Bounded Model Checking

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Automatic System Verification

Introduction

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  - it costed $\approx$ US $475 million;
  - big investment in formal verification.

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- distinctive features:
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  - exhaustive;
  - it generates a counterexample trace if the specification does not hold.

Linear Temporal Logic

We consider LTL model checking.

- LTL syntax:

$$p \mid \neg\phi \mid \phi_1 \vee \phi_2 \mid \phi_1 \wedge \phi_2$$
$$\mid \mathsf{X}\,\phi_1 \mid \phi_1\,\mathcal{U}\,\phi_2$$
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- shortcuts:
  - $\phi_1\,\mathcal{R}\,\phi_2 \equiv \neg(\neg\phi_1\,\mathcal{U}\,\neg\phi_2)$,
  - $\mathsf{F}\,\phi_1 \equiv \top\,\mathcal{U}\,\phi_1$
  - $\mathsf{G}\,\phi_1 \equiv \neg\,\mathsf{F}\,\neg\phi_1$

Linear Temporal Logic

- Semantics. LTL formulas are interpreted over infinite *state sequences* $\sigma = \langle \sigma_0, \sigma_1, \ldots \rangle \in (2^{\Sigma})^{\omega}$ of sets of propositions $\sigma_i \in 2^{\Sigma}$:

$$
\begin{array}{lll}
\sigma \models_i p & \text{iff} & p \in \sigma_i \\
\sigma \models_i \mathsf{X}\,\phi & \text{iff} & \sigma \models_{i+1} \phi \\
\sigma \models_i \phi_1 \,\mathcal{U}\, \phi_2 & \text{iff} & \text{there exists } j \geq i \text{ such that} \\
 & & \sigma \models_j \phi_2 \text{ and } \sigma \models_k \phi_1 \text{ for all} \\
 & & i \leq k < j
\end{array}
$$

  ...

- LTL model checking:
  - decide if $\mathcal{M}, s \models \phi$, where $\mathcal{M} = (S, I, T, L)$ is a Kripke structure, $s \in I$ is an initial state and ϕ is an LTL formula; in many contexts, you may find the notation: $\mathcal{M}, s \models A\phi$;
  - PSPACE-complete.

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$$\mathcal{M}, s \not\models \phi$$

State-space Explosion Problem

- the previous algorithm belongs to the class of explicit model checking algorithms:
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  is exponential in n;
- the size of system that could be verified by explicit model checkers was restricted to $\approx 10^6$ states.

Tackling the explosion...

Three main techniques have been proposed:

- BDD-based symbolic model checking, next lessons
- partial order reduction, next lessons :-)
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They allowed for the verification of systems with $> 10^{20}$ states.

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- why not in DNF?

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- there are several efficient algorithms for solving SAT (*e.g.*, DPLL, CDCL...) along with many heuristics (*e.g.*, 2 watching literals, glue clauses...)
- some numbers:
  - $> 100{\cdot}000$ variables;
  - $> 1{\cdot}000{\cdot}000$ clauses;

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The corresponing symbolic Kripke structure is the tuple $(\bar{s}, f_I, f_T, \{f_{p_1}, \ldots, f_{p_k}\})$.

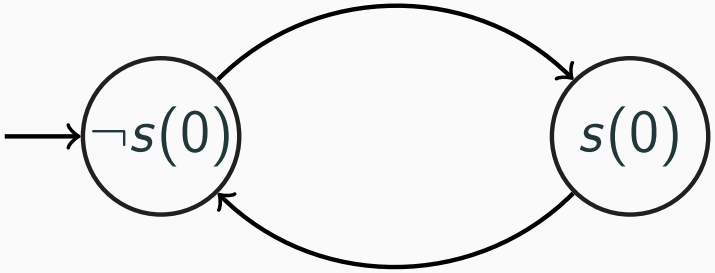
Symbolic Transition Systems

- we will write simply $\mathcal{M} = (S, I, T, L)$, meaning a symbolic transition system
- a path (or trace) $\pi = m_0, m_1, \ldots$ is an infinite sequence of assignment to the state variables such that:
  - $m_0 \models I(s)$;
  - $m_i, m'_{i+1} \models T(s, s')$ holds, for all $i \geq 0$.

  where $\bar{s}' := \{s'(0), \ldots, s'(n)\}$.

Example 1

simple-example.smv

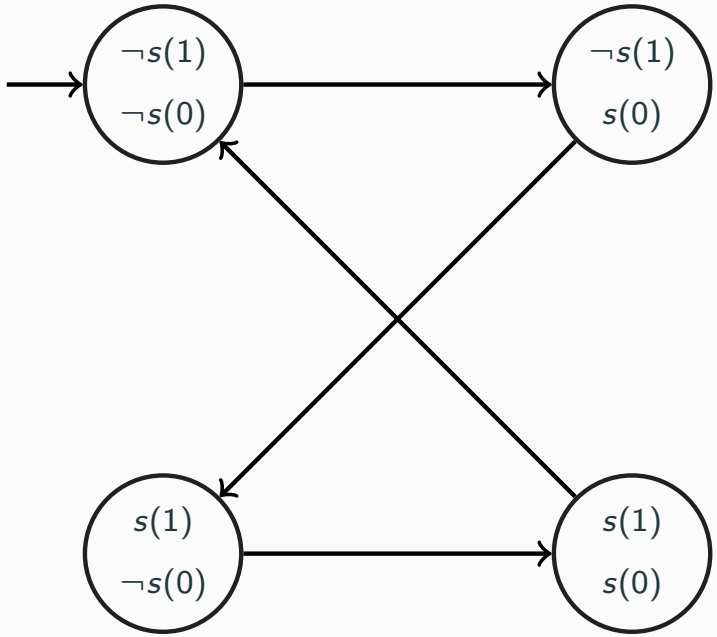


Example 1 - SMV

```
1  MODULE main
2  VAR
3     s0 : boolean;
4  INIT
5     !s0;
6  TRANS
7     s0 <-> next(!s0);
```

Example - 2

modulo-4-counter.smv



Example 2 - SMV

```
1  MODULE main
2  VAR
3     s0 : boolean;
4     s1 : boolean;
5  INIT
6     !s0 & !s1;
7  TRANS
8     (next(s0) <-> !s0)
9       &
10    (next(s1) <-> ((s0 & !s1) | (!s0 & s1)));
```

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- recall that we can reduce $\mathcal{M}, s \models \psi$ to checking the emptiness of $\mathcal{M} \times \mathcal{A}_{\neg\psi}$;
  - the universal problem $\mathcal{M}, s \models A\psi$ is reduced to the existential problem $\mathcal{M}, s \models E\phi$, where $\phi := \neg\psi$;
- Bounded Model Checking (BMC) solves the problem $\mathcal{M}, s \models E\phi$ by proceeding incrementally:
  - we start with $k = 0$;
  - check if there exists and execution π of $\mathcal{M}$ of length k that satisfies ϕ; encode this problem into a SAT instance and call a SAT-solver;
  - if so, we have found a counterexample to ψ; if not, $k{+}{+}$.

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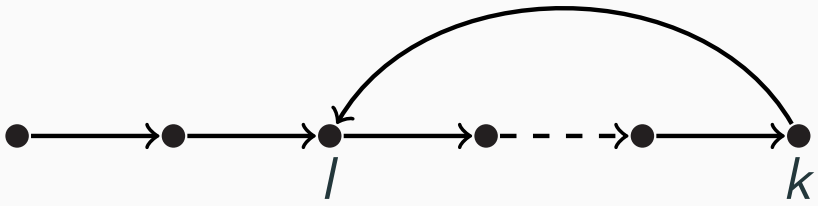
Crucial observation:

- a finite trace can still represent an infinite state sequence, if it contains a loop-back.

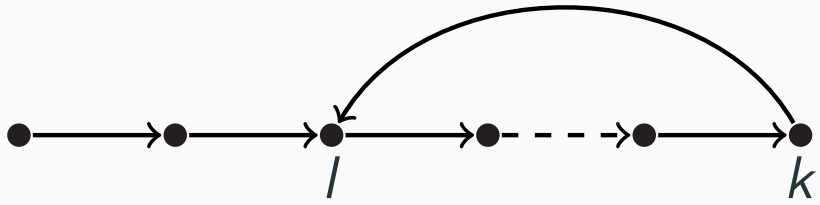


BMC

Given a finite trace π of the system $\mathcal{M}$, BMC distinguishes between two cases:

- either π contains a loop-back (π is lasso-shaped): apply standard LTL semantics to check if $\pi \models \phi$;
- or π is loop-free: apply bounded semantics.

k-loop, aka Lasso-Shaped Models



Definition (k-loop)

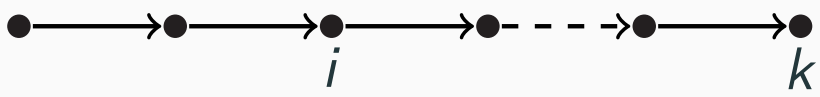
A path π is a (k, l)-loop, with $l \leq k$, if $T(\pi(k), \pi(l))$ holds and $\pi = u \cdot v^{\omega}$, where:

- $u = \pi(1) \dots \pi(l-1)$;
- $v = \pi(l) \dots \pi(k)$.

We call π a k-loop if there exists $l \leq k$ for which π is a (k, l)-loop.

If a path π is a k-loop, then we can check if $\pi \models \phi$ with the standard LTL semantics.

Bounded Semantics for LTL



If π is not a k-loop, we introduce bounded semantics for LTL.

Definition (Bounded semantics for LTL)

Let $k \geq 0$ and π a path that is not a k-loop. An LTL formula ϕ is valid along π with bound k, written $\pi \models_k^0 \phi$, iff:

- $\pi \models_k^i p \quad$ *iff* $\quad p \in L(\pi(i))$
- $\pi \models_k^i \neg p \quad$ *iff* $\quad p \notin L(\pi(i))$

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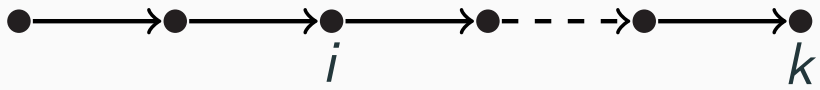

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- $\pi \models_k^i \phi_1 \vee \phi_2$ *iff* $\pi \models_k^i \phi_1$ *or* $\pi \models_k^i \phi_2$
- $\pi \models_k^i \phi_1 \wedge \phi_2$ *iff* $\pi \models_k^i \phi_1$ *and* $\pi \models_k^i \phi_2$

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- $\pi \models_k^i \mathsf{X}\,\phi_1$ *iff* $i < k$ *and* $\pi \models_k^{i+1} \phi_1$
- $\pi \models_k^i \phi_1\,\mathcal{U}\,\phi_2$ *iff* $\exists i \leq j \leq k$ *such that* $\pi \models_k^j \phi_2$ *and* $\forall i \leq n < j$ *it holds that* $\pi \models_k^n \phi_1$

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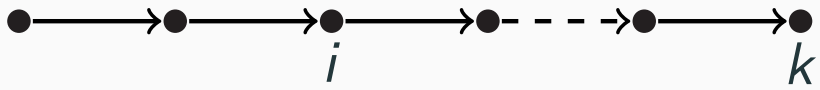

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- $\pi \models_k^i \mathsf{G}\,\phi_1$ *iff* *???*
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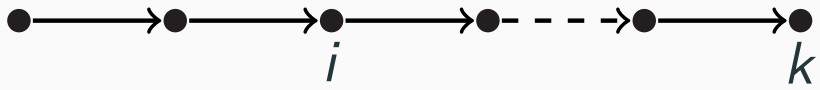
Bounded Semantics for LTL



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- $\pi \models_k^i \mathsf{G}\,\phi_1$ *is always false*
- $\pi \models_k^i \mathsf{F}\,\phi_1$ *iff* $\exists i \leq j \leq k$ *such that* $\pi \models_k^j \phi_1$

SAT-based encoding of BMC

Now we see how to reduce BMC to SAT.

- the first thing to do is to define a Boolean formula that encodes all the paths of $\mathcal{M}$ of length k.

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Definition (Unfolding of the Transition Relation)

For a Kripke structure $\mathcal{M}$ and $k \geq 0$, we define:

$$[\![\mathcal{M}]\!]_k := I(s_0) \wedge \bigwedge_{i=0}^{k-1} T(s_i, s_{i+1})$$

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What does a model of $[\![\mathcal{M}]\!]_k$ represent?

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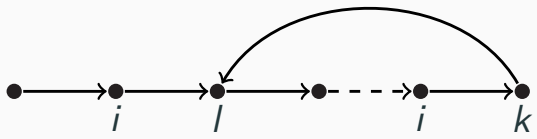
We have seen that BMC distinguishes between lasso-shaped (k-loop) and loop-free paths:

- we start with the encoding in case of k-loops.

Encoding of a loop



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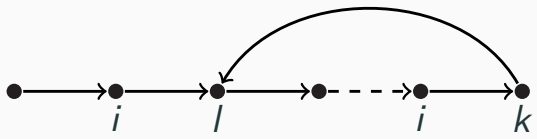

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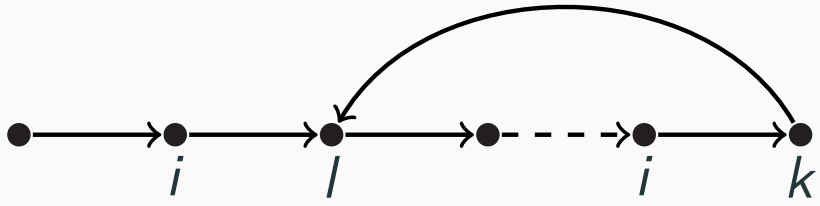
Definition (Successor in a Loop)

Let $l, i \leq k$ and π be a (k, l)-loop. We define the successor $succ(i)$ of i in π as:

- $succ(i) := i + 1$ *if* $i < k$;
- $succ(i) := l$ *if* $i = k$.

Encoding in case of Loop



Definition (Encoding of an LTL formula for a (k, l)-loop)

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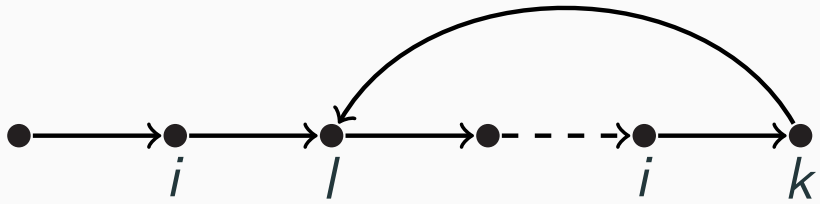
Encoding in case of Loop



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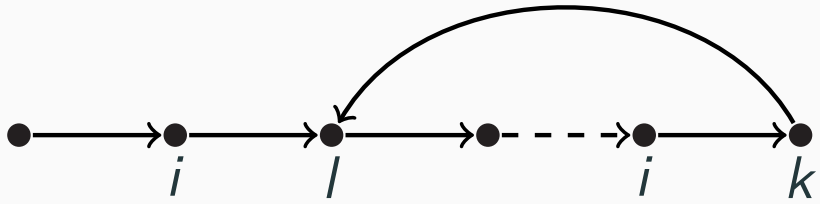

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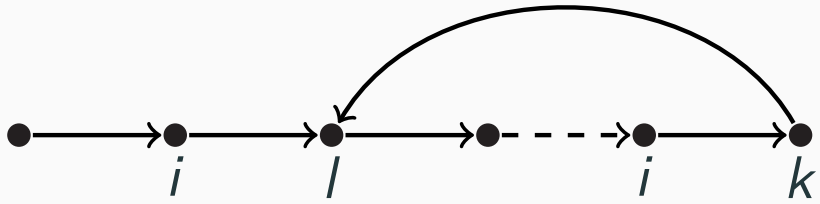

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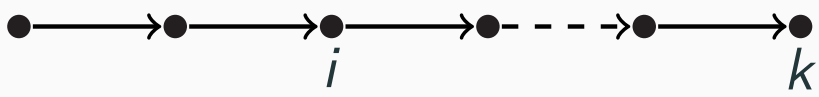
Encoding in case of NO Loops



Definition (Encoding of an LTL formula for a loop-free path)

Let ϕ be an LTL formula and $i, k \geq 0$. We define $[\![\phi]\!]_k^i$ recursively as follows:

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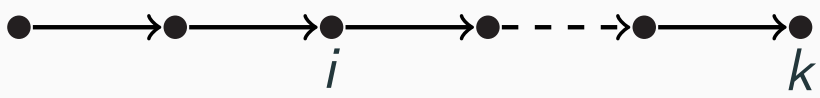
Encoding in case of NO Loops



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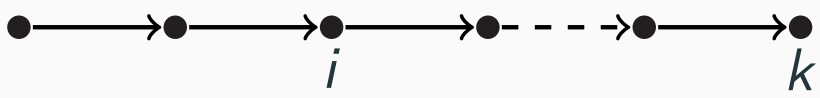

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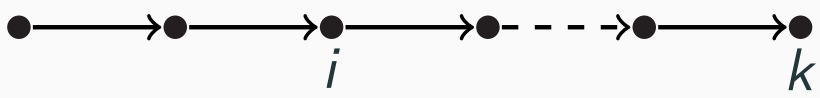

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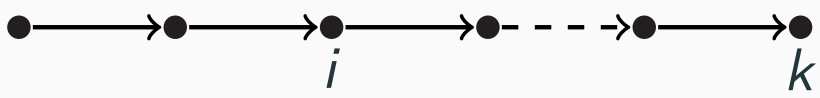

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Overall encoding

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Let ϕ be an LTL formula, $\mathcal{M}$ be a Kripke structure and $k \geq 0$:

$$[\![M, \phi]\!]_k := \underbrace{[\![\mathcal{M}]\!]_k}_{\substack{\text{encoding of} \\ \text{the machine}}} \wedge \Bigg(\underbrace{(\neg L_k \wedge [\![\phi]\!]_k^0)}_{\substack{\text{loop-free} \\ \text{models}}} \vee \underbrace{\bigvee_{l=0}^{k} ({}_l L_k \wedge {}_l[\![\phi]\!]_k^0)}_{\substack{\text{lasso-shaped} \\ \text{models}}} \Bigg)$$

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Theorem (Soundness)

$[\![\mathcal{M}, \phi]\!]_k$ is satisfiable iff $\mathcal{M} \models_k E\phi$.

Completeness

Algorithm:

- start with $k = 0$
- call a SAT-solver on $[\![\mathcal{M}, \phi]\!]_k$
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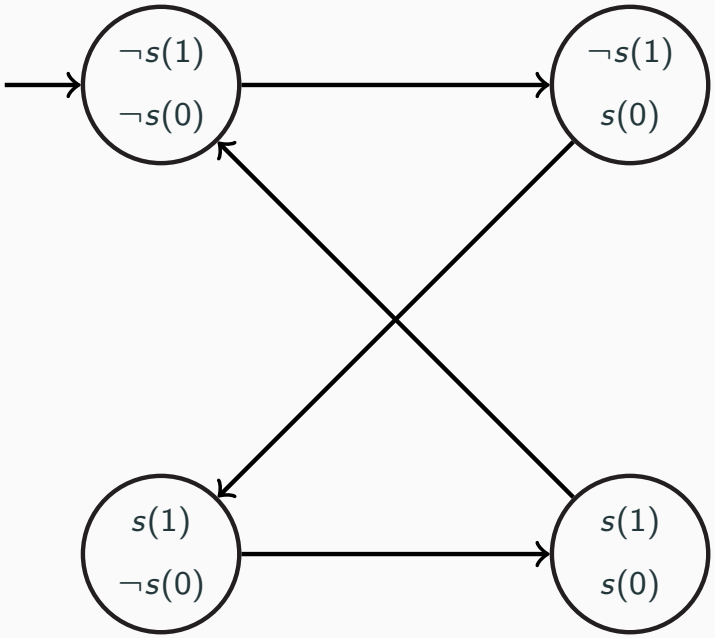
- the procedure does not terminate: BMC is not complete;
- BMC is mainly used as a bug finder, rather than as a prover.
- several techniques have been proposed to make BMC complete:
  - recurrence diameter
  - Simple Bounded Model Checking (SBMC)

Example

modulo-4-counter.smv



- $\phi_1 := \mathsf{G}\,\mathsf{F}(s(0) \wedge s(1))$ ✓
- $\phi_2 := \mathsf{F}\,\mathsf{G}(\neg s(0) \wedge \neg s(1))$ ✗

Solving LTL-SAT with BMC

LTL-SAT is the problem of establishing if, given an LTL formula ϕ, there exists an infinite state sequence σ such that $\sigma \models \phi$.

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BLACK

- we developed this tool based on the idea of *bounded satisfiability checking*
- BLACK = Bounded Ltl sAtisfiability ChecKer [1]

[1] https://github.com/black-sat/black