

## Chapter

*On the Higher-Order Sheffer Orthogonal Polynomial Sequences*

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# On the Sheffer A-Type 0 Orthogonal Polynomial Sequences and Related Results

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## Abstract

In this chapter, we present a rigorous development of I. M. Sheffer's characterization of the *A-Type 0* orthogonal polynomial sequences. We first develop the results that led to the main theorem that characterizes the general *A-Type 0* polynomial sequences via a linear generating function. From there, we develop the additional theory that Sheffer utilized in order to determine which *A-Type 0* polynomial sequences are also orthogonal. We then address Sheffer's additional characterizations of *B-Type* and *C-Type*, as well as E.D. Rainville's  $\sigma$ -*Type* classification. Lastly, we cover J. Meixner's approach to the same characterization problem studied by Sheffer and then discuss an extension of Meixner's analysis by W.A. Al-Salam. Portions of the analysis addressed throughout this chapter are supplemented with informative concrete examples.

## References

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