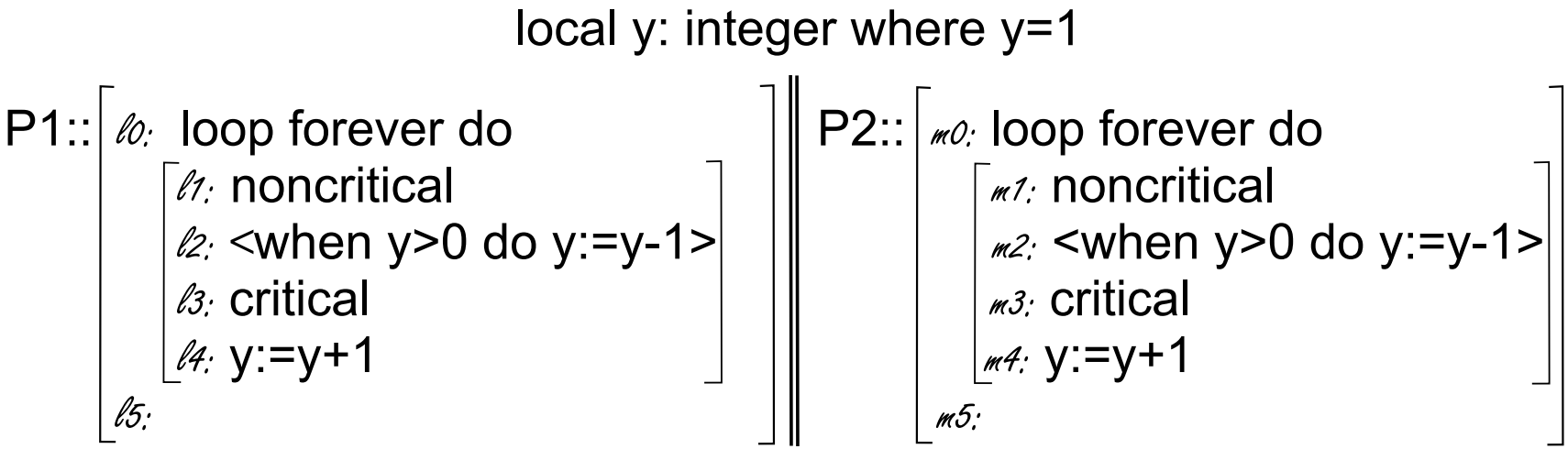
- If y does not remain constantly equal to 1, the computation necessarily has the following form:

σ: $< \pi = \{l_0, m_0\}, y = 1 > \rightarrow \ldots < \pi = \{l_2, m_2\}, y = 1 > \rightarrow^{m_2}$
$< \pi = \{l_2, m_3\}, y = 0 > \rightarrow^{m_3} < \pi = \{l_2, m_4\}, y = 0 > \rightarrow^{m_4}$
$< \pi = \{l_2, m_0\}, y = 1 > \rightarrow^{m_0} < \pi = \{l_2, m_1\}, y = 1 > \rightarrow^{m_1}$
$< \pi = \{l_2, m_2\}, y = 1 > \rightarrow \ldots$

Such a computation visits infinitely many times the state $< \pi = \{l_2, m_2\}, y = 1 >$ where the transition l_2 is enabled, which implies that the requirement of compassion is violated.

This allows us to conclude that the program satisfies the property of accessibility,

The mutual exclusion problem: the SPL program MUX_WHEN



The properties of exclusivity and accessibility

- It **satisfies** the property of **mutual exclusion**: it can be shown by means of an argument analogous to the one used for the previous program (the use of a grouped instruction in $l2$ and $m2$ is fundamental).
- It **does not satisfy** the property of **accessibility**: the previously described computation where $l2$ is enabled infinitely many times (but not constantly enabled) and never taken from a given point onwords is admissible (for $l2$ and $m2$ justice, and not compassion, is required).

The property of "communal" accessibility

- The proposed variant **satisfies** a weaker form of accessibility, called **communal accessibility**.

  Communal accessibility states that each state of a computation where a process reaches the end of its non-critical region, e.g., location $l2$ for $P1$, is followed by a state where some process, not necessarily the same, is within its critical region, that is, location $m3$ for $P2$ or location $l3$ for $P1$.

Producer-Consumer with a Bounded Buffer

local send, ack: channel [1..] of integer
where send=Λ , ack=[1,...,1]

Prod:: [local x, t: integer
ℓ_0: loop forever do
[ℓ_1: produce x
ℓ_2: ack $\Rightarrow$ t
ℓ_3: send $\Leftarrow$ x]]

$\|$

Cons:: [local y: integer
m_0: loop forever do
[m_1: send $\Rightarrow$ y
m_2: ack $\Leftarrow$ 1
m_3: consume y]]

where ack initially contains n occorrences of 1; $l2$ can be executed only if ack is not empty, $m1$ only if send is not empty.

The SPL program Fair-Merge

out b : channel of integer
local a1, a2 : channel of integer

P1:: [local x1: integer
ℓ0: loop forever do
[ℓ1: produce x1
ℓ2: a1 ⇐ 2•x1 + 1]]

||

P2:: [local x2: integer
k0: loop forever do
[k1: produce x2
k2: a2 ⇐ 2•x2]]

||

Merger:: [local y: integer
m0: loop forever do
m1: [m1a: a1 ⇒ y
or
m1b: a2 ⇒ y]
m2: b ⇐ y]

||

Cons:: [local z: integer
n0: loop forever do
[n1: b ⇒ z
n2: consume z]]