LETTERS TO THE EDITOR

A NOTE ON THE GENERATING FUNCTION OF LAGUERRE POLYNOMIALS

In teaching quantum mechanics, we sometimes need the generating functions of classical orthogonal polynomials, namely, the polynomials of Jacobi, Laguerre and Hermite. At the same time it may be instructive for students to become familiar in the course of evaluating generating function, with the operator calculus which is an indispensable ingredient of quantum mechanics.

Fujiwara and Inowe[1] used operator calculus and Fourier transform to obtain the usual generating function for the Laguerre polynomials. The purpose of this note is to give a different but direct method of obtaining the generating function for the Laguerre polynomials. Our method is simple and straightforward.

In the course of our discussion we need the following well-known rules. In what follows D_x denotes $\partial/\partial x$, δ denotes $(x\,\partial/\partial x)$, and so on.

$$F(\delta)\,[x^a\,f(x)] = x^a\,F(\delta + a),\ F_x \qquad (1)$$

$$x^{na}\,F(\delta)\,F(\delta + a)\ \dots\ F(\delta + (n-1)a)$$
$$= [x^a\,F(\delta)]^n, \qquad (2)$$

$$a^\delta\,f(x) = f(ax). \qquad (3)$$

Note the following interesting special case of (2). If $a = -1$, and $F(\delta) = \delta$, then we have

$$[D^x]^n = x^{-n}\,\delta\,(\delta - 1)\ \dots\ (\delta - n + 1). \qquad (4)$$

It is remarked that the treatment that follows is formal.

The Laguerre polynomials $L_n{}^a(x)$ are defined by the following Rodrigue's formula :

$$n!\,L_n{}^a(x) = e^x\,x^{-a}\,(D_x)^n\,[e^{-x}\,x^{n+a}].$$

This can be written as

$$n!\,L_n{}^a(x)$$
$$= e^x\,\lim_{y \to 1}\left(-\frac{1}{x}D_y\right)^a D_x{}^n\,[x^n\,e^{-yx}]$$
$$\hspace{6cm}[\text{use}\,(4)]$$
$$= e^x\,\lim_{y \to 1}\left(-\frac{1}{x}D_y\right)^a x^{-n}\,\delta\,(\delta - 1)\ ..$$
$$\times\,(\delta - n + 1)\,[x^n\,e^{-xy}] \qquad [\text{use}\,(1)]$$
$$= e^x\,\lim_{y \to 1}\left(-\frac{1}{x}D_y\right)^a (\delta + 1)_n\,e^{-yx}.$$

Accordingly, if $|t| < 1$ we have the following

$$\sum_{n=0}^{\infty}\,L_n{}^a(x)\,t^n$$
$$= e^x\,\lim_{y \to 1}\left(-\frac{1}{x}D_y\right)^a (1 - t)^{-\delta - 1}\,e^{-yx}$$

$$= e^x\,\lim_{y \to 1}\left(-\frac{1}{x}D_y\right)^a (1 - t)^{-1}$$
$$\times\left(\frac{1}{1 - t}\right)^\delta\,e^{-yx}.$$

Using result (3) we now get,

$$\sum_{n=0}^{\infty}\,L_n{}^a(x)\,t^n$$
$$= e^x\,\lim_{y \to 1}\left(-\frac{1}{x}D_y\right)^a (1 - t)^{-1}\,e^{-y(x/(1-t))}.$$

Thus we get the required generating function for the Laguerre polynomials in the form

$$\sum_{n=0}^{\infty}\,L_n{}^a(x)\,t^n$$
$$= e^x\,(1 - t)^{(-1-a)}\,.\,e^{-x\,\frac{1}{1-t}}$$
$$= (1 - t)^{-a-1}\,e^{-xt/1-t}.$$

It is remarked that by using operator calculus Thakare[2] has given the generating function for the classical orthogonal polynomials and Bessel polynomials in the unified form.

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1. Fujiwara, I. and Inowe, Z., "Operator calculus for evaluating generating functions," *Physica Norvegica*, 1971, 5, 199.

2. Thakare, N. K., "Generating function in the unified form for the classical orthogonal polynomials by using operator calculus" (to appear).

DETERMINATION OF HEAVY ATOM POSITIONS IN PROTEIN CRYSTALLOGRAPHY AN IMPROVED DIFFERENCE FOURIER SYNTHESIS INCORPORATING ANOMALOUS SCATTERING DATA

THE first step in the X-ray analysis of protein crystals using isomorphous replacement method consists in the determination of heavy atom positions in protein heavy atom derivatives. In the initial stages, Patterson techniques of one type or another are used for this purpose. However, once a set of phase angles for the structure factors from the native protein crystals is derived from one or more heavy atom derivatives, the heavy atom positions in yet another derivative can be determined