A GPU Implementation of Large Neighborhood Search for Solving Constraint Optimization Problems

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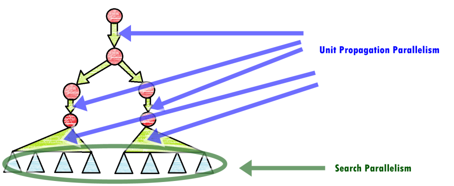
Introduction

- Every new desktop/laptop comes equipped with a powerful, programmable, graphic processor unit (GPU).
- For most of their life, however, there GPUs are absolutely idle (unless some kid is continuously playing with your PC)
- Auxiliary graphics cards can be bought with a very low price per computing core
- Their HW design is made for certain applications

Introduction

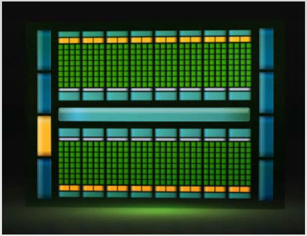
- In the last years we have experienced the use of GPUs for SAT solvers, exploiting parallelism either for deterministic computation or for non-deterministic search [CILC 2012–JETAI 2014]



- We have also used GPU for an ad-hoc implementation of LS solver for the protein structure prediction problem [ICPP13]
- We present here how we have converted our previous experience in the developing of a constraint solver with LNS.

GPUs, in few minutes

A GPU is a parallel machine with a lot of computing cores, with shared and local memories, able to schedule the execution of a large number of threads.

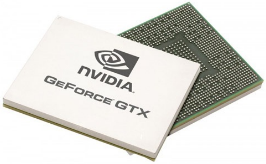


However, things are not that easy. Cores are organized hierarchically, and slower than CPUs, memories have different behaviors, . . . *it's not easy to obtain a good speed-up*

Do not reason as: 394 cores $\Rightarrow \sim 400\times$

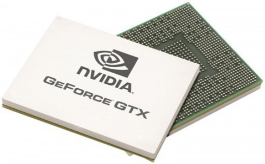
$10\times$ would be great!!!

CUDA: Compute Unified Device Architecture



CUDA: Compute Unified Device Architecture

HOST



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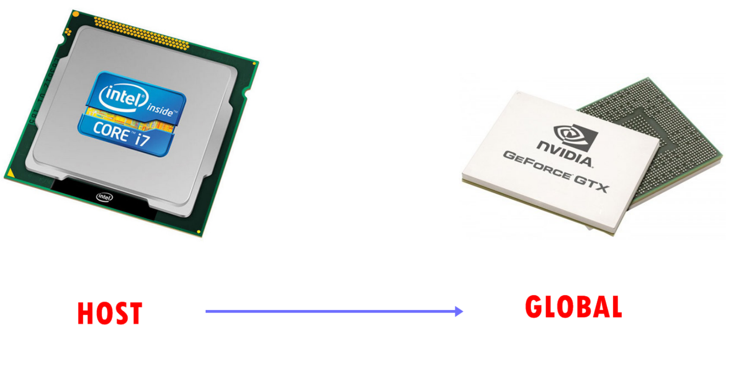

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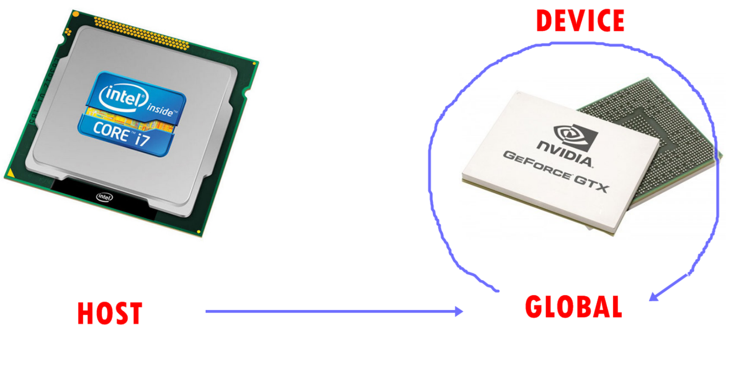

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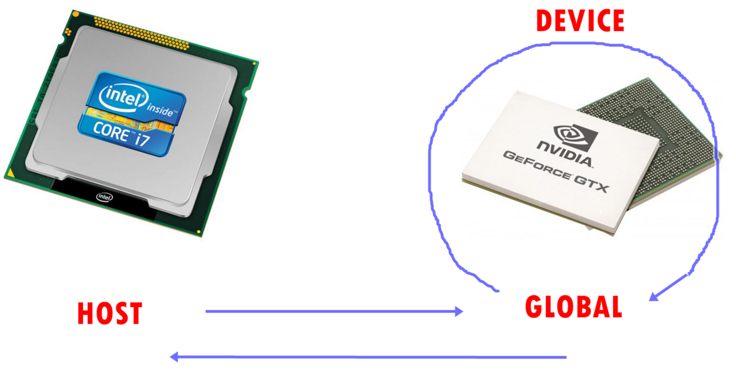

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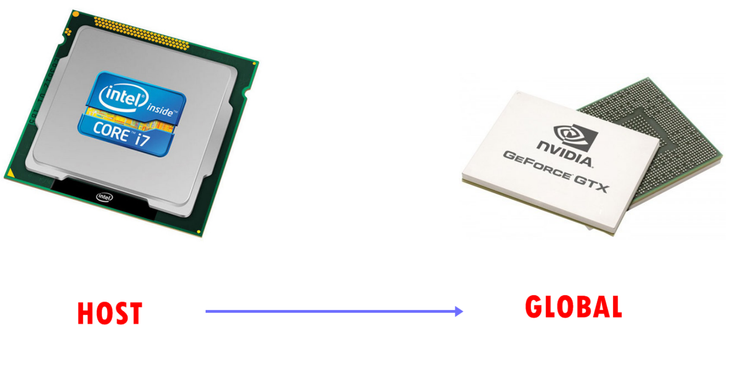

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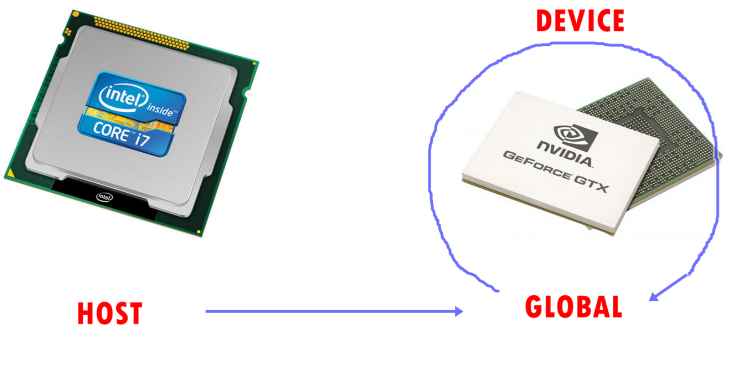
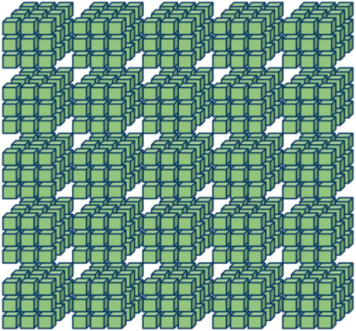


CUDA: Grids, Blocks, threads

- When a global (*kernel*) function is invoked, the number of parallel executions is established
- The set of all these executions is called a *grid*.
- A grid is organized in *blocks*
- A block is organized in a number of *threads*.
- The thread is therefore the *basic parallel unit* and it has a unique identifier (an integer number, a pair, or a triple):
  - its block `blockIdx` and
  - its position in the block `threadIdx`.
- This identifier is typically used to address different portions of a matrix
- The scheduler works with sets of 32 threads (warp) per time. A warp used SIMD (Single Instruction Multiple Data) in a warp: this must be exploited!

CUDA: Host, Global, Device



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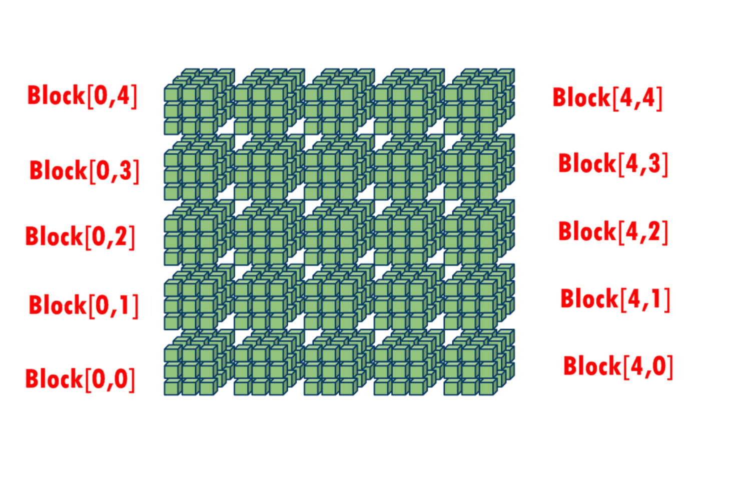

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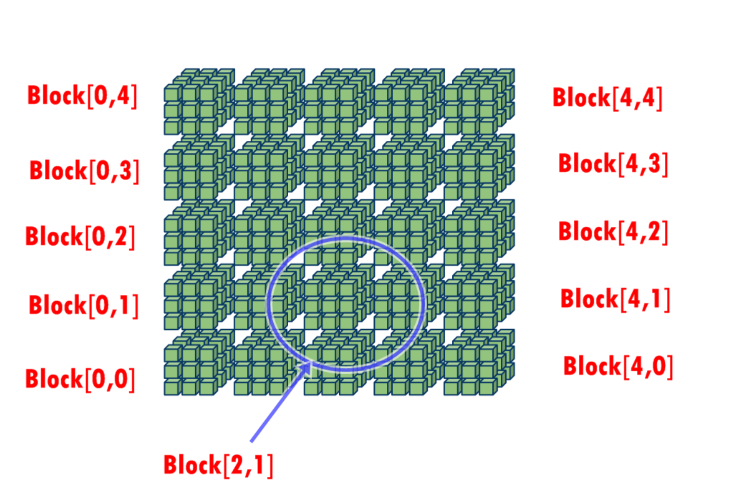

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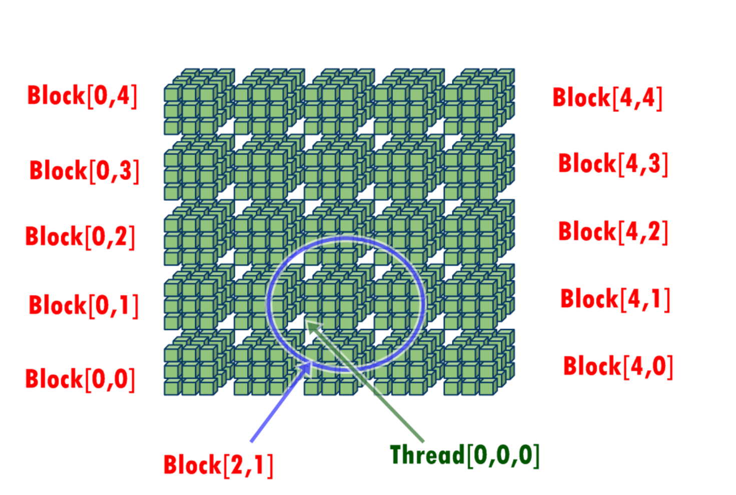

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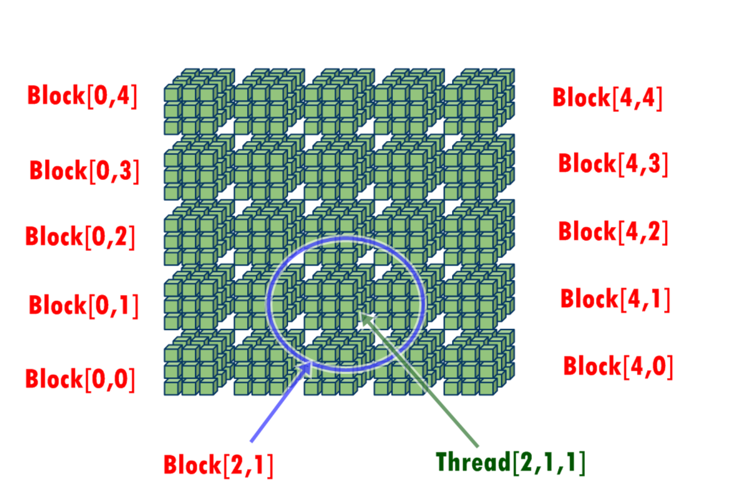
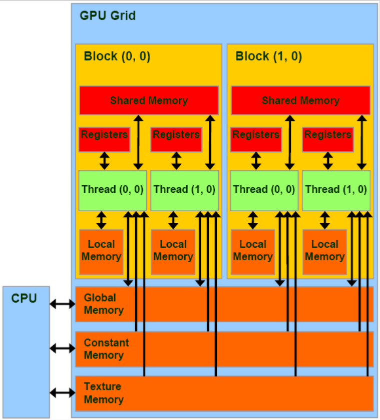


CUDA: Memories

The device memory architecture is rather involved, with 6 different types of memory (plus a new feature in CUDA 6)



The Solver iNVIDIOSO

NVIDIa-based cOnstraint SOlver

- Modeling Language: *MiniZinc*, to define a COP $\langle \vec{X}, \vec{D}, C, f \rangle$
- Translation from MiniZinc to *FlatZinc* is made by standard front-end (available in the MiniZinc distribution)
- We implemented *propagators* for “simple” FlatZinc constraints (most of them!)
- plus specific propagators for *some* global constraints
- There is a *device* function for each propagator (plus some alternatives)
- MiniZinc is becoming the *standard* constraint modeling language (e.g., for competitions)

The Solver iNVIDIOSO

Recent and current work

- We are exploiting GPUs for constraint propagation (more effective for “complex” constraints)
- We have comparable running time w.r.t. state of the art propagators (JaCoP, Gecode) but sensible speed-ups for some global constraints such as *table [PADL2014]*

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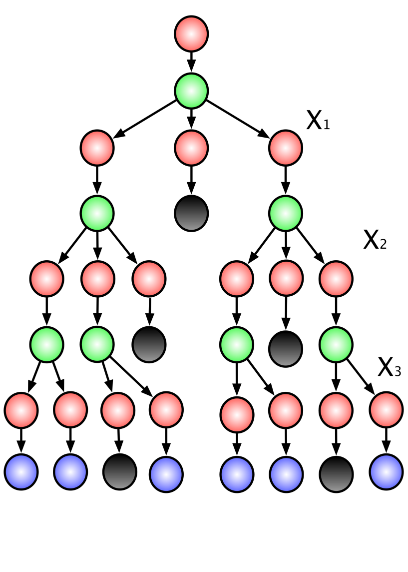
- We have not (yet) implemented a real-complete parallel search (GPU SIMT is not made for that even if SAT experiments show that for suitable sizes it can work)
- Rather, we have implemented a Large Neighborhood Search (LNS) on GPU *[this contribution]*
- LNS hybridizes Constraint Programming and Local Search for solving optimization problems (COPs).
- Exploring a neighborhood for improving assignments fits with GPU parallelism

Small and Large Neighboorhood with CP



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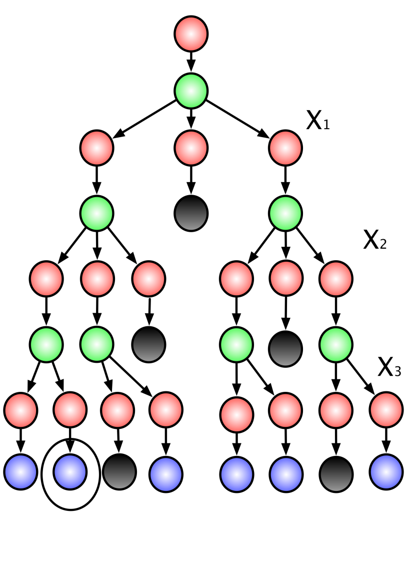

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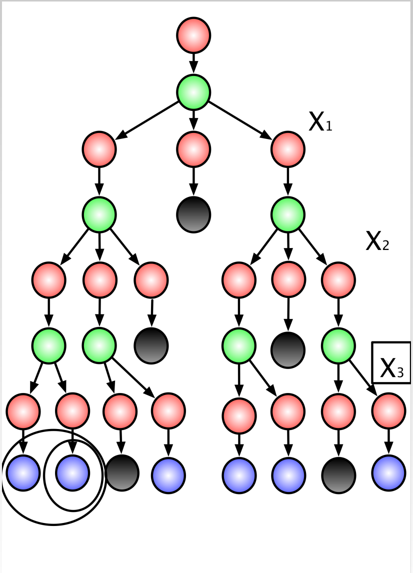

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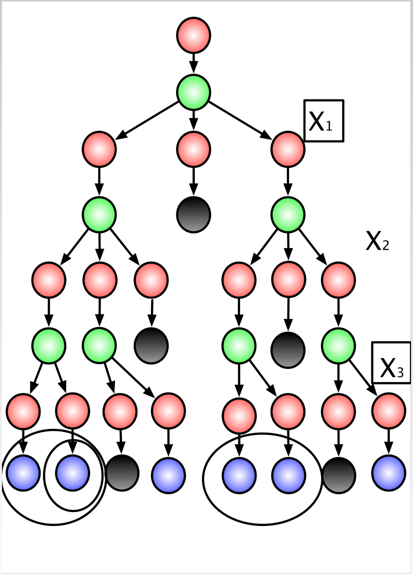

Large Neighborhood Search

- Given a solution $\vec{s}$ for the COP $\langle \vec{X}, \vec{D}, C, f \rangle$ we can "unassign" some of the variables, say $\mathcal{N} \subseteq \vec{X}$
- The set of values for $\mathcal{N}$ that are a solution of the COP constitutes a *neighborhood* of $\vec{s}$ (including $\vec{s}$)
- Given the COP, $\mathcal{N}$ identifies uniquely a neighborhood (that should be explored)

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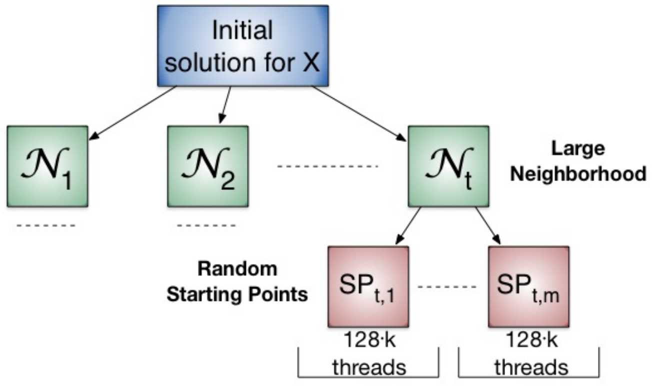
- With GPUs we can consider many (large) neighborhoods in parallel each of them randomly chosen
- For each of them we consider different "starting points" (randomly chosen) from which starting the exploration of the neighborhood.
- We use parallelism to implement local search (and constraint propagation) within each neighborhood considering each starting point to cover (sample) large parts of the search space.

LNS: implementation

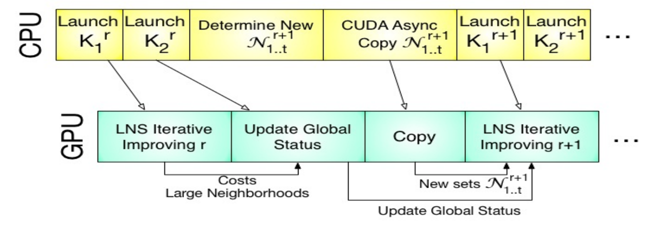
Parallelizing local search



LNS: implementation

Some details

- All constraints and initial domains are communicated to the GPU *once, at the beginning of the computation*
- The CPU calls a sequence of kernels K_i^r with $t \cdot m$ blocks (t subsets, m fixed number of initial assignments). r ranges with the number of improving steps.
- A block contains $128k$ threads ($1 \leq k \leq 8$ fixed)—$4k$ warps
- CPU and GPU work in parallel



LNS: implementation

Within each block

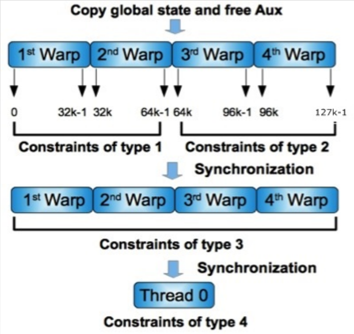
A block contains 128k threads, i.e., 4k warps (for simplicity assume now $k = 1$)

VARIABLES:

- FD (from the model)
- OBJ (one)
- AUX (for the obj function)

CONSTRAINTS:

- involving FD only
- involving FD and 1 AUX
- involving 2 or more AUX
- involving OBJ



LS techniques implemented

Random Labeling: randomly assigns values to the variables of the neighborhoods (i.e., MonteCarlo), possibly propagating constraints after each single assignment

Random Permutation: random permutation of the starting points

Two-exchange Permutations: swaps the values of all the pairs of variables in a neighborhood

Gibbs Sampling: Markov Chain Monte Carlo algorithm, used to solve a maximum a-posteriori estimation problem. Let s be the current solution and ν its cost. *for each* variable x in $\mathcal{N}$, choose a *random* candidate $d \in D^x \setminus \{s(x)\}$; then determine the new value ν' of the cost function, and accept or reject the candidate d with probability $\frac{\nu'}{\nu}$. Repeat p times.

Iterated Conditional Mode: similar to Gibbs but it performs gradient descent (hill climbing)

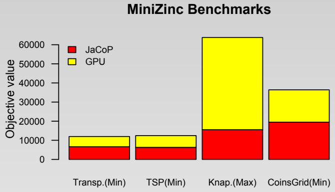
Complete exploration: try all possible combinations of assignments (unpractical!)

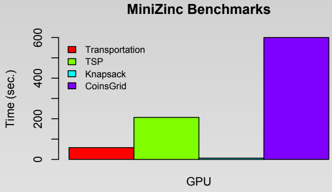
LNS: results

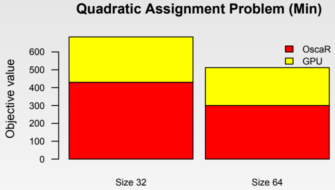
- LNS is developed for GPU.
- However, we have made some tests comparing the implementation with a CPU implementation of the same technique.
- Detailed results on the paper. *Speed-up from 2.5x to 40x* (best results on random labeling and on complete assignment)
- Let us see a comparison with JaCoP and Oscar

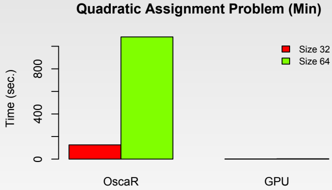
LNS: results

Graphs are one over the other, not behind!





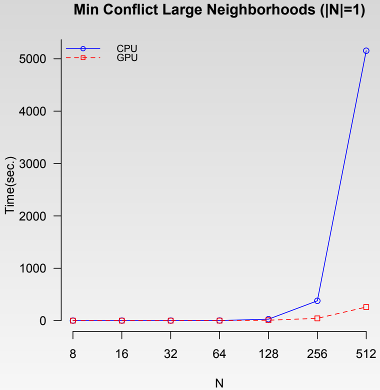


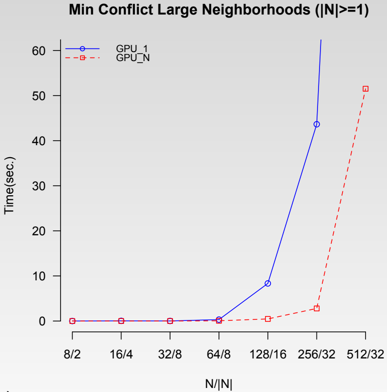


We tested CoinsGrid on OscaR (LNS). Both tools reach the timeout (600 s); we compute 25036 while Oscar 123262 (5x).

NEW: Some results on work in progress

LEFT: standard implementation of min-conflict ($|\mathcal{N}_i| = 1$)
RIGHT: Min Conflict Heuristic on Large Neighborhoods.





Tests on N queens (naive modeling)

Conclusions

- We have developed a constraint solver running (mostly) on GPU.
- Speed-up wrt sequential implementation
- Comparable with state-of-the-art solvers in the worst case, faster when (some) global constraints or LNS is used
- We are working for moving all computation to the GPU
- and/or we will try to exploit the (new, in CUDA 6) Unified Memory
- Standard (complete) search options and other basic constraints are now implemented
- GPUs will be used for parallel “search look-ahead” for choosing dynamically the most promising search strategy for a complete search
- The parallel propagation of other global constraints (e.g., alldifferent, circuit, cumulative, sets) will be soon investigated

Extra slides

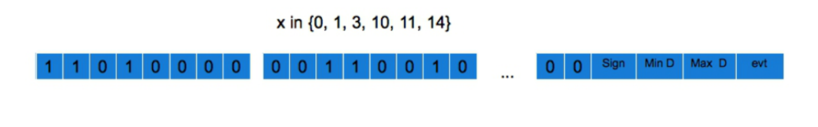
Just in case . . .

Some Remarks

- Heuristics chosen for the test with JaCoP are those that perform better for JaCoP (combination of first-fail/indomain_min, etc).
- TSP instances are on 240 cities (and some flux constraints)
- Knapsack instances are of 100 elements and made hard using an on-line generator (link in the paper) — few constraints.
- CoinsGrid problem instead has many constraints

Domain Representation

- Domain as a Bitset
- 4 extra variables are used: (1) sign, (2) min, (3) max, (4) event
- The use of bit-wise operators on domains reduces the differences between the GPU cores and the CPU cores



- Offsets are used (e.g. if $x \in \{-1000, -999\}$)
- The status is stored in a vector of nM integer (M a multiple of 32, n number of variables)