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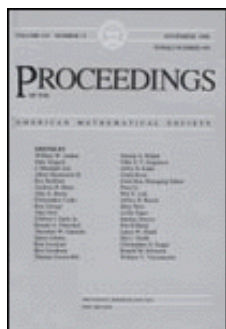
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Jacobi's Generating Function for Jacobi Polynomials

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PROCEEDINGS OF THE
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JACOBI'S GENERATING FUNCTION FOR JACOBI POLYNOMIALS

RICHARD ASKEY¹

ABSTRACT. An idea of Hermite is used to give a simple proof of Jacobi's
generating function for Jacobi polynomials.

One of the standard ways to prove the orthogonality of Legendre
polynomials is to take their generating function

$$(1 - 2xr + r^2)^{-1/2} = \sum_{n=0}^{\infty} P_n(x)r^n \quad (1)$$

and show that the integral

$$\int_{-1}^1 (1 - 2xr + r^2)^{-1/2} (1 - 2xs + s^2)^{-1/2} dx$$

is a function of the variable rs . See, for example, Courant-Hilbert [1, pp. 85–86]. This proof was given by Legendre [6, p. 250].

Jacobi gave a generating function for a more general set of orthogonal polynomials $P_n^{(\alpha, \beta)}(x)$. These polynomials can be defined by

$$(1-x)^\alpha (1+x)^\beta P_n^{(\alpha, \beta)}(x) = \frac{(-1)^n}{2^n n!} \frac{d^n}{dx^n} [(1-x)^{n+\alpha} (1+x)^{n+\beta}]. \quad (2)$$

It is easy to use (2) and integration by parts to prove

$$\int_{-1}^1 P_n^{(\alpha, \beta)}(x) P_m^{(\alpha, \beta)}(x) (1-x)^\alpha (1+x)^\beta dx = 0, \quad m \neq n, \alpha, \beta > -1. \quad (3)$$

Jacobi's generating function is

$$\sum_{n=0}^{\infty} P_n^{(\alpha, \beta)}(x) r^n = 2^{\alpha+\beta} R^{-1} (1-r+R)^{-\alpha} (1+r+R)^{-\beta} \quad (4)$$

where $R = (1 - 2xr + r^2)^{1/2}$. His original proof used Lagrange's extension of Taylor's theorem. A second proof of this generating function was given a few years later by Tchebychef [9]. His proof was modeled on Legendre's proof mentioned above. He seems to have found this generating function independently from Jacobi, since he does not mention Jacobi's paper. Tchebychef's proof is a very complicated one which involves a number of changes of variables to reduce the integral

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Abstract

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