AUTOMATED REASONING

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Università di Udine
CLPLAB

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Semantics of Logic Programs

```
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p(b).
```

A program (or in general, a first-order theory) P is a built from a list of symbols.
In this case constants a and b, and one unary predicate symbol p
We would like to assign a meaning (interpretation) to these symbols on a universe of objects.

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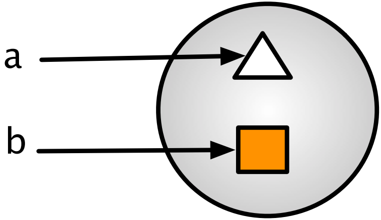

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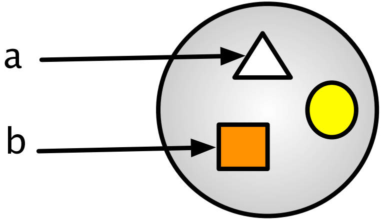

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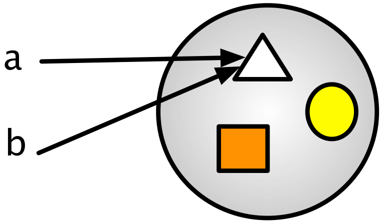
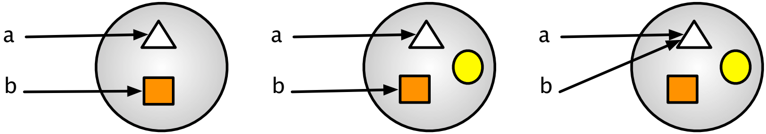


Different interpretations for constant symbols induce different interpretations for first-order formulas (with equality)



$a \neq b$ $\exists X\,(X \neq a \wedge X \neq b)$ $a = b$

```
p(a).
p(s(X)) ← p(X).
q(g(X,Y)) ← p(X),p(Y).
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Constant a and function symbols $s/1$ and $g/2$. Function symbols must be interpreted as functions ($g(x, y) = z$ means $(x, y, z) \in F$)

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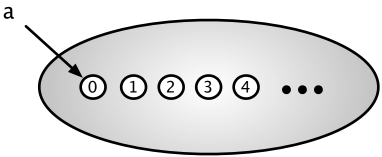

UNIVERSES AND INTERPRETATIONS

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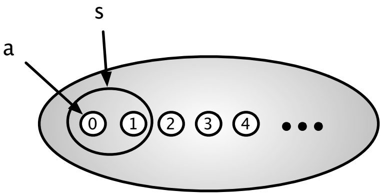

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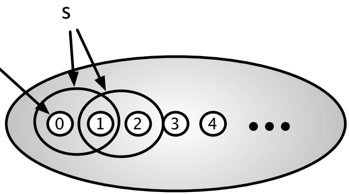

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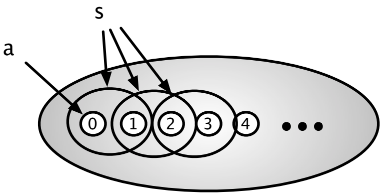

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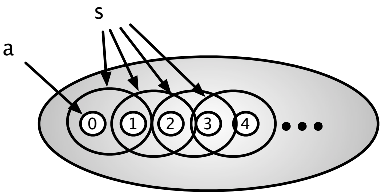

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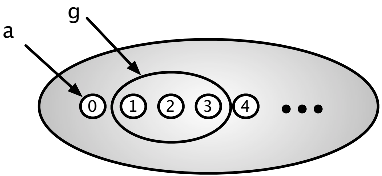

Universes and Interpretations

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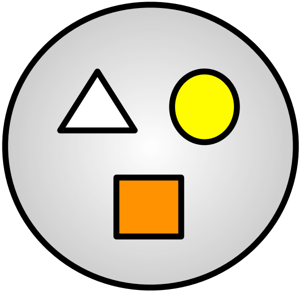

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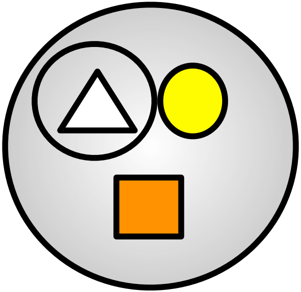

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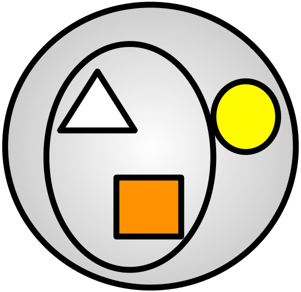

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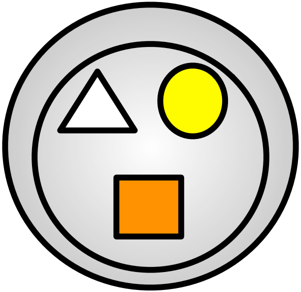

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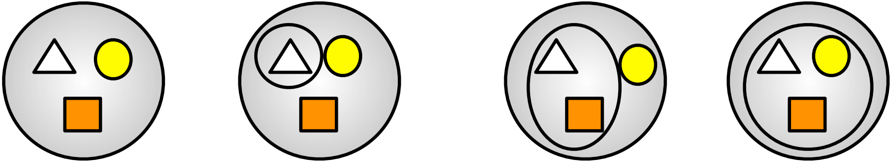

$\forall X \, \neg p(X)$

$\exists X \, p(X) \wedge \exists X \exists Y (X \neq Y \wedge \neg p(X) \wedge \neg p(Y))$

$\exists X \, (\neg p(X)) \wedge \exists X \exists Y (X \neq Y \wedge p(X) \wedge p(Y))$

$\forall X \, p(X)$

Models

Some of the various interpretations of constant, function, and predicate symbols can be models of the program (or of the theory) P.

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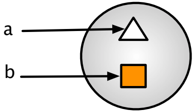

Let us denote $\overline{a}$ =triangle and $\overline{b}$ =square. Let use denote with P, Q, R the interpretations of the predicate symbols p, q, r.

An interpretation that satisfies the logical meaning of all the formulas of P is a model.

$P = \{\overline{a}\}, Q = \{\overline{b}\}, R = \{\overline{a}, \overline{b}\}$ is a model.
$P = \{\overline{a}, \overline{b}\}, Q = \{\overline{b}\}, R = \{\overline{a}\}$ is NOT a model.

An atom $q(t_1, \ldots, t_n)$ is a logical consequence of a program/theory P if $(\overline{t_1}, \ldots, \overline{t_n}) \in Q$ in all (interpretations that are) models of P.
We say that $P \models q(t_1, \ldots, t_n)$.

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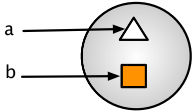

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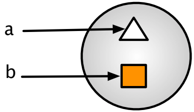

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MODELS

The set of logical consequences seems to be what we expect from the program.

The questions are: Does it exist always? If yes, how to compute it?