A foundational view of co-LP

Davide Ancona[1] and Agostino Dovier[2]

[1]Università di Genova

[2]Università di Udine

CILC 2013
Catania
September 2013

Motivation

- Several proposal by Gupta et al. on conductive logic programming, conductive logic programming with negation, conductive logic programming with constraints, applications of conductive logic programming, (from now on, simply co-LP)
- ...

Motivation

- Several proposal by Gupta et al. on conductive logic programming, conductive logic programming with negation, conductive logic programming with constraints, applications of conductive logic programming, (from now on, simply co-LP)
- ...
- Some serious issues about the semantics
- Some issues about the proposed co-SLD procedure
- Some issues (easy to check) on the completeness of the interpreter
- Some (inherited) issues about its correctness if negation is used

Outline

- Formal results on decidability for co-LP
- A simple operational semantics for co-LP
- Correctness based on the semantics of infinite tree LP (Jaffar, Stuckey)
- Completeness?

Fixpoints
Quick refresh

Let P be a definite clause ground program and I a set of atoms. Then

$$T_P(I) = \{a : (a \leftarrow b_1, \ldots, b_n) \in P \wedge \{b_1, \ldots, b_n\} \subseteq I\}$$

Fixpoints

Quick refresh

Let P be a definite clause ground program and I a set of atoms. Then

$$T_P(I) = \{a : (a \leftarrow b_1, \ldots, b_n) \in P \wedge \{b_1, \ldots, b_n\} \subseteq I\}$$

```
p.
q :- q.
r :- p, q, s.
```

Fixpoints
Quick refresh

Let P be a definite clause ground program and I a set of atoms. Then

$$T_P(I) = \{a : (a \leftarrow b_1, \ldots, b_n) \in P \land \{b_1, \ldots, b_n\} \subseteq I\}$$

```
p.
q :- q.
r :- p, q, s.
```

$T_P(\emptyset) = \{p\}$,

Fixpoints
Quick refresh

Let P be a definite clause ground program and I a set of atoms. Then

$$T_P(I) = \{a : (a \leftarrow b_1, \ldots, b_n) \in P \wedge \{b_1, \ldots, b_n\} \subseteq I\}$$

```
p.
q :- q.
r :- p, q, s.
```

$T_P(\emptyset) = \{p\}, T_P(\{p\}) = \{p\} = lfp(T_P)$

Fixpoints

Quick refresh

Let P be a definite clause ground program and I a set of atoms. Then

$$T_P(I) = \{a : (a \leftarrow b_1, \dots, b_n) \in P \wedge \{b_1, \dots, b_n\} \subseteq I\}$$

```
p.
q :- q.
r :- p, q, s.
```

$T_P(\emptyset) = \{p\}, T_P(\{p\}) = \{p\} = \mathit{lfp}(T_P)$

$T_P(\{p, q, r, s\}) = \{p, q, r\}$

Fixpoints
Quick refresh

Let P be a definite clause ground program and I a set of atoms. Then

$$T_P(I) = \{a : (a \leftarrow b_1, \ldots, b_n) \in P \wedge \{b_1, \ldots, b_n\} \subseteq I\}$$

```
p.
q :- q.
r :- p, q, s.
```

$T_P(\emptyset) = \{p\}, T_P(\{p\}) = \{p\} = \mathit{lfp}(T_P)$

$T_P(\{p, q, r, s\}) = \{p, q, r\}, T_P(\{p, q, r\}) = \{p, q\}$

Fixpoints

Quick refresh

Let P be a definite clause ground program and I a set of atoms. Then

$$T_P(I) = \{a : (a \leftarrow b_1, \ldots, b_n) \in P \wedge \{b_1, \ldots, b_n\} \subseteq I\}$$

```
p.
q :- q.
r :- p, q, s.
```

$T_P(\emptyset) = \{p\}, T_P(\{p\}) = \{p\} = \mathit{lfp}(T_P)$

$T_P(\{p, q, r, s\}) = \{p, q, r\}, T_P(\{p, q, r\}) = \{p, q\},$
$T_P(\{p, q\}) = \{p, q\} = \mathit{gfp}(T_P)$

co-LP in a nutshell
Syntax

- Let us focus on the *pure* co-LP (Gupta et al. 1996)
- A co-LP program is a *definite clause program*.
- Namely a set of definite clauses

$$A \leftarrow B_1, \dots, B_n$$

  where $n \geq 0$ and A and B_i are f.o. atomic formulas (atoms)
- The "standard" semantics of Logic Programming is based on $lfp(T_P)$: a r.e. complete set, in general.
- The semantics of co-LP, instead is based on the greatest fix point

co-LP in a nutshell

Syntax

- Let us focus on the *pure* co-LP (Gupta et al. 1996)
- A co-LP program is a *definite clause program*.
- Namely a set of definite clauses

$$A \leftarrow B_1, \ldots, B_n$$

  where $n \geq 0$ and A and B_i are f.o. atomic formulas (atoms)
- The "standard" semantics of Logic Programming is based on $\mathit{lfp}(T_P)$: a r.e. complete set, in general.
- The semantics of co-LP, instead is based on the greatest fix point
- By the way, since the idea is to capture perpetual processes, this fix point is computed on the extension of the Herbrand Universe that consider *infinite* terms, as well.

co-LP in a nutshell

Notions from Lloyd, 1987

- complete Herbrand Universe co-U_P: the set of finite and infinite terms built over functional symbols and variables
  - rational terms: can be represented by a *finite system of term equations*
    Example: $\Omega = s(s(s(\cdots)))$ is represented by $X = s(X)$
  - non rational terms: cannot be represented by a finite system of term equations. Example: $[\underline{0}, \underline{1}, \underline{2}, \underline{3}, \ldots]$ ($\underline{0} = \emptyset, \underline{n+1} = s(\underline{n})$)
- complete Herbrand base co-B_P: the set of all (possibly infinite, ground) atoms built on predicate symbols and terms in co-U_P
- complete ground program co-$ground(P)$: the set of all instances of clauses of P where all variables are replaced by (possibly infinite) terms in co-U_P

co-LP in a nutshell

gfp-based semantics

- *model-theoretical semantics* of a definite clause program P
    - $T_P^{\mathrm{co}} : \wp(\text{co-}B_P) \longrightarrow \wp(\text{co-}B_P)$
      $T_P^{\mathrm{co}}(I) = \{a : (a \leftarrow b_1, \ldots, b_n) \in \text{co-ground}(P) \wedge \{b_1, \ldots, b_n\} \subseteq I\}$
    - $P \models_{\mathrm{co}} a$ ($a \in \text{co-}B_P$) if and only if $a \in \mathit{gfp}(T_P^{\mathrm{co}})$
    - $P \models_{\mathrm{co}} A$ (A atom possibly with variables) if and only if for all tree substitutions $\gamma : \mathrm{FV}(A) \longrightarrow \text{co-}U_P$, $P \models_{\mathrm{co}} A\gamma$ holds

Iterated T_P^{co}

$$
\begin{array}{lcll}
T_P^{co} \uparrow 0 & = & \emptyset & \\
T_P^{co} \uparrow \alpha & = & T_P^{co}(T_P^{co} \uparrow (\alpha - 1)) & \text{if } \alpha \text{ is a successor ordinal} \\
T_P^{co} \uparrow \alpha & = & \bigcup_{\beta<\alpha} T_P^{co} \uparrow \beta & \text{if } \alpha \text{ is a limit ordinal} \\
\hline
T_P^{co} \downarrow 0 & = & \text{co-}B_P & \\
T_P^{co} \downarrow \alpha & = & T_P^{co}(T_P^{co} \downarrow (\alpha - 1)) & \text{if } \alpha \text{ is a successor ordinal} \\
T_P^{co} \downarrow \alpha & = & \bigcap_{\beta<\alpha} T_P^{co} \downarrow \beta & \text{if } \alpha \text{ is a limit ordinal}
\end{array}
$$

Important property: $gfp(T_P^{co}) = T_P^{co} \downarrow \omega$

Iterated T_P^{co}

$$
\begin{array}{lcll}
T_P^{co} \uparrow 0 & = & \emptyset & \\
T_P^{co} \uparrow \alpha & = & T_P^{co}(T_P^{co} \uparrow (\alpha - 1)) & \text{if } \alpha \text{ is a successor ordinal} \\
T_P^{co} \uparrow \alpha & = & \bigcup_{\beta<\alpha} T_P^{co} \uparrow \beta & \text{if } \alpha \text{ is a limit ordinal} \\
\hline
T_P^{co} \downarrow 0 & = & \text{co-}B_P & \\
T_P^{co} \downarrow \alpha & = & T_P^{co}(T_P^{co} \downarrow (\alpha - 1)) & \text{if } \alpha \text{ is a successor ordinal} \\
T_P^{co} \downarrow \alpha & = & \bigcap_{\beta<\alpha} T_P^{co} \downarrow \beta & \text{if } \alpha \text{ is a limit ordinal}
\end{array}
$$

Important property: $gfp(T_P^{co}) = T_P^{co} \downarrow \omega$

Remark1: this property does not hold for T_P and finite terms

Remark2: this property does not hold for T_P^{co} if $\neq$ is allowed in the clauses

SLD with rational terms

Jaffar and Stuckey generalized SLD derivation — 1986 for Prolog II

Main ideas: unification without occurs check and use of a constraint store. For instance $P = p(X) \leftarrow p(s(X))$.

- $T_P \uparrow \omega = T_P^{co} \uparrow \omega = lfp(T_P^{co}) = \emptyset$
- $T_P^{co} \downarrow \omega = gfp(T_P^{co}) = \text{co-}B_P = \{p(\Omega), p(\underline{0}), p(\underline{1}), p(\underline{2}), p(\underline{3}), \dots\}$

1. Infinite derivation for $p(\Omega)$

$$\begin{array}{l}\langle\{X = s(X)\} \,\square\, p(X)\rangle \vdash_{\infty} \\ \langle\{X = s(X), X_1 = X\} \,\square\, p(s(X_1))\rangle \vdash_{\infty} \\ \langle\{X = s(X), X_1 = X, X_2 = s(X_1)\} \,\square\, p(s(X_2))\rangle \vdash_{\infty} \dots\end{array}$$

2. Infinite derivation for $p(0)$

$$\begin{array}{l}\langle\emptyset \,\square\, p(0)\rangle \vdash_{\infty} \\ \langle\{X_1 = 0\} \,\square\, p(s(X_1))\rangle \vdash_{\infty} \\ \langle\{X_1 = 0, X_2 = s(X_1)\} \,\square\, p(s(X_2))\rangle \vdash_{\infty} \dots\end{array}$$

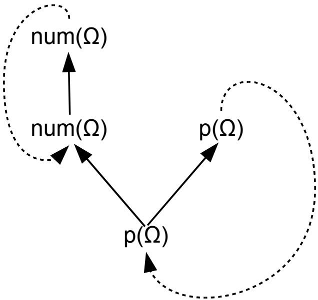
Operational semantics of co-LP

Gupta et. al. 2006

- Based on a state transition system which builds rational proof trees
- Example:

$$num(s(X)) \leftarrow num(X).$$
$$p(s(X)) \leftarrow num(X), p(s(X)).$$

Proof tree for $p(\Omega)$:



Operational semantics of co-LP

Gupta et. al. 2006, formally

- a state is a pair (T, E), where T is a finite tree with nodes labeled with atoms, and E is a system of term equations
- a state (T, E) transitions to another state (T', E') by transition rule R of program P whenever:
  1. R is a definite clause of the form $p(t'_0, \ldots, t'_n) \leftarrow B_1, \ldots, B_m$ and $E' = \{t_1 = t'_1, \ldots, t_n = t'_n\} \cup E$ is solvable, and T' is obtained from T according to the following case analysis of m:
     1. $m = 0$ implies T' is obtained from T by removing a leaf labeled $p(t_1, \ldots, t_n)$ and the maximum number of its ancestors, such that the result is still a tree.
     2. $m > 0$ implies T' is obtained from T by adding children $B_1, \ldots, B_m$ to a leaf labeled with $p(t_1, \ldots, t_n)$.
  2. R is of the form $\nu(m)$, a leaf N in T is labeled with $p(t_1, \ldots, t_n)$, the proper ancestor of N at depth m is labeled with $p(t'_1, \ldots, t'_n)$, $E' = \{t_1 = t'_1, \ldots, t_n = t'_n\} \cup E$ is solvable, then T' is obtained from T by removing N and the maximum number of its ancestors, such that the result is still a tree.

Operational semantics of co-LP

Our proposal

- *hypothetical goal* (Bonatti, Pontelli, Son): $\langle E \,\square\, (A_1, S_1), \ldots, (A_n, S_n)\rangle$, where A_i are atoms and S_i are the associated hypotheses (set of atoms)
- derivation step from $G = \langle E \,\square\, (A_1, S_1), \ldots, (A_n, S_n)\rangle$ to G' for P: select atom $A_i = p(s_1, \ldots, s_n)$, with hypotheses S_i and apply one of the following rules:
  1. let $p(t_1, \ldots, t_n) \leftarrow B_1, \ldots, B_m$ be a renaming of a clause in P with fresh variables, and let $E' = E \cup \{s_1 = t_1, \ldots, s_n = t_n\}$ be solvable. Then $G' = \langle E' \,\square\, (A_1, S_1), \ldots, (A_{i-1}, S_{i-1}), (B_1, S'), \ldots, (B_m, S'), (A_{i+1}, S_{i+1}), \ldots, (A_n, S_n)\rangle$ where $S' = S_i \cup \{p(s_1, \ldots, s_n)\}$.
  2. let $p(t_1, \ldots, t_n) \in S_i$ be such that $E' = E \cup \{s_1 = t_1, \ldots, s_n = t_n\}$ is solvable. Then $G' = \langle E' \,\square\, (A_1, S_1), \ldots, (A_{i-1}, S_{i-1}), (A_{i+1}, S_{i+1}), \ldots, (A_n, S_n)\rangle$.

Operational semantics of co-LP

Our proposal

- *hypothetical goal* (Bonatti, Pontelli, Son): $\langle E \,\square\, (A_1, S_1), \ldots, (A_n, S_n)\rangle$, where A_i are atoms and S_i are the associated hypotheses (set of atoms)
- derivation step from $G = \langle E \,\square\, (A_1, S_1), \ldots, (A_n, S_n)\rangle$ to G' for P: select atom $A_i = p(s_1, \ldots, s_n)$, with hypotheses S_i and apply one of the following rules:
  1. let $p(t_1, \ldots, t_n) \leftarrow B_1, \ldots, B_m$ be a renaming of a clause in P with fresh variables, and let $E' = E \cup \{s_1 = t_1, \ldots, s_n = t_n\}$ be solvable. Then $G' = \langle E' \,\square\, (A_1, S_1), \ldots, (A_{i-1}, S_{i-1}), (B_1, S'), \ldots, (B_m, S'), (A_{i+1}, S_{i+1}), \ldots, (A_n, S_n)\rangle$ where $S' = S_i \cup \{p(s_1, \ldots, s_n)\}$.
  2. let $p(t_1, \ldots, t_n) \in S_i$ be such that $E' = E \cup \{s_1 = t_1, \ldots, s_n = t_n\}$ is solvable. Then $G' = \langle E' \,\square\, (A_1, S_1), \ldots, (A_{i-1}, S_{i-1}), (A_{i+1}, S_{i+1}), \ldots, (A_n, S_n)\rangle$.
- a SWI-Prolog meta-interpreter has been implemented directly from the 2 rules given above

Operational semantics of co-LP

Our proposal

$$num(s(X)) \leftarrow num(X).$$
$$p(s(X)) \leftarrow num(X), p(s(X)).$$

Example of successful derivation:

$$\langle \{X = s(X)\} \,\square\, (p(X), \emptyset)\rangle \vdash_{\text{co}}$$

$$\langle \{X = s(X), X = s(X_1)\} \,\square\, (num(X_1), \{p(X)\}), (p(s(X_1)), \{p(X)\})\rangle \vdash_{\text{co}}$$

$$\langle \{X = s(X), X = s(X_1), X_1 = s(X_2)\} \,\square\, (num(X_2), \{p(X), num(X_1)\}), (p(s(X_1)), \{p(X)\})\rangle \vdash_{\text{co}}$$

$$\langle \{X = s(X), X = s(X_1), X_1 = s(X_2), X_2 = X_1\} \,\square\, (p(s(X_1)), \{p(X)\})\rangle \vdash_{\text{co}}$$

$$\langle \{X = s(X), X = s(X_1), X_1 = s(X_2), X_2 = X_1, s(X_1) = X\} \,\square\, \epsilon\rangle \vdash_{\text{co}}$$

Correctness

JS86 + “Pumping Lemma”

- Let P be a definite clause program. If there is a successful (hence finite) $\vdash_{\text{co}}$ derivation for $\langle E \,\Box\, (A, \emptyset)\rangle$ with c.a.s. θ, then $P \models_{\text{co}} A\gamma$ for every term substitution γ solution of $E\theta$.
- proof sketch:
  - if only rule 1 is applied, then the derivation is equivalent to a $\vdash_{\infty}$ derivation, and correctness directly follows from Jaffar and Stuckey results
  - if rule 2 is employed at least once, the proof is similar to that of the *pumping lemma*: a finite successful derivation can be transformed into an infinite derivation using only rule 1, which is, therefore, equivalent to a $\vdash_{\infty}$ derivation

Decidability issues

$$P = p(X) \leftarrow p(s(X)).$$

- $T_P^{\text{co}} \downarrow \omega = gfp(T_P^{\text{co}}) = \text{co-}B_P$
- the derivation for $p(\Omega)$ is finite and successful
- the derivation for $p(0)$ is infinite!
- is it possible to define a correct operational semantics for which there exists a finite successful derivation for $p(0)$?

Decidability issues

$$P = p(X) \leftarrow p(s(X)).$$

- $T_P^{\text{co}} \downarrow \omega = gfp(T_P^{\text{co}}) = \text{co-}B_P$
- the derivation for $p(\Omega)$ is finite and successful
- the derivation for $p(0)$ is infinite!
- is it possible to define a correct operational semantics for which there exists a finite successful derivation for $p(0)$?
- Maybe, but unfortunately this is not possible in general!

Formal results on (un)decidability

- $\Upsilon(S)$ denotes the subset of S containing only rational terms
- Theorem:
  1. $\Upsilon(T_P^{co} \uparrow \omega)$ is recursively enumerable complete
  2. $\Upsilon(\text{co-}B_P \setminus T_P^{co} \downarrow \omega)$ is recursively enumerable complete (hence, $\Upsilon(T_P^{co} \downarrow \omega)$ is productive).

  Proof of (1): follows from known results.

  Proof of (2): standard reduction from $\bar{K}$ (building a suitable Prolog program s.t. $x \in \bar{K}$ iff $p(\underline{x}) \in gfp(T_P^{co})$)

Formal results on (un)decidability

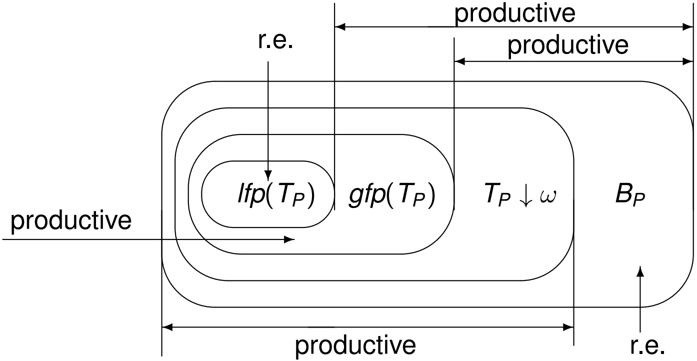
- $\Upsilon(S)$ denotes the subset of S containing only rational terms
- Theorem:
  1. $\Upsilon(T_P^{co} \uparrow \omega)$ is recursively enumerable complete
  2. $\Upsilon(\text{co-}B_P \setminus T_P^{co} \downarrow \omega)$ is recursively enumerable complete (hence, $\Upsilon(T_P^{co} \downarrow \omega)$ is productive).

  Proof of (1): follows from known results.

  Proof of (2): standard reduction from $\bar{K}$ (building a suitable Prolog program s.t. $x \in \bar{K}$ iff $p(\underline{x}) \in gfp(T_P^{co})$)

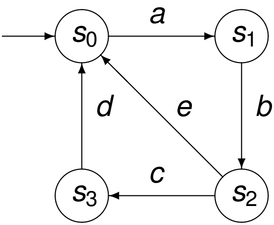
  Corollary: even when the semantics is restricted to rational terms, no complete procedure exists for establishing whether $P \models_{co} a$; however, in absence of $\neq$ symbols, there exists a complete procedure for establishing whether $P \not\models_{co} a$.

A famous picture



Example

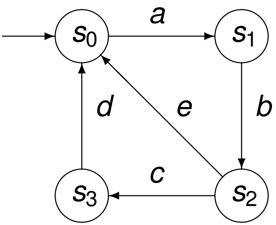
Büchi ω-automata



$delta(s0, a, s1).$
$delta(s1, b, s2).$
$delta(s2, c, s3).$
$delta(s2, e, s0).$
$delta(s3, d, s0).$
$automata([X|T], S) :-$
$\quad delta(S, X, S1),$
$\quad automata(T, S1).$

Example

Büchi ω-automata



$delta(s0, a, s1).$
$delta(s1, b, s2).$
$delta(s2, c, s3).$
$delta(s2, e, s0).$
$delta(s3, d, s0).$
$automata([X|T], S) :-$
$\quad delta(S, X, S1),$
$\quad automata(T, S1).$

```
?- meta((automata(A,s0))).
A = [a, b, c, d|A] ;
A = [a, b, e|A] ;
A = [a, b, c, d, a, b, e|A] ;
A = [a, b, e, a, b, c, d|A] ;
...
```

Outline

- Formal results on decidability for co-LP
- A simple operational semantics for co-LP
- Correctness based on the semantics of infinite tree LP (Jaffar, Stuckey) + Pumping Lemma
- Completeness is impossible!
- Can be used for correctly detecting (some) properties

Outline

- Formal results on decidability for co-LP
- A simple operational semantics for co-LP
- Correctness based on the semantics of infinite tree LP (Jaffar, Stuckey) + Pumping Lemma
- Completeness is impossible!
- Can be used for correctly detecting (some) properties
- What about negation? And constraints?

Thank you

(We're not selling co-LP, just explaining it)