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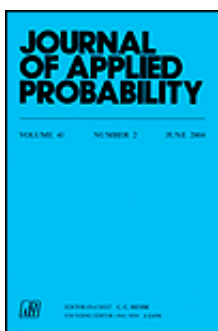


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## On a Probabilistic Analogue of the Fibonacci Sequence

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### ON A PROBABILISTIC ANALOGUE OF THE FIBONACCI SEQUENCE

C. C. HEYDE,\* *CSIRO Division of Mathematics and Statistics, Canberra*

#### Abstract

One of the earliest population models to be studied gives rise to the Fibonacci

sequence and has a history dating back more than 750 years. A stochastic version of the model is discussed in this paper, its basic defining property being  $E(X_n | X_{n-1}, \dots, X_0) = X_{n-1} + X_{n-2}$  a.s. The process  $\{X_n\}$  mimics many of the standard properties of the Fibonacci sequence. In particular, under mild additional conditions,  $X_{n+1}X_n^{-1} \rightarrow \alpha$  a.s. as  $n \rightarrow \infty$  where  $\alpha$  is the 'golden ratio'  $\frac{1}{2}(1 + \sqrt{5})$ .

POPULATION PROCESSES; GROWTH RATES; FIBONACCI NUMBERS; GOLDEN RATIO; GENERALIZED MARTINGALES

One of the oldest population growth models to be studied gives rise to the Fibonacci sequence. This is the following rabbit-breeding problem proposed by Leonardo of Pisa in his book *Liber Abacci* which was written in 1202 and has survived in its second edition of 1228. A closed population begins with one newborn pair of rabbits. Every month a pair of rabbits produces another pair and the rabbits begin to bear young 2 months after their birth. It is easily seen that the number of pairs  $u_n$  at time  $n$  is given by the recurrence relation

$$u_n = u_{n-1} + u_{n-2}$$

and the initial conditions  $u_1 = u_2 = 1$ . The sequence  $\{u_n\}$  is known as the Fibonacci sequence (and the  $u_n$  as the Fibonacci numbers). It has been subjected to intensive scrutiny, partly through the Fibonacci Association (founded by mathematicians interested in the sequence) and in its journal, the *Fibonacci Quarterly*. The literature associated with the sequence is enormous.

Amongst the many interesting properties of the Fibonacci sequence is the result that

$$\lim_{n \rightarrow \infty} u_{n+1}u_n^{-1} = \alpha = \frac{1}{2}(1 + \sqrt{5}).$$

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## Abstract

One of the earliest population models to be studied gives rise to the Fibonacci sequence and has a history dating back more than 750 years. A stochastic version of the model is discussed in this paper, its basic defining property being  $E(X_n | X_{n-1}, \dots, X_0) = X_{n-1} + X_{n-2}$  a.s. The process  $\{X_n\}$  mimics many of the standard properties of the Fibonacci sequence. In particular, under mild additional conditions,  $X_{n+1}X_n^{-1} \rightarrow \alpha$  a.s. as  $n \rightarrow \infty$  where  $\alpha$  is the 'golden ratio'  $\frac{1}{2}(1 + \sqrt{5})$ .

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