

University of Udine
Department of Mathematics, Computer Science and Physics

MODELLING MULTI-AGENT EPISTEMIC PLANNING IN ASP



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1. Multi-Agent Epistemic Planning
2. Possibilities
3. The action language $m\mathcal{A}^p$
4. PLATO
5. Conclusions

Chapter 1

Multi-Agent Epistemic Planning

Introduction



Epistemic Reasoning

Reasoning not only about agents' *perception of the world* but also about agents' *knowledge* and/or *beliefs* of her and others' beliefs.

Introduction



2

Epistemic Reasoning

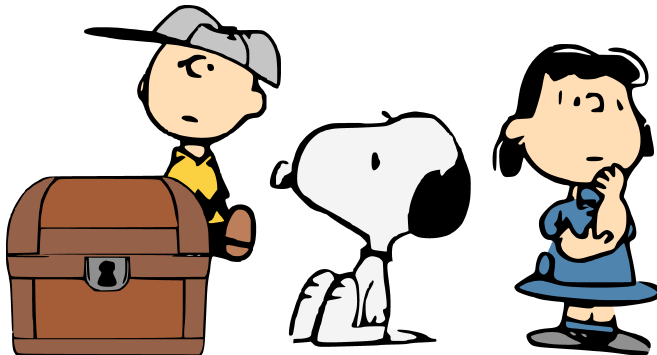
Reasoning not only about agents' *perception of the world* but also about agents' *knowledge* and/or *beliefs* of her and others' beliefs.

Multi-agent Epistemic Planning Problem [BA11]

Finding *plans* where the goals can refer to:

- the state of the world
- the knowledge and/or the beliefs of the agents

An Example

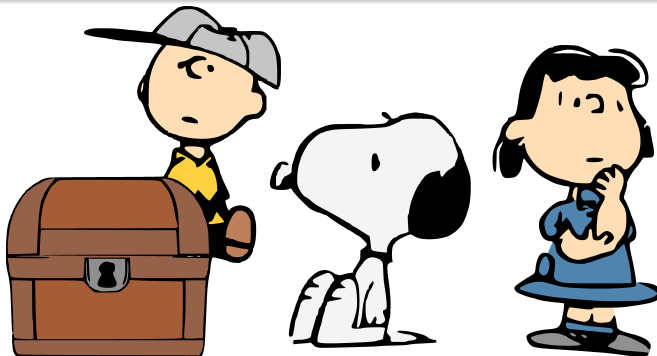


An Example



Initial State

- Snoopy and Charlie are **looking** while Lucy is \neg **looking**
- No one knows the **coin position**.

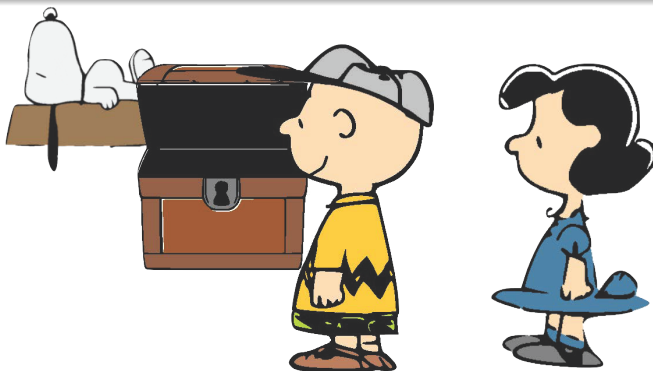


An Example



Goal State

- Charlie knows the **coin position**
- Lucy knows that Charlie knows the **coin position**
- Snoopy does not know anything about the plan execution



Notations



Given a set of agents \mathcal{AG}

Belief formulae

where $ag \in \mathcal{AG}$, $\alpha \subseteq \mathcal{AG}$

We use the operators \mathbf{B}_{ag} and \mathbf{C}_{α} to model the beliefs and the common knowledge of the agents.

Notations



5

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We use the operators \mathbf{B}_{ag} and \mathbf{C}_{α} to model the beliefs and the common knowledge of the agents.

Properties of \mathbf{B}_{ag}

KD45 and **S5** Axioms

Given the fluent formulae ϕ , ψ and the worlds i , j

$$D \neg \mathcal{R}_i \perp \quad \mathcal{BK}$$

$$K (\mathcal{R}_i \phi \wedge \mathcal{R}_i (\phi \implies \psi)) \implies \mathcal{R}_i \psi \quad \mathcal{BK}$$

$$T \mathcal{R}_i \phi \implies \phi \quad \mathcal{K}$$

$$4 \mathcal{R}_i \phi \implies \mathcal{R}_i \mathcal{R}_i \phi \quad \mathcal{BK}$$

$$5 \neg \mathcal{R}_i \phi \implies \mathcal{R}_i \neg \mathcal{R}_i \phi \quad \mathcal{BK}$$

Chapter 2

Possibilities



- Introduced by Gerbrandy and Groeneveld [GG97]
- Used to represent *multi-agent information change*
- Based on *non-well-founded sets*



Possibility [GG97]

Let \mathcal{AG} be a set of agents and \mathcal{F} a set of propositional variables:

- A *possibility* u is a function that assigns to each propositional variable $l \in \mathcal{F}$ a truth value $u(l) \in \{0, 1\}$ and to each agent $ag \in \mathcal{AG}$ a set of possibilities $u(ag) = \sigma$ (*information state*).

Intuitively ...

- The possibility u is a possible interpretation of the world and of the agents' beliefs
- $u(l)$ specifies the truth value of the literal l
- $u(ag)$ is the set of all the interpretations the agent ag considers possible in u
- Representable with graphs: we will use graph terminology

Chapter 3

The action language $m\mathcal{A}^\rho$

Action types



We introduced the action language $m\mathcal{A}^p$ in [Fab+20]

- Used to describe MEP problems
- Uses possibilities as states
- Actions preconditions: belief formulae



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Three types of actions:

- *Ontic*: modifies some fluents of the world
Charlie *opens* the box



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Charlie *opens* the box
- *Sensing*: senses the true value of a fluent
Charlie *peeks* inside the box



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Three types of actions:

- Ontic: modifies some fluents of the world
Charlie *opens* the box
- Sensing: senses the true value of a fluent
Charlie *peeks* inside the box
- **Announcement**: announces the fluent to other agents
Charlie *announces* the coin position



Observability Relations



An *execution* of an action might change or not an agents' belief accordingly to her degree of awareness

Action type	Full observers	Partial Observers	Oblivious
Ontic	✓		✓
Sensing	✓	✓	✓
Announcement	✓	✓	✓

Chapter 4

PLATO



PLATO, e**P**istemic mu**L**ti-agent **A**nswer se**T** programming s**O**lver:

- Declarative encoding in ASP of MEP
- Based on the language $m\mathcal{A}^p$
- Main components: *initial state generation*, *entailment*, *transition function*
- Exploits *clingo*'s multi-shot capabilities [Geb+19]
- Formal proof of correctness

Encoding possibilities



Let u be a possibility.

ASP encoding: possibilities

We encode u with the atom `possible_world(T_u, R_u, P_u)`, where:

- T_u tells us when u was created
- R_u is the *repetition* of u
- P_u is the numerical index of u

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ASP encoding: pointed possibility

If u is the possibility that represents the *real configuration* of the world, we encode it with the atom `pointed(T_u, R_u, P_u)`.

When the context is clear we will use *only* P_u instead of (T_u, R_u, P_u) .

Encoding possibilities



Let u, v be two possibilities, let AG be an agent and let F be a fluent.

ASP encoding: information states

We encode $v \in u(AG)$ with the atom $\text{believes}(P_u, P_v, AG)$.

Encoding possibilities



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ASP encoding: information states

We encode $v \in u(AG)$ with the atom $\text{believes}(P_u, P_v, AG)$.

ASP encoding: interpretations

We encode $u(F) = 1$ with the atom $\text{holds}(P_u, F)$.



Given a possibility P and a belief formula F .

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```
entails      (P,      F)      :- holds(P,F), fluent(F).
entails      (P,      neg(F))  :- not entails(P,F).
entails      (P,      and(F1,F2)) :- entails(P,F1), entails(P,F2).
entails      (P,      or(F1,F2)) :- entails(P,F1).
entails      (P,      or(F1,F2)) :- entails(P,F2).
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entails      (P,    or(F1,F2))   :- entails(P,F2).
not_entails  (P1,    b(AG,F))    :- not entails(P2,F), believes(P1,P2,AG).
entails      (P,    b(AG,F))    :- not not_entails(P, b(AG,F)).
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entails      (P,      b(AG,F))   :- not not_entails(P, b(AG,F)).

not_entails  (P1,    c(AGS,F))   :- not entails(P2,F), reaches(P1,P2,AGS).
entails      (P,      c(AGS,F))   :- not not_entails(P, c(AGS,F)).
  
```

Ontic actions



Let *open* be an ontic action such that

- It sets the fluent *opened* to true
- Only **Charlie** and **Snoopy** are attentive

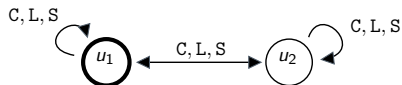


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$$u_1(\mathcal{F}) = \{\text{head}, \text{looking}_{\text{Charlie}}, \text{looking}_{\text{Snoopy}}\}$$

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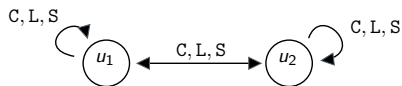
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$\text{pw}(u'_i) :- \text{pointed}(u_1), \text{reaches}(u_1, u_i, \text{AGS}), \text{fully_obs}(\text{AGS}). \quad (i \in \{1, 2\})$



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u'_1

u'_2

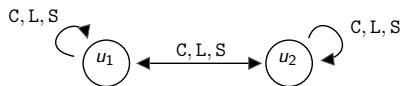
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$\text{believes}(u'_i, u'_j, \text{AG}) :- \text{pw}(u'_i), \text{pw}(u'_j), \text{fully_obs}(\text{AG}), \quad (i, j \in \{1, 2\})$
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Ontic actions

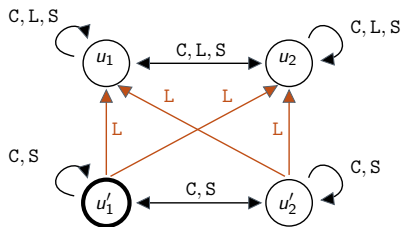


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$\text{believes}(u'_i, u_j, \text{AG}) :- \text{pw}(u'_i), \text{oblivious}(\text{AG}), \quad (i, j \in \{1, 2\})$
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Sensing/Announcement actions



Let *peek* be an ontic action such that

- Charlie senses the fluent *heads*
- Only Charlie and Snoopy are attentive



Sensing/Announcement actions

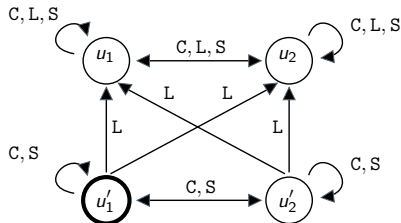


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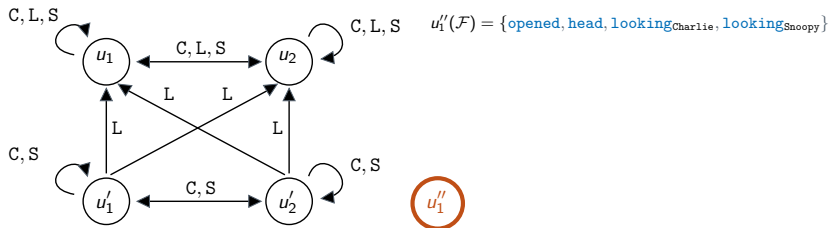
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$\text{pw}(u_1'') :- \text{pointed}(u_1'), \text{reaches}(u_1', u_1', \text{AGS}), \text{fully_obs}(\text{AGS}),$
 $\text{holds_sensed}(u_1, \text{peek}).$



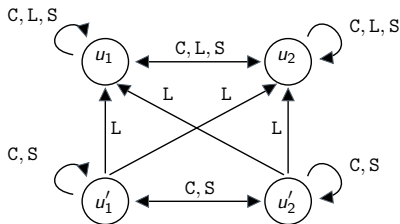
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Let *peek* be an ontic action such that

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$\text{pw}(u_2'') :- \text{pointed}(u_1'), \text{reaches}(u_1', u_2', \text{AGS}), \text{not_oblivious}(\text{AGS}).$



$u_1''(\mathcal{F}) = \{\text{opened}, \text{head}, \text{looking}_{\text{Charlie}}, \text{looking}_{\text{Snoopy}}\}$

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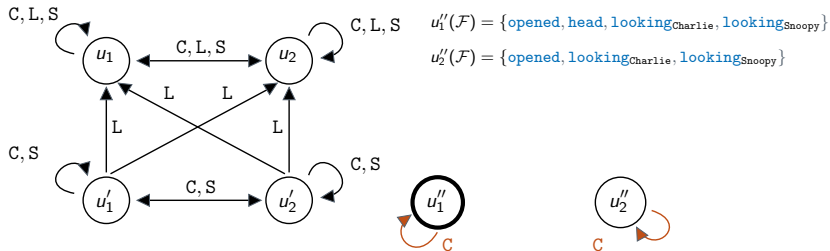
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$\text{believes}(u''_i, u''_j, AG) :-$ $\text{pw}(u''_i), \text{pw}(u''_j), \text{pw}(u'_i), \text{pw}(u'_j),$ $(i, j \in \{1, 2\})$
 $\text{believes}(u'_i, u'_j, AG), \text{fully_obs}(AG),$
 $\text{holds_sensed}(u_i, \text{peek}) = \text{holds_sensed}(u_j, \text{peek}).$



Sensing/Announcement actions

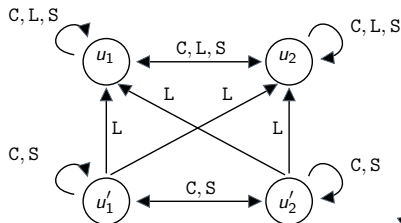


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$$\text{believes}(u'_i, u'_j, AG), \text{partially_obs}(AG).$$


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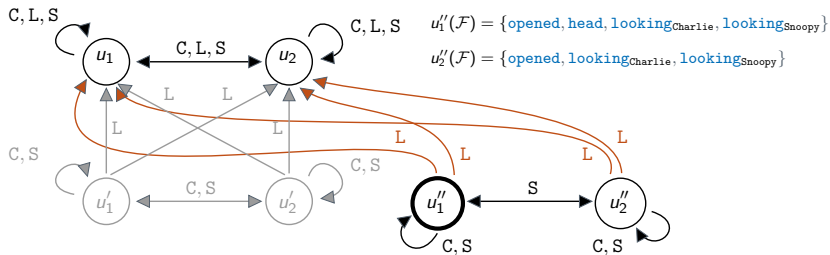

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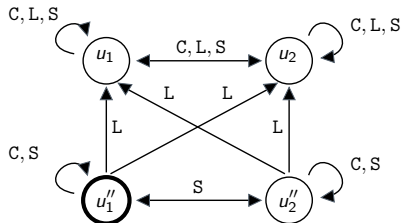
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Correctness w.r.t. $m\mathcal{A}^\rho$ 

Let u, v be two possibilities and ψ be a belief formula.

Entailment correctness

For each u , we have that $\forall \psi \quad u \models_\Phi \psi$ iff $u \models_\Gamma \psi$.

Initial state generation correctness

For each u, v such that u is the initial state in $m\mathcal{A}^\rho$ and v is the initial state in PLATO then $\forall \psi \quad u \models_\Phi \psi$ iff $v \models_\Gamma \psi$.

Transition function correctness

Let a be an action instance. For each u, v such that $\forall \psi \quad u \models_\Phi \psi$ iff $v \models_\Gamma \psi$, then $\forall \psi \quad \Phi(a, u) \models_\Phi \psi$ iff $\Gamma(a, v) \models_\Gamma \psi$.

Experimental evaluation



SC: $ \mathcal{AG} = 9$, $ \mathcal{F} = 12$, $ \mathcal{A} = 14$				
L	many	frumpy	K-BIS	P-MAR
4	.24	.24	.03	.007
6	2.56	2.49	.16	.04
8	36.79	38.34	4.23	.30
9	204.52	146.343	5.79	.83
10	T0	839.27	7.36	1.78

Gr: $ \mathcal{AG} = 3$, $ \mathcal{F} = 9$, $ \mathcal{A} = 24$				
L	Total	Ground	Solve	Atoms
3	.97	.60	.36	28'615
4	4.25	2.24	2.01	42'022
5	32.83	2.52	30.31	71'482
6	211.69	5.27	206.41	140'305
7	1066.80	16.94	1049.86	302'623

Experimental evaluation



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CC.1: $ \mathcal{AG} = 2$, $ \mathcal{F} = 10$, $ \mathcal{A} = 16$					CC.2: $ \mathcal{AG} = 3$, $ \mathcal{F} = 13$, $ \mathcal{A} = 24$			
L	single	multi	K-BIS	P-MAR	single	multi	K-BIS	P-MAR
3	48.74	6.52	.08	.02	153.76	14.07	.13	.03
4	188.32	8.74	.16	.03	T0	28.02	.54	.10
5	T0	7.68	1.14	.16	T0	16.13	4.89	.60
6	1222.67	10.83	4.42	0.64	T0	14.84	12.66	1.71
7	T0	30.08	16.06	2.61	T0	56.48	142.06	12.37

Chapter 5

Conclusions

Conclusions



- Exploited a *declarative* approach to implement Multi-Agent Epistemic Planning
- Improved readability and code maintenance
- Straightforward semantical adaptations
- Results comparable to the imperative approach
- Formal *proof* of correctness

Future works



- Enhancement of the entailment rules
- Implementation of heuristics
- Formal proof of equivalence between $m\mathcal{A}^*$ and $m\mathcal{A}^p$
- We are using PLATO to implement novel concepts in MEP, such as *trust*, *lies* and *misconceptions*

The end



Thank You
for the attention

References I

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- [Mui+15] Christian J. Muise et al. “Planning Over Multi-Agent Epistemic States: A Classical Planning Approach”. In: *Proc. of AAAI*. 2015, pp. 3327–3334.

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- [Son+14] Tran Cao Son et al. “Finitary S5-theories”. In: *European Workshop on Logics in Artificial Intelligence*. Springer. 2014, pp. 239–252.

Ohter Planners

To the best of our knowledge EFP 2.0 is the only *comprehensive* epistemic multi-agent planner.

Other planners with the best results in the literature are:

- RP-MEP [Mui+15]: translates into classical planning. Only deals with a finite level of nested beliefs.
- MEPK [Hua+17]: does not support dynamic common knowledge.

Domains I

- ▶ *Assembly Line* (**AL**): two agents are responsible for processing a different part of a product. They can fail in processing their part and inform the other of the status of her task. The agents decide to *assemble* the product or *restart*. Goal: the agents must assemble the product. We change the *depth* of the belief formulae.
- ▶ *Coin in the Box* (**CB**). $n \geq 3$ agents are in a room. There is a closed box containing a coin. None of the agents know the coin position. One agent has the key. An agent may look inside the box to sense the state of the coin and also share the result.

Domains II

- ▶ *Collaboration and Communication (CC)*. $n \geq 2$ agents move along a corridor with $k \geq 2$ rooms in which $m \geq 1$ boxes can be located. Agents can determine if a certain box is in the room they are in. They can communicate information about the boxes' position. Agents may move only to adjacent rooms.
- ▶ *Grapevine*. $n \geq 2$ agents are located in $k \geq 2$ rooms. Each agent ag knows a “secret” (s_{ag}). Agents can move to an adjacent room and share their secret within the same room.
- ▶ *Selective Communication (SC)*. $n \geq 2$ agents within one of the $k \geq 2$ rooms in a corridor. Agents can move to an adjacent room. In only one of the rooms, agents may acquire some information q and may communicate it to others.

Finitary S5 Theories

Finitary S5-theory [Son+14]

Let ϕ be a fluent formula and let $i \in \mathcal{AG}$ be an agent. A *finitary S5-theory* is a collection of formulae of the form:

$$(i) \phi \quad (ii) C \phi \quad (iii) C (B_i \phi \vee B_i \neg \phi) \quad (iv) C (\neg B_i \phi \wedge \neg B_i \neg \phi)$$

Each fluent $f \in \mathcal{F}$ must appear in at least one of the formulae (ii)–(iv) (for at least one agent $i \in \mathcal{AG}$).

A finitary S5-theory has *finitely many* S5-models up to equivalence.

Initial state generation

Given

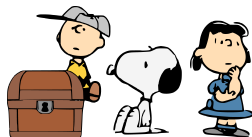
- $\mathcal{AG} = \{\text{Snoopy}, \text{Charlie}, \text{Lucy}\}$
- $\mathcal{F} = \{\text{opened}, \text{head}, \text{looking}_{\text{ag}}\} \text{ ag} \in \mathcal{AG}$



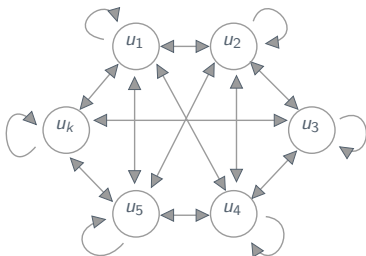
Initial state generation

Given

- $\mathcal{AG} = \{\text{Snoopy, Charlie, Lucy}\}$
- $\mathcal{F} = \{\text{opened, head, looking}_{\text{ag}}\} \text{ ag} \in \mathcal{AG}$



Consider a formula of a finitary **S5** theory.

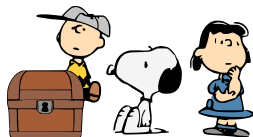


Formula type:

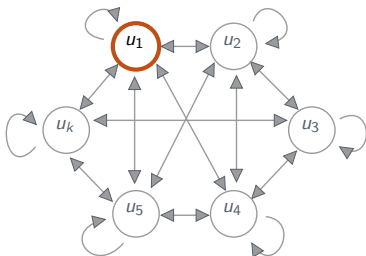
Initial state generation

Given

- $\mathcal{AG} = \{\text{Snoopy, Charlie, Lucy}\}$
- $\mathcal{F} = \{\text{opened, head, looking}_{\text{ag}}\} \text{ ag} \in \mathcal{AG}$



Consider a formula of a finitary **S5** theory.



Formula type:

- (i) ϕ

Initial state generation

Given

- $\mathcal{AG} = \{\text{Snoopy, Charlie, Lucy}\}$
- $\mathcal{F} = \{\text{opened, head, looking}_{\text{ag}}\} \text{ ag} \in \mathcal{AG}$



Consider a formula of a finitary **S5** theory.



$$u_1(\mathcal{F}) = \{\text{head, looking}_{\text{Charlie}, \text{looking}_{\text{Snoopy}}}\}$$

$$u_2(\mathcal{F}) = \{\text{looking}_{\text{Charlie}, \text{looking}_{\text{Snoopy}}}\}$$

Formula type:

- (i) ϕ
- (ii) $\mathcal{C} \phi$

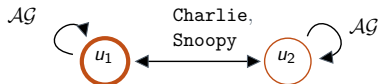
Initial state generation

Given

- $\mathcal{AG} = \{\text{Snoopy, Charlie, Lucy}\}$
- $\mathcal{F} = \{\text{opened, head, looking}_{\text{ag}}\} \text{ ag} \in \mathcal{AG}$



Consider a formula of a finitary **S5** theory.



$$u_1(\mathcal{F}) = \{\text{head, looking}_{\text{Charlie}, \text{looking}_{\text{Snoopy}}}\}$$

$$u_2(\mathcal{F}) = \{\text{looking}_{\text{Charlie}, \text{looking}_{\text{Snoopy}}}\}$$

Formula:

$$C (B_{\text{Lucy}} \text{head} \vee B_{\text{Lucy}} \neg \text{head})$$

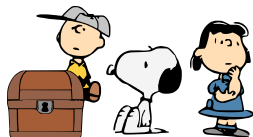
Formula type:

- (i) ϕ
- (ii) $C \phi$
- (iii) $C (B_i \phi \vee B_i \neg \phi)$

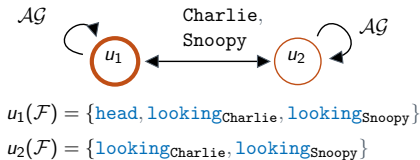
Initial state generation

Given

- $\mathcal{AG} = \{\text{Snoopy, Charlie, Lucy}\}$
- $\mathcal{F} = \{\text{opened, head, looking}_{\text{ag}} \mid \text{ag} \in \mathcal{AG}\}$



Consider a formula of a finitary **S5** theory.



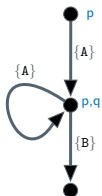
Formula type:

- (i) ϕ
- (ii) $C \phi$
- (iii) $C (B_i \phi \vee B_i \neg \phi)$
- (iv) $C (\neg B_i \phi \wedge \neg B_i \neg \phi)$

From Possibilities to Kripke Structures

Considering a possibility

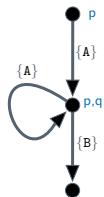
A possibility



From Possibilities to Kripke Structures

Considering a possibility
 → Can be expressed as a *system of equations*

A possibility



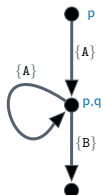
Its system of equation

$$\left\{ \begin{array}{ll} w(p) = 1 & w(q) = 0 \\ v(p) = 1 & v(q) = 1 \\ u(p) = 0 & u(q) = 0 \\ w(A) = \{v\} & w(B) = \{\emptyset\} \\ v(A) = \{v\} & v(B) = \{u\} \\ u(A) = \{\emptyset\} & u(B) = \{\emptyset\} \end{array} \right.$$

From Possibilities to Kripke Structures

Considering a possibility
 → Can be expressed as a *system of equations*
 → Systems of equations have unique solutions

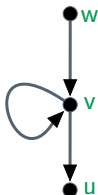
A possibility



Its system of equation

$$\left\{ \begin{array}{ll} w(p) = 1 & w(q) = 0 \\ v(p) = 1 & v(q) = 1 \\ u(p) = 0 & u(q) = 0 \\ w(A) = \{v\} & w(B) = \{\emptyset\} \\ v(A) = \{v\} & v(B) = \{u\} \\ u(A) = \{\emptyset\} & u(B) = \{\emptyset\} \end{array} \right.$$

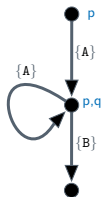
The solution



From Possibilities to Kripke Structures

- Considering a possibility
- Can be expressed as a *system of equations*
 - Systems of equations have unique solutions
 - The solution decorates a Kripke structure

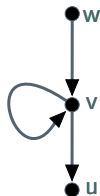
A possibility



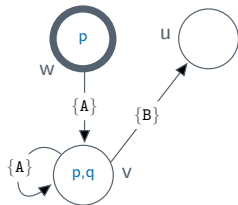
Its system of equation

$$\left\{ \begin{array}{ll} w(p) = 1 & w(q) = 0 \\ v(p) = 1 & v(q) = 1 \\ u(p) = 0 & u(q) = 0 \\ w(A) = \{v\} & w(B) = \{\emptyset\} \\ v(A) = \{v\} & v(B) = \{u\} \\ u(A) = \{\emptyset\} & u(B) = \{\emptyset\} \end{array} \right.$$

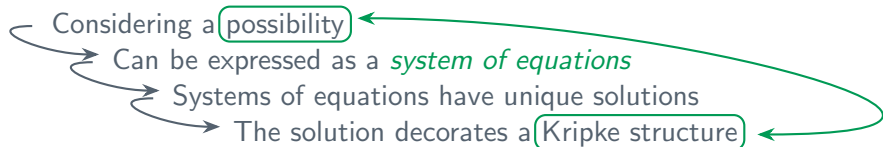
The solution



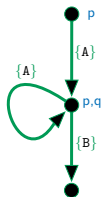
Relative Kripke Structure



From Possibilities to Kripke Structures



A possibility



Its system of equation

$$\begin{cases} w(p) = 1 & w(q) = 0 \\ v(p) = 1 & v(q) = 1 \\ u(p) = 0 & u(q) = 0 \\ w(A) = \{v\} & w(B) = \{\emptyset\} \\ v(A) = \{v\} & v(B) = \{u\} \\ u(A) = \{\emptyset\} & u(B) = \{\emptyset\} \end{cases}$$

The solution



Relative Kripke Structure

