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Research Article

Some Properties of a Sequence Similar to Generalized Euler Numbers

Haiqing Wang and Guodong Liu

Department of Mathematics, Huizhou University, Huizhou, Guangdong 516007, China

Correspondence should be addressed to Guodong Liu; gdliu@pub.huizhou.gd.cn

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We introduce the sequence $\{U_n^{(x)}\}$ given by generating function $(1/(e^t+e^{-t}-1))^x=\sum_{n=0}^{\infty}U_n^{(x)}(t^n/n!)$ $(|t|<(1/3)\pi, 1^x:=1)$ and establish some explicit formulas for the sequence $\{U_n^{(x)}\}$. Several identities involving the sequence $\{U_n^{(x)}\}$, Stirling numbers, Euler polynomials, and the central factorial numbers are also presented.

1. Introduction and Definitions

For a real or complex parameter α, the generalized Euler polynomials $E_n^{(\alpha)}(x)$ are defined by the following generating function (see [1–4])

$$\left(\frac{2}{e^t+1}\right)^{\alpha}e^{xt}=\sum_{n=0}^{\infty}E_n^{(\alpha)}(x)\frac{t^n}{n!}\quad\left(|t|<\pi, 1^{\alpha}:=1\right). \tag{1}$$

Obviously, we have

$$E_n^{(1)}(x)=E_n(x)\quad\left(n\in\mathbb{N}_0:=\mathbb{N}\cup\{0\}\right), \tag{2}$$

in terms of the classical Euler polynomials $E_n(x)$, $\mathbb{N}$ being the set of positive integers. The classical Euler numbers E_n are given by the following:

$$E_n=2^nE_n\left(\frac{1}{2}\right)\quad\left(n\in\mathbb{N}_0\right). \tag{3}$$

The so-called the generalized Euler numbers $E_{2n}^{(x)}$ are defined by (see [3, 5])

$$\left(\frac{2}{e^t+e^{-t}}\right)^{x}=\sum_{n=0}^{\infty}E_{2n}^{(x)}\frac{t^{2n}}{(2n)!}\quad\left(|t|<\frac{\pi}{2}, 1^{x}:=1\right). \tag{4}$$

In fact, $E_{2n}^{(k)}$ $(k\in\mathbb{Z})$ are the Euler numbers of order k, $\mathbb{Z}$ being the set of integers. The numbers $E_{2n}^{(1)}=E_{2n}$ are the ordinary Euler numbers.

Zhi-Hong Sun introduces the sequence $\{U_n\}$ similar to Euler numbers as follows (see [6, 7]):

$$U_0=1,\qquad U_n=-2\sum_{k=1}^{[n/2]}\binom{n}{2k}U_{n-2k},\quad(n\geq 1), \tag{5}$$

where (and in what follows) $[x]$ is the greatest integer not exceeding x.

Clearly, $U_{2n-1}=0$ for $n\geq 1$. The first few values of U_{2n} are shown below

$$U_2=-2,\quad U_4=22,\quad U_6=-602,\quad U_8=30742,$$
$$U_{10}=-2523002,\quad U_{12}=303692662. \tag{6}$$

The sequence $\{U_n\}$ is related to the classical Bernoulli polynomials $B_n(x)$ (see [8–11]) and the classical Euler polynomials $E_n(x)$. Zhi-Hong Sun gets the generating function of

$\{U_n\}$ and deduces many identities involving $\{U_n\}$. As example, (see [6]),

$$\frac{1}{e^t+e^{-t}-1}=\sum_{n=0}^{\infty}U_n\frac{t^n}{n!}=\sum_{n=0}^{\infty}U_{2n}\frac{t^{2n}}{(2n)!}\quad\left(|t|<\frac{1}{3}\pi\right),\tag{7}$$

$$\frac{1}{2\cos t-1}=\sum_{n=0}^{\infty}(-1)^nU_{2n}\frac{t^{2n}}{(2n)!}\quad\left(|t|<\frac{1}{3}\pi\right),\tag{8}$$

$$U_{2n}=3^{2n}E_{2n}\left(\frac{1}{3}\right).\tag{9}$$

Similarly, we can define the generalized sequence $\{U_n^{(x)}\}$. For a real or complex parameter x, the generalized sequence $\{U_n^{(x)}\}$ is defined by the following generating function:

$$\left(\frac{1}{e^t+e^{-t}-1}\right)^x=\sum_{n=0}^{\infty}U_n^{(x)}\frac{t^n}{n!}\quad\left(|t|<\frac{1}{3}\pi,1^x:=1\right).\tag{10}$$

Obviously,

$$U_0^{(x)}=1,\qquad U_n^{(1)}=U_n\quad(n\in\mathbb{N}).\tag{11}$$

By using (10), we can obtain

$$U_n^{(k)}=n!\sum_{\substack{v_1,\ldots,v_k\in\mathbb{N}_0}}^{(v_1+\cdots+v_k=n)}\frac{U_{v_1}\cdots U_{v_k}}{v_1!\cdots v_k!}\quad(k\in\mathbb{N}).\tag{12}$$

We now return to the Stirling numbers $s(n,k)$ of the first kind, which are usually defined by (see [2, 5, 8, 11, 12])

$$x(x-1)(x-2)\cdots(x-n+1)=\sum_{k=0}^{n}s(n,k)x^k\tag{13}$$

or by the following generating function:

$$(\log(1+x))^k=k!\sum_{n=k}^{\infty}s(n,k)\frac{x^n}{n!}.\tag{14}$$

It follows from (13) or (14) that

$$s(n,k)=s(n-1,k-1)-(n-1)s(n-1,k)\tag{15}$$

and that

$$\begin{gathered}s(n,0)=0\quad(n\in\mathbb{N}),\qquad s(n,n)=1\quad(n\in\mathbb{N}_0),\\ s(n,1)=(-1)^{n-1}(n-1)!\quad(n\in\mathbb{N}),\\ s(n,k)=0\quad(k>n\text{ or }k<0).\end{gathered}\tag{16}$$

The central factorial numbers $T(n,k)$ are given by the following expansion formula (see [3, 5, 13]):

$$x^n=\sum_{k=0}^{n}T(n,k)x\left(x-1^2\right)\times\left(x-2^2\right)\cdots\left(x-(k-1)^2\right)\tag{17}$$

or by means of the generating function

$$\left(e^x+e^{-x}-2\right)^k=(2k)!\sum_{n=k}^{\infty}T(n,k)\frac{x^{2n}}{(2n)!}.\tag{18}$$

It follows from (17) or (18) that

$$T(n,k)=T(n-1,k-1)+k^2T(n-1,k),\tag{19}$$

with

$$\begin{gathered}T(0,0)=1,\qquad T(n,0)=0\quad(n\in\mathbb{N}),\\ T(n,1)=1\quad(n\in\mathbb{N}).\end{gathered}\tag{20}$$

We also find from (18) that

$$\begin{gathered}T(n,2)=\frac{1}{4}\left(4^{n-1}-1\right),\\ T(n,3)=\frac{9^n}{360}-\frac{4^n}{60}+\frac{1}{24}\quad(n\in\mathbb{N}).\end{gathered}\tag{21}$$

The main purpose of this paper is to prove some formulas for the generalized sequence $\{U_n^{(x)}\}$ and $E_n(x)$. Some identities involving the sequence $\{U_n^{(x)}\}$, Stirling numbers $s(n,k)$, and the central factorial numbers $T(n,k)$ are deduced.

2. Main Results

Theorem 1. *Let $n\ge k$ $(n,k\in\mathbb{N})$ and*

$$q(n,k)=(-1)^k\sum_{j=k}^{n}\frac{(2j)!}{j!}T(n,j)s(j,k).\tag{22}$$

Then,

$$U_{2n}^{(x)}=\sum_{k=1}^{n}q(n,k)x^k.\tag{23}$$

Remark 2. By (15), (19), (20), and Theorem 1, we know that $U_{2n}^{(x)}$ is a polynomial of x with integral coefficients. For example, by setting $n=1,2,3,4$ in Theorem 1, we get

$$\begin{gathered}U_2^{(x)}=-2x,\qquad U_4^{(x)}=10x+12x^2,\\ U_6^{(x)}=-182x-300x^2-120x^3,\\ U_8^{(x)}=6970x+13692x^2+8400x^3+1680x^4.\end{gathered}\tag{24}$$

Taking $x=1$ in Theorem 1, we can obtain the following.

Corollary 3. *Let $n\in\mathbb{N}$. Then,*

$$U_{2n}=\sum_{j=0}^{n}(-1)^j(2j)!T(n,j).\tag{25}$$

From Corollary 3, we may immediately deduce the following results.

Corollary 4. *Let $n \in \mathbb{N}$. Then,*

$$\begin{aligned} U_{2n} &\equiv -2 \pmod{24}, \\ U_{2n} &\equiv -2 + 24T(n,2) \pmod{720}, \\ U_{2n} &\equiv -2 + 24T(n,2) - 720T(n,3) \pmod{40320}. \end{aligned} \tag{26}$$

Theorem 5. *Let $n \geq k$ $(n, k \in \mathbb{N})$. Then,*

$$\begin{aligned} U_{2n} &= \sum_{k=1}^{n} q(n,k), \\ U_{2n} &= 2\sum_{k=1}^{[n/2]} q(n,2k) - 2 \\ &= 2\sum_{k=1}^{[(n-1)/2]} q(n,2k+1) + 2. \end{aligned} \tag{27}$$

Theorem 6. *Let $n \geq k$ $(n, k \in \mathbb{N})$. Suppose also that $q(n,k)$ is defined by* (22). *Then,*

$$\begin{aligned} k!q(n,k) = {} & (2n)!3^{2n-k} \\ & \times \sum_{\nu_1,\dots,\nu_k\in\mathbb{N}}^{(\nu_1+\cdots+\nu_k=n)} \left(E_{2\nu_1-1}(0) - E_{2\nu_1-1}\left(\frac{2}{3}\right)\right) \\ & \cdots \left(E_{2\nu_k-1}(0) - E_{2\nu_k-1}\left(\frac{2}{3}\right)\right) \\ & \times \left((2\nu_1)!\cdots(2\nu_k)!\right)^{-1}. \end{aligned} \tag{28}$$

Theorem 7. *Let $n \in \mathbb{N}$. Then,*

$$-2\sum_{k=0}^{n-1}\binom{2n-1}{2k}U_{2k} = 3^{2n-1}\left(E_{2n-1}(0) - E_{2n-1}\left(\frac{2}{3}\right)\right). \tag{29}$$

Theorem 8. *Let $n \in \mathbb{N}$. Then,*

$$U_{n+1} = \sum_{k=0}^{n-1}\binom{n}{k}\left(\left(1-2^{n-k}\right)U_{k+1} - 2^{n-k}U_k\right). \tag{30}$$

Theorem 9. *Let $n \in \mathbb{N}_0$. Then,*

$$\sum_{n=0}^{\infty}\frac{1}{(n+1)!}U_n = \frac{1}{\sqrt{3}}\log\frac{2e-1-\sqrt{3}}{2\left(2-\sqrt{3}\right)e-5+3\sqrt{3}}. \tag{31}$$

3. Proofs of Theorems

Proof of Theorem 1. By (10), (13), and (18), we have

$$\begin{aligned} \sum_{n=0}^{\infty}U_{2n}^{(x)}\frac{t^{2n}}{(2n)!} &= \left(\frac{1}{e^t+e^{-t}-1}\right)^x \\ &= \left(\frac{1}{1+(e^t+e^{-t}-2)}\right)^x \\ &= \sum_{j=0}^{\infty}(-1)^j\binom{x+j-1}{j}\left(e^t+e^{-t}-2\right)^j \\ &= \sum_{j=0}^{\infty}(-1)^j\binom{x+j-1}{j}(2j)!\sum_{n=j}^{\infty}T(n,j)\frac{t^{2n}}{(2n)!} \\ &= \sum_{n=0}^{\infty}\frac{t^{2n}}{(2n)!}\sum_{j=0}^{n}(-1)^j(2j)!\binom{x+j-1}{j}T(n,j), \end{aligned} \tag{32}$$

which readily yields

$$\begin{aligned} U_{2n}^{(x)} &= \sum_{j=0}^{n}(-1)^j(2j)!\binom{x+j-1}{j}T(n,j) \\ &= \sum_{j=0}^{n}(-1)^j(2j)!T(n,j)\frac{1}{j!}x(x+1)\cdots(x+j-1) \\ &= \sum_{j=0}^{n}\frac{(2j)!}{j!}T(n,j)\sum_{k=1}^{j}(-1)^k s(j,k)x^k \\ &= \sum_{k=1}^{n}(-1)^k\sum_{j=k}^{n}\frac{(2j)!}{j!}T(n,j)s(j,k)x^k \\ &= \sum_{k=1}^{n}q(n,k)x^k. \end{aligned} \tag{33}$$

This completes the proof of Theorem 1. □

Proof of Theorem 5. By (10), we have

$$\sum_{n=0}^{\infty}U_{2n}^{(-1)}\frac{t^{2n}}{(2n)!} = e^t+e^{-t}-1 = 2\sum_{n=0}^{\infty}\frac{t^{2n}}{(2n)!}-1, \tag{34}$$

and $U_0^{(x)} = 1$, thus

$$\sum_{n=1}^{\infty}U_{2n}^{(-1)}\frac{t^{2n}}{(2n)!} = e^t+e^{-t}-1 = 2\sum_{n=1}^{\infty}\frac{t^{2n}}{(2n)!}. \tag{35}$$

By Theorem 1 and comparing the coefficient of $t^{2n}/(2n)!$ on both sides of (35), we get

$$\sum_{k=1}^{n}q(n,k)(-1)^k = U_{2n}^{(-1)} = 2. \tag{36}$$

Again, by taking $x = 1$ in Theorem 1, we have

$$\sum_{k=1}^{n} q(n,k) = U_{2n}. \tag{37}$$

By (36) and (37), we immediately obtain (27). This completes the proof of Theorem 5. □

Proof of Theorem 6. By applying Theorem 1, we have

$$k! q(n,k) = \frac{d^k}{dx^k}\left\{U_n^{(x)}\right\}\bigg|_{x=0}. \tag{38}$$

On the other hand, it follows from (10) that

$$\sum_{n=k}^{\infty} \frac{d^k}{dx^k}\left\{U_n^{(x)}\right\}\bigg|_{x=0}\frac{t^{2n}}{(2n)!} = \left(\log\left(\frac{1}{e^t + e^{-t} - 1}\right)\right)^k. \tag{39}$$

By using (38) and (39), we find that

$$k!\sum_{n=k}^{\infty} q(n,k)\frac{t^{2n}}{(2n)!} = \left(\log\left(\frac{1}{e^t + e^{-t} - 1}\right)\right)^k. \tag{40}$$

We now note that

$$\begin{aligned}
&\frac{d}{dt}\left\{\log\left(\frac{1}{e^t + e^{-t} - 1}\right)\right\} \\
&\quad = \frac{e^{-t} - e^t}{e^t + e^{-t} - 1} \\
&\quad = \frac{e^{-t} - e^t}{2}\left(\frac{2e^t}{e^{3t}+1} + \frac{2e^{-t}}{e^{-3t}+1}\right) \\
&\quad = \frac{1}{2}\left(\left(\frac{2}{e^{3t}+1} - \frac{2}{e^{-3t}+1}\right) - \left(\frac{2e^{2t}}{e^{3t}+1} - \frac{2e^{-2t}}{e^{-3t}+1}\right)\right) \\
&\quad = \frac{1}{2}\left(\sum_{n=0}^{\infty} E_n(0)\frac{(3t)^n}{n!} - \sum_{n=0}^{\infty} E_n(0)\frac{(-3t)^n}{n!}\right) \\
&\qquad - \frac{1}{2}\left(\sum_{n=0}^{\infty} E_n\left(\frac{2}{3}\right)\frac{(3t)^n}{n!} - \sum_{n=0}^{\infty} E_n\left(\frac{2}{3}\right)\frac{(-3t)^n}{n!}\right) \\
&\quad = \sum_{n=0}^{\infty} 3^{2n+1}\left(E_{2n+1}(0) - E_{2n+1}\left(\frac{2}{3}\right)\right)\frac{t^{2n+1}}{(2n+1)!}.
\end{aligned} \tag{41}$$

Hence,

$$\begin{aligned}
\log\frac{1}{e^t + e^{-t} - 1} &= \sum_{n=0}^{\infty} 3^{2n+1}\left(E_{2n+1}(0) - E_{2n+1}\left(\frac{2}{3}\right)\right)\frac{t^{2n+2}}{(2n+2)!} \\
&= \sum_{n=1}^{\infty} 3^{2n-1}\left(E_{2n-1}(0) - E_{2n-1}\left(\frac{2}{3}\right)\right)\frac{t^{2n}}{(2n)!}
\end{aligned} \tag{42}$$

yields

$$\begin{aligned}
&k!\sum_{n=k}^{\infty} q(n,k)\frac{t^{2n}}{(2n)!} \\
&\quad = \left(\sum_{n=1}^{\infty} 3^{2n-1}\left(E_{2n-1}(0) - E_{2n-1}\left(\frac{2}{3}\right)\right)\frac{t^{2n}}{(2n)!}\right)^k \\
&\quad = \sum_{n=k}^{\infty}\frac{t^{2n}}{(2n)!}(2n)!3^{2n-k} \\
&\qquad \times \sum_{\substack{(v_1+\cdots+v_k=n)\\ v_1,\ldots,v_k\in\mathbb{N}}}\left(E_{2v_1-1}(0) - E_{2v_1-1}\left(\frac{2}{3}\right)\right) \\
&\qquad\qquad \cdots\left(E_{2v_k-1}(0) - E_{2v_k-1}\left(\frac{2}{3}\right)\right) \\
&\qquad\qquad \times\left((2v_1)!\cdots(2v_k)!\right)^{-1}.
\end{aligned} \tag{43}$$

Comparing the coefficient of $t^{2n}/(2n)!$ on both sides of (43), we immediately get (28). This completes the proof of Theorem 6. □

Proof of Theorem 7. Consider

$$\begin{aligned}
\frac{d}{dt}\left\{\log\left(\frac{1}{e^t + e^{-t} - 1}\right)\right\} &= \frac{e^{-t} - e^t}{e^t + e^{-t} - 1} \\
&= \sum_{n=0}^{\infty} U_{2n}\frac{t^{2n}}{(2n)!}\left(-2\sum_{n=0}^{\infty}\frac{t^{2n+1}}{(2n+1)!}\right) \\
&= -2\sum_{n=0}^{\infty}\sum_{k=0}^{n}\binom{2n+1}{2k}U_{2k}\frac{t^{2n+1}}{(2n+1)!}.
\end{aligned} \tag{44}$$

Thus,

$$\log\frac{1}{e^t + e^{-t} - 1} = -2\sum_{n=1}^{\infty}\sum_{k=0}^{n-1}\binom{2n-1}{2k}U_{2k}\frac{t^{2n}}{(2n)!}. \tag{45}$$

By (42) and (45) we obtain (29). This completes the proof of Theorem 7. □

Proof of Theorem 8. By using (7), we have

$$\sum_{n=1}^{\infty} U_n\frac{t^{n-1}}{(n-1)!} = \frac{e^{-t} - e^t}{(e^t + e^{-t} - 1)^2}. \tag{46}$$

Thus

$$\begin{aligned}
\left(e^{2t} - e^t + 1\right)\sum_{n=1}^{\infty} U_n\frac{t^{n-1}}{(n-1)!} &= \left(1 - e^{2t}\right)\sum_{n=0}^{\infty} U_n\frac{t^n}{n!}, \\
\sum_{n=0}^{\infty}\left(2^n - 1\right)\frac{t^n}{n!}\sum_{n=0}^{\infty} U_{n+1}\frac{t^n}{n!} + \sum_{n=0}^{\infty} U_{n+1}\frac{t^n}{n!} & \\
= \sum_{n=0}^{\infty} U_n\frac{t^n}{n!} - \sum_{n=0}^{\infty} 2^n\frac{t^n}{n!}\sum_{n=0}^{\infty} U_n\frac{t^n}{n!}. &
\end{aligned} \tag{47}$$

That is,

$$\sum_{n=0}^{\infty}\sum_{k=0}^{n}\binom{n}{k}\left(2^{n-k}-1\right)U_{k+1}\frac{t^n}{n!}+\sum_{n=0}^{\infty}U_{n+1}\frac{t^n}{n!}$$
$$=\sum_{n=0}^{\infty}U_n\frac{t^n}{n!}-\sum_{n=0}^{\infty}\sum_{k=0}^{n}\binom{n}{k}2^{n-k}U_k\frac{t^n}{n!}. \tag{48}$$

Comparing the coefficient of $t^n/n!$ on both sides of (48), we get the following:

$$U_{n+1}-U_n=\sum_{k=0}^{n}\binom{n}{k}\left(\left(1-2^{n-k}\right)U_{k+1}-2^{n-k}U_k\right). \tag{49}$$

By (49) we immediately obtain (30). This completes the proof of Theorem 8. □

Proof of Theorem 9. By integrating (7) with respect to t from 0 to 1, we have

$$\sum_{n=0}^{\infty}\frac{1}{(n+1)!}U_n=\int_0^1\frac{1}{e^t+e^{-t}-1}dt$$
$$=\int_0^1\frac{1}{e^{2t}-e^t+1}de^t=\int_1^e\frac{1}{x^2-x+1}dx. \tag{50}$$

By (50) and $\int(1/(ax^2+bx+c))dx=(1/\sqrt{b^2-4ac})\log|(2ax+b-\sqrt{b^2-4ac})/(2ax+b+\sqrt{b^2-4ac})|+c$ (c is constant), we have (31). This completes the proof of Theorem 9. □

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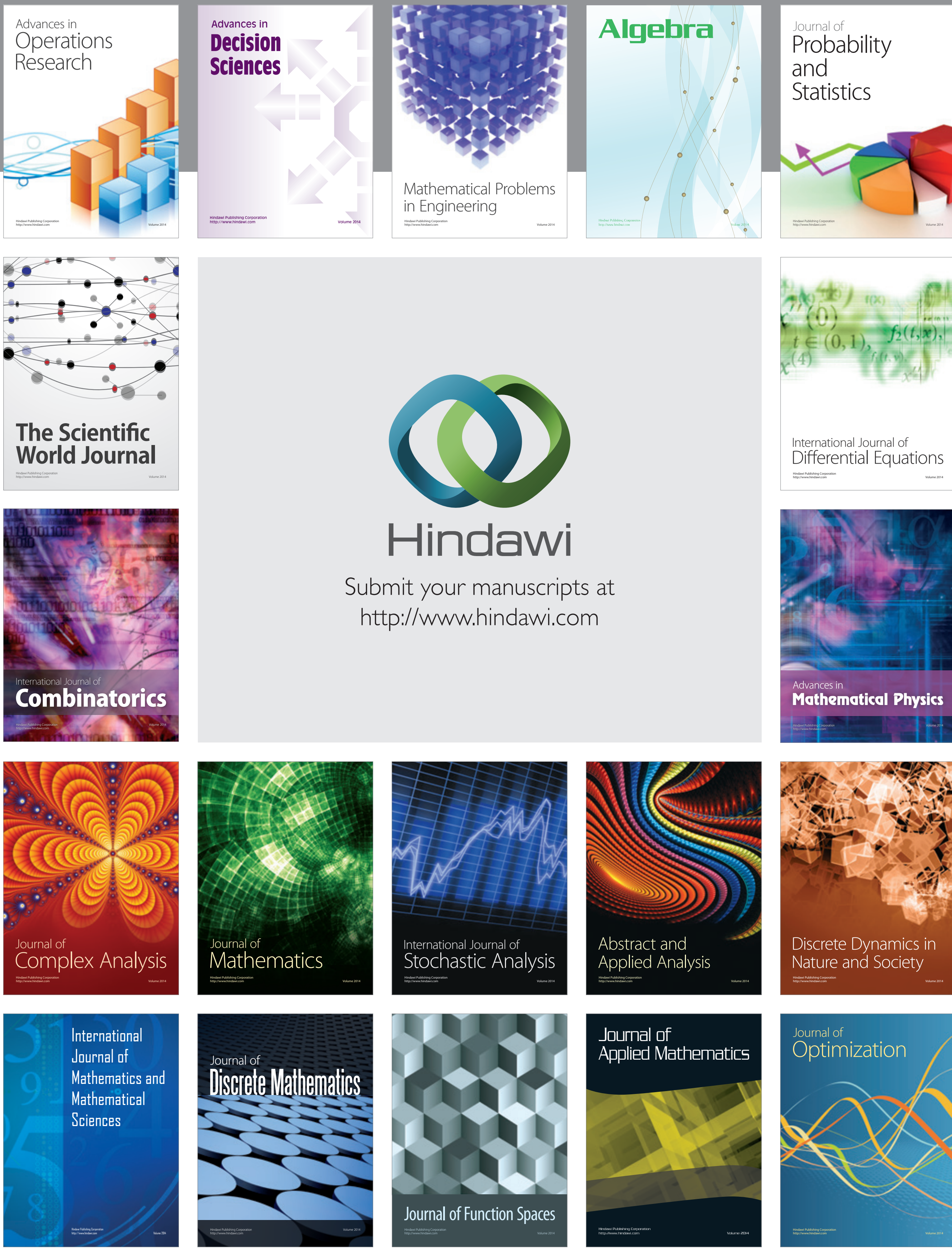
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