## DETERMINANT COMPUTATIONS FOR SOME CLASSES OF TOEPLITZ-HANKEL MATRICES

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*Abstract.* The purpose of this paper is to compute the asymptotics of determinants of finite sections of operators that are trace class perturbations of Toeplitz operators. For example, we consider the asymptotics in the case where the matrices are of the form  $(a_{i-j} \pm a_{i+j+1-k})_{i,j=0...N-1}$  with *k* fixed. We will show that this example as well as some general classes of operators have expansions that are similar to those that appear in the Strong Szegö Limit Theorem. We also obtain exact identities for some of the determinants that are analogous to the one derived independently by Geronimo and Case and by Borodin and Okounkov for finite Toeplitz matrices. These problems were motivated by certain statistical quantities that appear in random matrix theory.

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