Hindawi Publishing Corporation
Discrete Dynamics in Nature and Society
Volume 2012, Article ID 169348, 8 pages
doi:10.1155/2012/169348

Research Article

q-Analogues of the Bernoulli and Genocchi Polynomials and the Srivastava-Pintér Addition Theorems

N. I. Mahmudov

Eastern Mediterranean University, Gazimagusa, TRNC, Mersin 10, Turkey

Correspondence should be addressed to N. I. Mahmudov, nazim.mahmudov@emu.edu.tr

Received 24 April 2012; Revised 5 July 2012; Accepted 23 July 2012

Academic Editor: Lee Chae Jang

The main purpose of this paper is to introduce and investigate a new class of generalized Bernoulli and Genocchi polynomials based on the q-integers. The q-analogues of well-known formulas are derived. The q-analogue of the Srivastava-Pintér addition theorem is obtained.

1. Introduction

Throughout this paper, we always make use of the following notation: $\mathbb{N}$ denotes the set of natural numbers, $\mathbb{N}_0$ denotes the set of nonnegative integers, $\mathbb{R}$ denotes the set of real numbers, and $\mathbb{C}$ denotes the set of complex numbers.

The q-shifted factorial is defined by

$$(a;q)_0 = 1, \quad (a;q)_n = \prod_{j=0}^{n-1}\left(1-q^j a\right), \quad n \in \mathbb{N}, \quad (a;q)_\infty = \prod_{j=0}^{\infty}\left(1-q^j a\right), \quad |q| < 1, \ a \in \mathbb{C}. \tag{1.1}$$

The q-numbers and q-numbers factorial is defined by

$$[a]_q = \frac{1-q^a}{1-q} \quad (q \neq 1); \quad [0]_q! = 1; \quad [n]_q! = [1]_q[2]_q \cdots [n]_q \quad n \in \mathbb{N}, \ a \in \mathbb{C}, \tag{1.2}$$

respectively. The q-polynomial coefficient is defined by

$$\begin{bmatrix} n \\ k \end{bmatrix}_q = \frac{(q;q)_n}{(q;q)_{n-k}(q;q)_k}. \tag{1.3}$$

The q-analogue of the function $(x+y)^n$ is defined by

$$(x+y)_q^n := \sum_{k=0}^{n} \begin{bmatrix} n \\ k \end{bmatrix}_q q^{(1/2)k(k-1)} x^{n-k} y^k, \quad n \in \mathbb{N}_0. \tag{1.4}$$

In the standard approach to the q-calculus two exponential function are used:

$$\begin{aligned} e_q(z) &= \sum_{n=0}^{\infty} \frac{z^n}{[n]_q!} = \prod_{k=0}^{\infty} \frac{1}{(1-(1-q)q^k z)}, \quad 0 < |q| < 1,\ |z| < \frac{1}{|1-q|}, \\ E_q(z) &= \sum_{n=0}^{\infty} \frac{q^{(1/2)n(n-1)} z^n}{[n]_q!} = \prod_{k=0}^{\infty} \left(1 + (1-q)q^k z\right), \quad 0 < |q| < 1,\ z \in \mathbb{C}. \end{aligned} \tag{1.5}$$

From this form we easily see that $e_q(z)E_q(-z) = 1$. Moreover,

$$D_q e_q(z) = e_q(z), \qquad D_q E_q(z) = E_q(qz), \tag{1.6}$$

where D_q is defined by

$$D_q f(z) := \frac{f(qz) - f(z)}{qz - z}. \tag{1.7}$$

The previous q-standard notation can be found in [1].

Carlitz has introduced the q-Bernoulli numbers and polynomials in [2]. Srivastava and Pintér proved some relations and theorems between the Bernoulli polynomials and Euler polynomials in [3]. They also gave some generalizations of these polynomials. In [4–6], Kim et al. investigated some properties of the q-Euler polynomials and Genocchi polynomials. They gave some recurrence relations. In [7], Cenkci et al. gave the q-extension of Genocchi numbers in a different manner. In [5], Kim gave a new concept for the q-Genocchi numbers and polynomials. In [8], Simsek et al. investigated the q-Genocchi zeta function and l-function by using generating functions and Mellin transformation. We also recall the definitions of the q-Bernoulli and the q-Genocchi polynomials of higher order (see [2, 9–12]):

$$\begin{aligned} (-t)^{\alpha} \sum_{n=0}^{\infty} \frac{\left([\alpha]_q\right)_n}{[n]_q!} q^{n+x} e^{t[n+x]_q} &= \sum_{n=0}^{\infty} B_{n,q}^{(\alpha)}(x) \frac{t^n}{n!}, \\ (2t)^{\alpha} \sum_{n=0}^{\infty} \frac{\left([\alpha]_q\right)_n}{[n]_q!} (-1)^n q^{n+x} e^{t[n+x]_q} &= \sum_{n=0}^{\infty} G_{n,q}^{(\alpha)}(x) \frac{t^n}{n!}. \end{aligned} \tag{1.8}$$

We propose the following definitions. We define the q-Bernoulli and the q-Genocchi polynomials of higher order in two variables x and y, using two q-exponential functions, which helps us easily prove some properties of these polynomials and q-analogue of the Srivastava and Pintér addition theorem.

Definition 1.1. The q-Bernoulli numbers $\mathfrak{B}_{n,q}^{(\alpha)}$ and polynomials $\mathfrak{B}_{n,q}^{(\alpha)}(x,y)$ in x,y of order α are defined by means of the generating function functions:

$$\begin{gathered}\left(\frac{t}{e_q(t)-1}\right)^{\alpha}=\sum_{n=0}^{\infty}\mathfrak{B}_{n,q}^{(\alpha)}\frac{t^n}{[n]_q!},\quad |t|<2\pi,\\ \left(\frac{t}{e_q(t)-1}\right)^{\alpha}e_q(tx)E_q(ty)=\sum_{n=0}^{\infty}\mathfrak{B}_{n,q}^{(\alpha)}(x,y)\frac{t^n}{[n]_q!},\quad |t|<2\pi.\end{gathered}\tag{1.9}$$

Definition 1.2. The q-Genocchi numbers $\mathfrak{G}_{n,q}^{(\alpha)}$ and polynomials $\mathfrak{G}_{n,q}^{(\alpha)}(x,y)$ in x,y are defined by means of the generating functions:

$$\begin{gathered}\left(\frac{2t}{e_q(t)+1}\right)^{\alpha}=\sum_{n=0}^{\infty}\mathfrak{G}_{n,q}^{(\alpha)}\frac{t^n}{[n]_q!},\quad |t|<\pi,\\ \left(\frac{2t}{e_q(t)+1}\right)^{\alpha}e_q(tx)E_q(ty)=\sum_{n=0}^{\infty}\mathfrak{G}_{n,q}^{(\alpha)}(x,y)\frac{t^n}{[n]_q!},\quad |t|<\pi.\end{gathered}\tag{1.10}$$

It is obvious that

$$\begin{gathered}\mathfrak{B}_{n,q}^{(\alpha)}=\mathfrak{B}_{n,q}^{(\alpha)}(0,0),\qquad \lim_{q\to1^-}\mathfrak{B}_{n,q}^{(\alpha)}(x,y)=B_n^{(\alpha)}(x+y),\qquad \lim_{q\to1^-}\mathfrak{B}_{n,q}^{(\alpha)}=B_n^{(\alpha)},\\ \mathfrak{G}_{n,q}^{(\alpha)}=\mathfrak{G}_{n,q}^{(\alpha)}(0,0),\qquad \lim_{q\to1^-}\mathfrak{G}_{n,q}^{(\alpha)}(x,y)=G_n^{(\alpha)}(x+y),\qquad \lim_{q\to1^-}\mathfrak{G}_{n,q}^{(\alpha)}=G_n^{(\alpha)}.\end{gathered}\tag{1.11}$$

Here $B_n^{(\alpha)}(x)$ and $E_n^{(\alpha)}(x)$ denote the classical Bernoulli, and Genocchi polynomials of order α are defined by

$$\left(\frac{t}{e^t-1}\right)^{\alpha}e^{tx}=\sum_{n=0}^{\infty}B_n^{(\alpha)}(x)\frac{t^n}{n!},\qquad \left(\frac{2}{e^t+1}\right)^{\alpha}e^{tx}=\sum_{n=0}^{\infty}G_n^{(\alpha)}(x)\frac{t^n}{n!}.\tag{1.12}$$

The aim of the present paper is to obtain some results for the q-Genocchi polynomials (properties of the q-Bernoulli polynomials are studied in [13]). The q-analogues of well-known results, for example, Srivastava and Pintér [3], can be derived from these q-identities. It should be mentioned that probabilistic proofs the Srivastava-Pintér addition theorems were given recently in [14]. The formulas involving the q-Stirling numbers of the second kind, q-Bernoulli polynomials and q-Bernstein polynomials, are also given. Furthermore some special cases are also considered.

The following elementary properties of the q-Genocchi polynomials $\mathfrak{E}_{n,q}^{(\alpha)}(x,y)$ of order α are readily derived from Definition 1.2. We choose to omit the details involved.

Property 1.3. Special values of the q-Genocchi polynomials of order α:

$$\mathfrak{E}_{n,q}^{(0)}(x,0) = x^n, \qquad \mathfrak{E}_{n,q}^{(0)}(0,y) = q^{(1/2)n(n-1)}y^n. \tag{1.13}$$

Property 1.4. Summation formulas for the q-Genocchi polynomials of order α:

$$\begin{gathered}
\mathfrak{E}_{n,q}^{(\alpha)}(x,y) = \sum_{k=0}^{n} \begin{bmatrix} n \\ k \end{bmatrix}_q \mathfrak{E}_{k,q}^{(\alpha)}(x+y)_q^{n-k}, \qquad \mathfrak{E}_{n,q}^{(\alpha)}(x,y) = \sum_{k=0}^{n} \begin{bmatrix} n \\ k \end{bmatrix}_q \mathfrak{E}_{n-k,q}^{(\alpha-1)} \mathfrak{E}_{k,q}(x,y), \\
\mathfrak{G}_{n,q}^{(\alpha)}(x,y) = \sum_{k=0}^{n} \begin{bmatrix} n \\ k \end{bmatrix}_q q^{(n-k)(n-k-1)/2} \mathfrak{G}_{k,q}^{(\alpha)}(x,0) y^{n-k} = \sum_{k=0}^{n} \begin{bmatrix} n \\ k \end{bmatrix}_q \mathfrak{G}_{k,q}^{(\alpha)}(0,y) x^{n-k}, \\
\mathfrak{G}_{n,q}^{(\alpha)}(x,0) = \sum_{k=0}^{n} \begin{bmatrix} n \\ k \end{bmatrix}_q \mathfrak{G}_{k,q}^{(\alpha)} x^{n-k}, \qquad \mathfrak{G}_{n,q}^{(\alpha)}(0,y) = \sum_{k=0}^{n} \begin{bmatrix} n \\ k \end{bmatrix}_q q^{(n-k)(n-k-1)/2} \mathfrak{G}_{k,q}^{(\alpha)} y^{n-k}.
\end{gathered} \tag{1.14}$$

Property 1.5. Difference equations:

$$\begin{gathered}
\mathfrak{G}_{n,q}^{(\alpha)}(1,y) + \mathfrak{G}_{n,q}^{(\alpha)}(0,y) = 2[n]_q \mathfrak{G}_{n-1,q}^{(\alpha-1)}(0,y), \\
\mathfrak{G}_{n,q}^{(\alpha)}(x,0) + \mathfrak{G}_{n,q}^{(\alpha)}(x,-1) = 2[n]_q \mathfrak{G}_{n-1,q}^{(\alpha-1)}(x,-1).
\end{gathered} \tag{1.15}$$

Property 1.6. Differential relations:

$$D_{q,x} \mathfrak{G}_{n,q}^{(\alpha)}(x,y) = [n]_q \mathfrak{G}_{n-1,q}^{(\alpha)}(x,y), \qquad D_{q,y} \mathfrak{G}_{n,q}^{(\alpha)}(x,y) = [n]_q \mathfrak{G}_{n-1,q}^{(\alpha)}(x,qy). \tag{1.16}$$

Property 1.7. Addition theorem of the argument:

$$\mathfrak{E}_{n,q}^{(\alpha+\beta)}(x,y) = \sum_{k=0}^{n} \begin{bmatrix} n \\ k \end{bmatrix}_q \mathfrak{E}_{n-k,q}^{(\alpha)}(x,0) \mathfrak{E}_{k,q}^{(\beta)}(0,y). \tag{1.17}$$

Property 1.8. Recurrence relationships:

$$\mathfrak{G}_{n,q}^{(\alpha)}\left(\frac{1}{m}, y\right) + \sum_{k=0}^{n} \begin{bmatrix} n \\ k \end{bmatrix}_q \left(\frac{1}{m} - 1\right)_q^{n-k} \mathfrak{G}_{k,q}^{(\alpha)}(0,y) = 2[n]_q \sum_{k=0}^{n-1} \begin{bmatrix} n-1 \\ k \end{bmatrix}_q \left(\frac{1}{m} - 1\right)_q^{n-1-k} \mathfrak{G}_{k,q}^{(\alpha-1)}(0,y). \tag{1.18}$$

2. Explicit Relationship between the q-Genocchi and the q-Bernoulli Polynomials

In this section we prove an interesting relationship between the q-Genocchi polynomials $\mathfrak{G}_{n,q}^{(\alpha)}(x,y)$ of order α and the q-Bernoulli polynomials. Here some q-analogues of known results will be given. We also obtain new formulas and their some special cases in the following.

Theorem 2.1. *For $n \in \mathbb{N}_0$, the following relationship*

$$\mathfrak{G}_{n,q}^{(\alpha)}(x,y) = \sum_{k=0}^{n} \frac{1}{m^{n-k-1}[k+1]_q} \left[2[k+1]_q \sum_{j=0}^{k} \begin{bmatrix} k \\ j \end{bmatrix}_q \frac{1}{m^{k-j}} \mathfrak{G}_{j,q}^{(\alpha-1)}(x,-1) - \sum_{j=0}^{k+1} \begin{bmatrix} k+1 \\ j \end{bmatrix}_q \frac{1}{m^{k+1-j}} \mathfrak{G}_{j,q}^{(\alpha)}(x,-1) - \mathfrak{G}_{k+1,q}^{(\alpha)}(x,0) \right] \mathfrak{B}_{n-k,q}(0,my) \tag{2.1}$$

holds true between the q-Genocchi and the q-Bernoulli polynomials.

Proof. Using the following identity:

$$\left(\frac{2t}{e_q(t)+1}\right)^{\alpha} e_q(tx) E_q(ty) = \left(\frac{2t}{e_q(t)+1}\right)^{\alpha} e_q(tx) \cdot \frac{e_q(t/m)-1}{t} \cdot \frac{t}{e_q(t/m)-1} \cdot E_q\left(\frac{t}{m} my\right), \tag{2.2}$$

we have

$$\begin{aligned}
\sum_{n=0}^{\infty} \mathfrak{G}_{n,q}^{(\alpha)}(x,y) \frac{t^n}{[n]_q!} &= \frac{m}{t} \sum_{n=0}^{\infty} \left(\sum_{k=0}^{n} \begin{bmatrix} n \\ k \end{bmatrix}_q \frac{1}{m^{n-k}} \mathfrak{G}_{k,q}^{(\alpha)}(x,0) - \mathfrak{G}_{n,q}^{(\alpha)}(x,0) \right) \frac{t^n}{[n]_q!} \sum_{n=0}^{\infty} \mathfrak{B}_{n,q}(0,my) \frac{t^n}{m^n [n]_q!} \\
&= \sum_{n=1}^{\infty} \left(\sum_{k=0}^{n} \begin{bmatrix} n \\ k \end{bmatrix}_q \frac{1}{m^{n-1-k}} \mathfrak{G}_{k,q}^{(\alpha)}(x,0) \right. \\
&\qquad \left. - m \mathfrak{G}_{n,q}^{(\alpha)}(x,0) \right) \frac{t^{n-1}}{[n]_q!} \sum_{n=0}^{\infty} \mathfrak{B}_{n,q}(0,my) \frac{t^n}{m^n [n]_q!} \\
&= \sum_{n=0}^{\infty} \left(\sum_{k=0}^{n+1} \begin{bmatrix} n+1 \\ k \end{bmatrix}_q m^k \mathfrak{G}_{k,q}^{(\alpha)}(x,0) \right. \\
&\qquad \left. - m^{n+1} \mathfrak{G}_{n+1,q}^{(\alpha)}(x,0) \right) \frac{t^n}{m^n [n+1]_q!} \sum_{n=0}^{\infty} \mathfrak{B}_{n,q}(0,my) \frac{t^n}{m^n [n]_q!} \\
&= \sum_{n=0}^{\infty} \sum_{k=0}^{n} \frac{1}{m^n [k+1]_q} \left(\sum_{j=0}^{k+1} \begin{bmatrix} k+1 \\ j \end{bmatrix}_q m^j \mathfrak{G}_{j,q}^{(\alpha)}(x,0) \right. \\
&\qquad \left. - m^{k+1} \mathfrak{G}_{k+1,q}^{(\alpha)}(x,0) \right) \mathfrak{B}_{n-k,q}(0,my) \frac{t^n}{[n]_q!}.
\end{aligned} \tag{2.3}$$

It remains to use Property 1.8. □

Since $\mathfrak{G}_{n,q}^{(\alpha)}(x,y)$ is not symmetric with respect to x and y, we can prove a different form of the previously mentioned theorem. It should be stressed out that Theorems 2.1 and 2.2 coincide in the limiting case when $q \to 1^-$.

Theorem 2.2. *For $n \in \mathbb{N}_0$, the following relationship*

$$\mathfrak{G}_{n,q}^{(\alpha)}(x,y) = \sum_{k=0}^{n} \begin{bmatrix} n \\ k \end{bmatrix}_q \frac{1}{m^{n-k-1}[k+1]_q} \Bigg[2[k+1]_q \sum_{j=0}^{k} \begin{bmatrix} k \\ j \end{bmatrix}_q \left(\frac{1}{m}-1\right)_q^{k-j} \mathfrak{G}_{j,q}^{(\alpha-1)}(0,y) - \sum_{j=0}^{k+1} \begin{bmatrix} k+1 \\ j \end{bmatrix}_q \left(\frac{1}{m}-1\right)_q^{k+1-j} \mathfrak{G}_{j,q}^{(\alpha)}(0,y) - \mathfrak{G}_{k+1,q}(0,y) \Bigg] \times \mathfrak{B}_{n-k,q}(mx,0) \tag{2.4}$$

holds true between the q-Genocchi and the q-Bernoulli polynomials.

Proof. The proof is based on the following identity:

$$\left(\frac{2t}{e_q(t)+1}\right)^{\alpha} e_q(tx)E_q(ty) = \left(\frac{2t}{e_q(t)+1}\right)^{\alpha} E_q(ty) \cdot \frac{e_q(t/m)-1}{t} \cdot \frac{t}{e_q(t/m)-1} \cdot e_q\left(\frac{t}{m}mx\right). \tag{2.5}$$

□

Next we discuss some special cases of Theorems 2.1 and 2.2. By noting that

$$\mathfrak{G}_{j,q}^{(0)}(0,y) = q^{(1/2)j(j-1)}y^j, \qquad \mathfrak{G}_{j,q}^{(0)}(x,-1) = (x-1)_q^j, \tag{2.6}$$

we deduce from Theorems 2.1 and 2.2 Corollary 2.3 below.

Corollary 2.3. *For $n \in \mathbb{N}_0$, $m \in \mathbb{N}$ the following relationship*

$$\mathfrak{G}_{n,q}(x,y) = \sum_{k=0}^{n} \begin{bmatrix} n \\ k \end{bmatrix}_q \frac{1}{m^{n-k-1}[k+1]_q} \Bigg[2[k+1]_q \sum_{j=0}^{k} \begin{bmatrix} k \\ j \end{bmatrix}_q \left(\frac{1}{m}-1\right)_q^{k-j} q^{(1/2)j(j-1)}y^j - \sum_{j=0}^{k+1} \begin{bmatrix} k+1 \\ j \end{bmatrix}_q \left(\frac{1}{m}-1\right)_q^{k+1-j} \mathfrak{G}_{j,q}(0,y) - \mathfrak{G}_{k+1,q}(0,y) \Bigg] \times \mathfrak{B}_{n-k,q}(mx,0),$$

$$\mathfrak{G}_{n,q}(x,y)=\sum_{k=0}^{n}\begin{bmatrix}n\\k\end{bmatrix}_q\frac{1}{m^{n-k-1}[k+1]_q}\left[2[k+1]_q\sum_{j=0}^{k}\begin{bmatrix}k\\j\end{bmatrix}_q\frac{1}{m^{k-j}}(x-1)_q^j\right.$$
$$\left.-\sum_{j=0}^{k+1}\begin{bmatrix}k+1\\j\end{bmatrix}_q\frac{1}{m^{k+1-j}}\mathfrak{G}_{j,q}(x,-1)-\mathfrak{G}_{k+1,q}(x,0)\right]$$
$$\times\mathfrak{B}_{n-k,q}(0,my) \tag{2.7}$$

holds true between the q-Bernoulli polynomials and q-Euler polynomials.

Corollary 2.4. *For $n\in\mathbb{N}_0$, $m\in\mathbb{N}$ the following relationship holds true:*

$$G_n(x+y)=\sum_{k=0}^{n}\binom{n}{k}\frac{2}{k+1}\Big((k+1)y^k-G_{k+1,q}(y)\Big)B_{n-k}(x), \tag{2.8}$$

$$G_n(x+y)=\sum_{k=0}^{n}\binom{n}{k}\frac{1}{m^{n-k-1}(k+1)}\left[2(k+1)G_k\left(y+\frac{1}{m}-1\right)\right.$$
$$\left.-G_{k+1}\left(y+\frac{1}{m}-1\right)-G_{k+1}(y)\right]B_{n-k,q}(mx) \tag{2.9}$$

between the classical Genocchi polynomials and the classical Bernoulli polynomials.

Note that the formula (2.9) is new for the classical polynomials.

In terms of the q-Genocchi numbers $\mathfrak{G}_{k,q}^{(\alpha)}$, by setting $y=0$ in Theorem 2.1, we obtain the following explicit relationship between the q-Genocchi polynomials $\mathfrak{G}_{k,q}^{(\alpha)}$ of order α and the q-Bernoulli polynomials.

Corollary 2.5. *The following relationship holds true:*

$$\mathfrak{G}_{n,q}^{(\alpha)}(x,0)=\sum_{k=0}^{n}\begin{bmatrix}n\\k\end{bmatrix}_q\frac{1}{m^{n-k-1}[k+1]_q}\left[2[k+1]_q\sum_{j=0}^{k}\begin{bmatrix}k\\j\end{bmatrix}_q\left(\frac{1}{m}-1\right)_q^{k-j}\mathfrak{G}_{j,q}^{(\alpha-1)}\right.$$
$$\left.-\sum_{j=0}^{k+1}\begin{bmatrix}k+1\\j\end{bmatrix}_q\left(\frac{1}{m}-1\right)_q^{k+1-j}\mathfrak{G}_{j,q}^{(\alpha)}-\mathfrak{G}_{k+1,q}^{(\alpha)}\right]\mathfrak{B}_{n-k,q}(mx,0). \tag{2.10}$$

Corollary 2.6. *For $n\in\mathbb{N}_0$ the following relationship holds true:*

$$\mathfrak{G}_{n,q}(x,y)=\sum_{k=0}^{n}\begin{bmatrix}n\\k\end{bmatrix}_q\frac{2}{[k+1]_q}\left[[k+1]_q q^{(1/2)k(k-1)}y^k-\mathfrak{G}_{k+1,q}(0,y)\right]\mathfrak{B}_{n-k,q}(x,0). \tag{2.11}$$

Corollary 2.7. *For $n \in \mathbb{N}_0$ the following relationship holds true:*

$$\mathfrak{G}_{n,q}(x,0) = -\sum_{k=0}^{n} \begin{bmatrix} n \\ k \end{bmatrix}_q \frac{2}{[k+1]_q} \mathfrak{G}_{k+1,q} \mathfrak{B}_{n-k,q}(x,0),$$
$$\mathfrak{G}_{n,q} = -\sum_{k=0}^{n} \begin{bmatrix} n \\ k \end{bmatrix}_q \frac{2}{[k+1]_q} \mathfrak{G}_{k+1,q} \mathfrak{B}_{n-k,q}. \tag{2.12}$$

References

[1] G. E. Andrews, R. Askey, and R. Roy, *Special Functions*, vol. 71, Cambridge University Press, Cambridge, UK, 1999.
[2] L. Carlitz, "q-Bernoulli numbers and polynomials," *Duke Mathematical Journal*, vol. 15, pp. 987–1000, 1948.
[3] H. M. Srivastava and Á. Pintér, "Remarks on some relationships between the Bernoulli and Euler polynomials," *Applied Mathematics Letters*, vol. 17, no. 4, pp. 375–380, 2004.
[4] T. Kim, "On the q-extension of Euler and Genocchi numbers," *Journal of Mathematical Analysis and Applications*, vol. 326, no. 2, pp. 1458–1465, 2007.
[5] T. Kim, "A note on the q-Genocchi numbers and polynomials," *Journal of Inequalities and Applications*, vol. 2007, Article ID 71452, 8 pages, 2007.
[6] T. Kim, "Note on q-Genocchi numbers and polynomials," *Advanced Studies in Contemporary Mathematics*, vol. 17, no. 1, pp. 9–15, 2008.
[7] M. Cenkci, M. Can, and V. Kurt, "q-extensions of Genocchi numbers," *Journal of the Korean Mathematical Society*, vol. 43, no. 1, pp. 183–198, 2006.
[8] Y. Simsek, I. N. Cangul, V. Kurt, and D. Kim, "q-Genocchi numbers and polynomials associated with q-Genocchi-type l-functions," *Advances in Difference Equations*, vol. 2008, Article ID 815750, 12 pages, 2008.
[9] L. Carlitz, "q-Bernoulli and Eulerian numbers," *Transactions of the American Mathematical Society*, vol. 76, pp. 332–350, 1954.
[10] J. Choi, P. J. Anderson, and H. M. Srivastava, "Some q-extensions of the Apostol-Bernoulli and the Apostol-Euler polynomials of order n, and the multiple Hurwitz zeta function," *Applied Mathematics and Computation*, vol. 199, no. 2, pp. 723–737, 2008.
[11] J. Choi, P. J. Anderson, and H. M. Srivastava, "Carlitz's q-Bernoulli and q-Euler numbers and polynomials and a class of generalized q-Hurwitz zeta functions," *Applied Mathematics and Computation*, vol. 215, no. 3, pp. 1185–1208, 2009.
[12] Q.-M. Luo and H. M. Srivastava, "q-extensions of some relationships between the Bernoulli and Euler polynomials," *Taiwanese Journal of Mathematics*, vol. 15, no. 1, pp. 241–257, 2011.
[13] N. I. Mahmudov, *A New Class of Generalized Bernoulli Polynomials and Euler Polynomials*, 2012.
[14] H. M. Srivastava and C. Vignat, "Probabilistic proofs of some relationships between the Bernoulli and Euler polynomials," *European Journal of Pure and Applied Mathematics*, vol. 5, no. 2, pp. 97–107, 2012.